Department of Electrical and Information Technology
Lund University

Digital Communications – advanced course (ETT055)

Laboratory manual

”Multi-path propagation, signal space, eye-diagram and the Viterbi algorithm”

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INTRODUCTION

In this laboratory session we study multi-path propagation, the receiver's signal space using the so-called eye-diagram, and the Viterbi algorithm.

The purpose of this laboratory session is to give an increased understanding of what the receiver should do to be able to optimally detect signals when the communication link is a multi-path channel.

HOME PROBLEMS

To be solved at home **before** this laboratory session.

Home Problem 1:

Answer the question on page 2.

Home Problem 2:

a) Consider Laboratory Exercise A and assume that $\alpha = 0.6$ och $\alpha_1 = -0.4$. Determine the optimal values, denoted c^* and c_1^*, that the ML-receiver uses.

Answer: $c^* =$ . . . ., $c_1^* =$ . . . .

b) Is it true that the ML-receiver in a) can use the values $c^* = 1$ and $c_1^* = \alpha/\alpha_1$?

Answer: . . . .

c) Determine c_1^* and c_2^* in Laboratory Exercise Part B1 on page 8 (compare with Figure 4.18 in the compendium).

Answer: $c_1^* =$ . . . ., $c_2^* =$ . . . .

Home Problem 3:

The decision variable $\xi[i]$ in Laboratory Exercise A contains a noise component. The variance of this noise component equals σ^2.

a) Determine σ^2. Answer: $\sigma^2 =$ . . . .

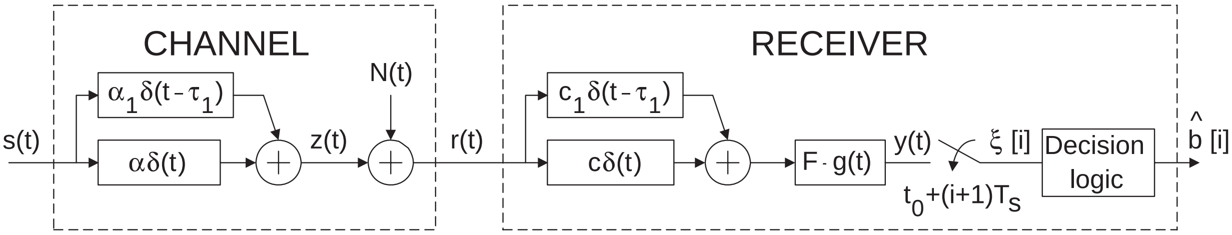
b) How much is σ^2 changed if c_1 is doubled? Answer: . . . .

Home Problem 4:

Solve Problem 8.31 in the compendium (2-ray multi-path, ISI and the VA).

LABORATORY EXERCISE A

Here we investigate the communication system below.



$s(t)$ is here a sequence of (coded, or uncoded) 4-ary PAM signal alternatives $\{s_j(t)\}_{j=0}^{3}$. The symbol rate is denoted R_s, the pulse shape is $g(t)$, and $N(t)$ is AWGN with power spectral density $R_N(f) = N_0/2$.

We assume that the symbol time T_s satisfies the relation $T_s \geq T + \tau_1$, where T is the duration of $g(t)$. This significantly simplifies for the receiver to make correct decisions.

WHY? . . . .

The communication link consists of a direct signal path, and one delayed (τ_1) signal path. The impulse response $v(t)$ of the receiver filter can be identified by finding the relationship between the input signal $r(t)$ and the output signal $y(t)$. It is seen that

$$v(t) = F \cdot (c\delta(t) + c_1\delta(t-\tau_1)) * g(t) = F \cdot (cg(t) + c_1 g(t-\tau_1)) \tag{1}$$

The parameter F is an arbitrary gain factor ($F \neq 0$). The choice of F does not change the error probability since the signal-to-noise ratio does not depend on F. However, the decision logic sometimes needs to adapt to the value of F.

In this laboratory session the value of F is chosen such that the energy E_v in the receiver filter $v(t)$ always is $E_v = 1$. This is obtained by chosing F as,

$$F = \left(\int_0^{\mathcal{T}+\tau_1} (cg(t) + c_1 g(t-\tau_1))^2 dt \right)^{-1/2} \tag{2}$$

The advantage with this choice of F is that the noise component in the decision variable $\xi[i]$ always has the variance $\sigma^2 = N_0/2$.

If the parameters c, c_1 och t_0 are chosen correct then the receiver above is identical with the ML-receiver (compare also with equation (4.146) and Figure 4.18 in the compendium).

The eye diagram

The so called eye diagram is obtained by plotting all possible noise-free signals $y(t)$ in the same diagram. In practice an eye diagram is obtained by connecting an oscilloscope with memory to $y(t)$. After a while, assuming a noise-free situation, the eye diagram (i.e. all possible output signals $y(t)$) is shown on the screen of the oscilloscope. The eye diagram gives us important information regarding the sensitivity to noise, jitter in the sampling time, and ISI. Furthermore, at the sampling time, there is a close connection between the eye diagram and the signal space!
All possible noise-free signals $y(t)$ can be expressed as,

$$y(t) = \sum_{i=-\infty}^{\infty} A_i x(t - iT_s) \tag{3}$$

where A_i denotes the sent amplitude, and where the over-all pulse shape $x(t)$ is,

$$x(t) = g(t) * h(t) * v(t) \tag{4}$$

For a well designed M-ary PAM communication system, the eye diagram will have M clear and easily found amplitude levels around the sampling times!

Laboratory Exercise Part A1: Random α and α_1

In this exercise the duration T of the pulse $g(t)$ is $T = T_s/2$. Furthermore, the delay τ_1 in the reflected signal path is $\tau_1 = T_s/4$, and both α and α_1 lie within the interval -1 to +1.

Observe! From a practical point of view, the value of the parameter c in the receiver is always equal to $c = 1$.

Execute the file uppga1!

Figure 1 on the screen shows the pulse shape $g(t)$.
Start with the noise-free situation by giving "n" as input to the program when it asks about "Brus".
Start with a guess of c_1 (e.g. $c_1 = -2$).

Figure 2 on the screen will then show a realization of the received noise-free signal $r(t) = z(t)$.
From this plot we can not determine the sent amplitude ($\pm A, \pm 3A$) since we do not know the actual values of α and α_1. The values of α and α_1 are chosen randomly when the program is started.
However, from the plot of $z(t)$ we can for most cases estimate the ratio α/α_1.

1. From the plot of $z(t)$ in Figure 2, estimate α/α_1.

   Answer: $\alpha/\alpha_1 =$ ....

   Figure 2 on the screen also shows the eye diagram that is obtained with your guess of c_1.

   A correct eye diagram in the receiver will have M clear and easily found amplitude levels around the sampling time "3"! Furthermore, the eye diagram should be symmetric around the sampling time "3"!

2. Did your guess of c_1 result in an ML-receiver?

   Answer: ....

   Figure 1 also shows the impulse response $v(t)$ for the receiver filter for your guess of c_1.

   By **not** ("n" when the program asks about "Avsluta") ending the program you can try other values of c_1 in the receiver and study the new eye diagrams (c_1 should be within the interval from -10 to +10).

3. Test two other values of c_1 and study the resulting eye diagrams. Are there minor or significant changes in the eye diagrams compared with your initial guess of c_1?

   Answer: ....

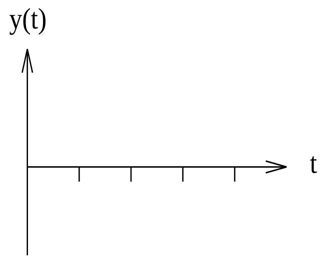
4. What is the relationship between the parameters c_1, α and α_1 for the ML-receiver if it is required that $c = 1$?

   Answer: $c_1^* =$ ....

5. Use Figure 1 and Figure 2, especially $z(t)$, to estimate the optimal choice of c_1, here denoted c_1^*, if a requirement is that $c = 1$.

   Answer: $c_1^* =$ ....

6. Sketch the eye diagram for the ML-receiver in 5.



7. There is a very close connection between the eye diagram in your sketch above and the signal space. Explain this connection!

   Answer: ....

8. Determine $D^2_{\min}$ for the ML-receiver in 5.

   Answer: $D^2_{\min} =$ ....

9. Assume that the noise component in $y(t)$ has the variance $\sigma^2 = 0.05$. Determine the exact value of the symbol error probability P_s for the ML-receiver in 5.

   Answer: $P_s =$ ....

10. Determine from the eye diagram if the receiver in 5 is robust against jitter in the sampling time.

    Answer: ....

11. Add noise $N(t)$ to the signal $z(t)$ and compare the two signals $r(t)$ and $y(t)$. Which of the signals $r(t)$ and $y(t)$ is less noisy, and why?

    Answer: ....

12. Is the noise margin in the eye diagram large enough?

    Answer: ....

13. Suggest a method how the receiver can learn the channel parameters α, α_1 and τ_1.

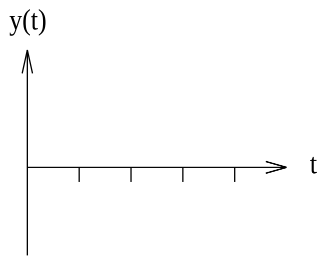
    Answer: ....

CONTACT THE LABORATORY ASSISTANT!

Laboratory Exercise Part A2: Fixed α and α_1

In this exercise $\alpha = 0.6$, $\alpha_1 = -0.4$ and $\tau_1 = T_s/4$. Furthermore, also here we assume that $c = 1$.

1. Execute the file **uppga2** and sketch the eye diagram for the ML-receiver.



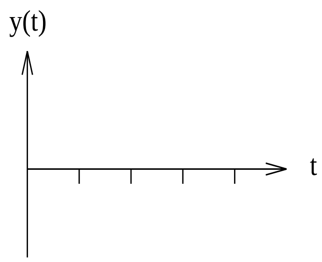
2. Assume now that the duration T of the pulse is reduced such that $T = T_s/1000$. Determine the signal alternatives $\{z_j(t)\}$ in this case, and suggest at least one advantage with this method.

   Answer: ....

Laboratory Exercise Part A3: Fixed α and α_1, sub-optimal receiver

In this exercise $\alpha = 0.6$, $\alpha_1 = -0.4$ and $\tau_1 = T_s/4$. Furthermore, here we assume that $c = 1$ and $c_1 = 0$.

1. Execute the file **uppga2** and sketch the eye diagram for this receiver.



2. What are the differences between this eye diagram and the eye diagram in Part A2, and which is best?

   Answer: ....

3. Are there any advantages by using $c_1 = 0$.

   Answer: ....

4. For which α and α_1 is this receiver optimal?

   Answer: ....

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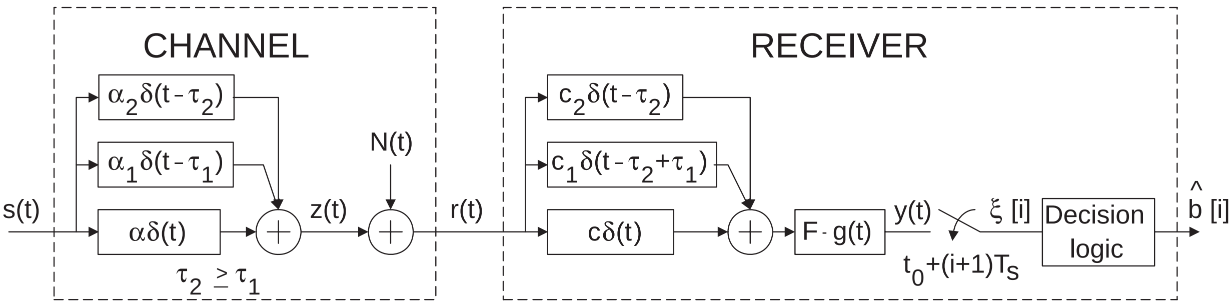
LABORATORY EXERCISE B: 3-ray channel

The communication system below will here be investigated (compare with Figure 4.18 in the compendium).



The signal $s(t)$ is also here 4-ary PAM, and $N(t)$ is AWGN. Here the symbol time T_s is chosen such that $T_s > T + \tau_2$ where T denotes the duration of the transmitted pulse $g(t)$. The parameter F is also here chosen such that the energy in the receiver filter is $E_v = 1$,

$$F = \left(\int_0^{\tau+\tau_2} (cg(t) + c_1 g(t - \tau_2 + \tau_1) + c_2 g(t - \tau_2))^2 dt \right)^{-1/2}$$

Laboratory Exercise Part B1: Random α, α_1 and α_2

In this exercise the duration T of the pulse $g(t)$ is $T = T_s/2$. Furthermore, $\tau_1 = T_s/4$ and $\tau_2 = 3T_s/8$. The parameters α, α_1 and α_2 all lie within the interval -1 to +1.

Observe! From a practical point of view, the value of the parameter c in the receiver is always equal to $c = 1$! c_1 and c_2 lie in the interval -10 to +10.

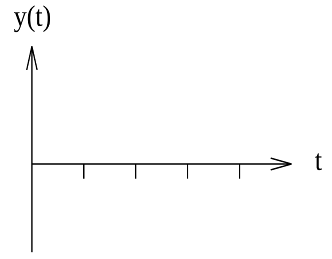
1. What is the relationship between the parameters c_1, c_2, α, α_1 and α_2 for the ML-receiver if it is required that $c = 1$?

   Answer: $c_1^* =$ . . . . $c_2^* =$ . . . .

2. Execute the file **uppgb1**. Use Figure 1 and Figure 2, especially $z(t)$, to estimate the optimal choice of c_1 and c_2 if a requirement is that $c = 1$.

   Answer: $c_1^* =$ . . . . $c_2^* =$ . . . .

3. Sketch the eye diagram for the ML-receiver in 2:



4. Assume that E_b/N_0 equals 9.6 dB. Determine the exact symbol error probability for the ML-receiver.

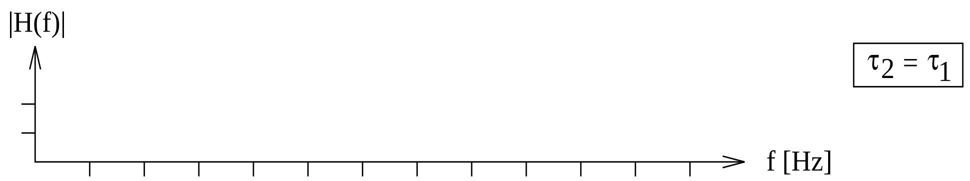
   Answer: ....

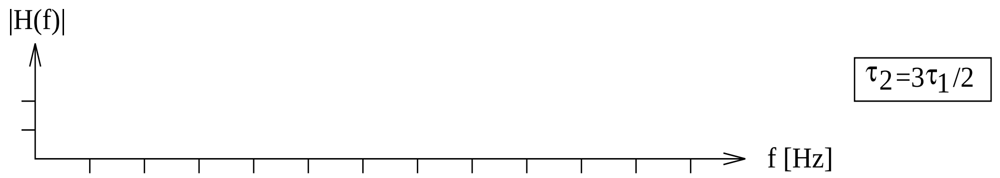
Laboratory Exercise Part B2: The transfer function $H(f)$

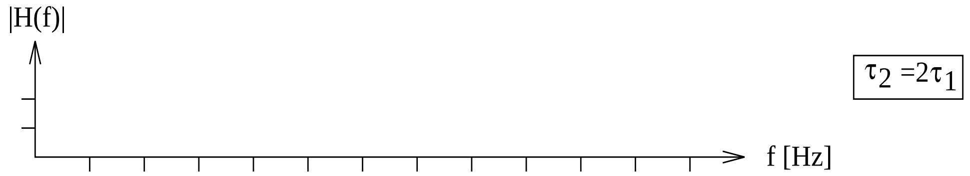
In this exercise $\alpha = 0.8$, $\alpha_1 = -0.4$, $\alpha_2 = -0.2$ and $\tau_1 = T_s/4$. The frequency function $H(f)$ for this 3-ray channel is

$$H(f) = \mathcal{F}\{h(t)\} = \alpha + \alpha_1 e^{-j2\pi f \tau_1} + \alpha_2 e^{-j2\pi f \tau_2}$$

5. Execute the file **uppgb2** and sketch $|H(f)|$ for the three cases below.







6. Does the transfer function $|H(f)|$ change if τ_2 is changed?

   Answer: ....

7. Suggest a suitable communication method in 5, and communication bandwidth, if the transmitter knows the transfer function $H(f)$.

   Answer: ....

8. Suggest a suitable communication method in 5, and communication bandwidth, if the transmitter does not know the transfer function $H(f)$.

   Answer: ....

9. Describe the output signal alternatives from the 3-ray multi-path channel if the input signal alternatives are "slow" QAM signal alternatives,

   Answer: ....

CONTACT THE LABORATORY ASSISTANT!

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