Final exam in

Digital Communications, Advanced Course

on March 7, 2008, 8–13.

Department of Electrical and Information Technology
Lund University

- During this final exam, You are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.

  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.

  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.
- If You want or if You do not want Your result to appear on the department's web site, please write so on the cover page of the exam.

Good Luck!

Problem 1: Assume 4 equally likely uncoded signal alternatives, AWGN channel with $R_N(f) = N_0/2$, and ML receiver. The signal space representation is:

$$
\begin{aligned}
z_{0,1} &= -2a, & z_{0,2} &= 0 \\
z_{1,1} &= 0, & z_{1,2} &= 0 \\
z_{2,1} &= -2a, & z_{2,2} &= 2a \\
z_{3,1} &= 0, & z_{3,2} &= 2a
\end{aligned}
$$

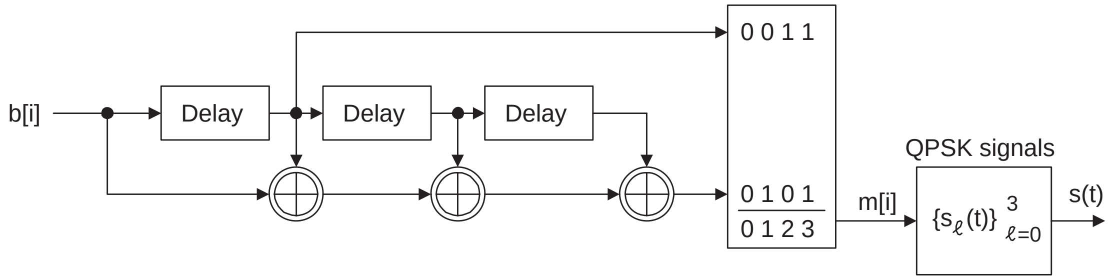
a) If the received noisy signal point $\boldsymbol{r}$ has $r_1 = -3a/2$ and $r_2 = a/2$ what is then the decision of the receiver?

b) Calculate the exact value of the symbol error probability if $\mathcal{E}_b/N_0$ is 9.6 dB.

c) What is meant by "rotation and attenuation" of QAM signals, and what are the consequences of this in the receiver?

(10 points)

Problem 2:



a) Construct the table that gives $\sigma[i+1]/m[i]$ for all inputs $b[i]$ and all states $\sigma[i]$.
Note: The notation and definitions used in the compendium must be used!

b) The communication bandwidth is W Hz. Suppose that the encoder above is replaced with a rate 7/8 encoder, and also that 256-ary QAM is used instead of QPSK.
How much is then the bit rate changed if the same communication bandwidth W is required?

c) What are the advantages and disadvantages with the given system compared with uncoded BPSK?

(10 points)

Problem 3: Determine for each of the five statements below if it is true or false.
Observe! As usual motivations to your answers must be given!

1. "The basic Shannon capacity formula, applied at large SNR, implies that the maximum bandwidth efficiency is increased by 1 bps/Hz if SNR is doubled."
2. "If we have a slow, frequency non-selective, Rayleigh fading channel then the concept of "attenuation and rotation" is not relevant."
3. "Frequency non-selective fading is better than frequency-selective fading."
4. "The bit rate used determines if a channel is slowly fading, or not."
5. "If 250 kbps is to be communicated within the bandwidth 25 kHz, and if $\mathcal{E}_b/N_0$ is 8 dB, then a rate 10/11 encoder in combination with 2048 signal alternatives is a suitable choice."

(10 points)

Problem 4: Here we investigate the consequences of a sudden mal-functioning (i.e. error) in the transmitter. Consider the TCM system in Figure 8.6a) in the compendium (rate 2/3 encoder and 8-ary PSK).

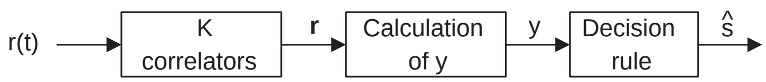
Assume now that sudddenly during communication the modulo two addition unit that adds $b_2[i-1]$ and $b_1[i]$ stops working, such that this unit starts to always produce the value "0" as future output values!
This will significantly change the performance of the communication system.

a) Analyse the situation and determine the consequences of this error at the transmitter side. Focus on the transmitted signal alternatives and the signal sequences.

b) Analyse the situation and determine the consequences of this error at the receiver side. Focus on consequences related to the Viterbi algorithm (there will be problems even if N_0 is very small!).
Note that we can not assume that the receiver knows anything about the error in the transmitter.
With correct operation at the transmitter side the VA will never produce any bit errors in the noise-less case (i.e. if no AWGN exists).
Will the VA produce bit errors in the noise-less case for the considered error in the transmitter?

(10 points)

Problem 5: Here we consider communication of M-ary PAM signals. The receiver has access to K distorted versions of the sent M-ary PAM signal, see the ML receiver below.



where $\boldsymbol{r}$ denotes the received noisy signal point,

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_K \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_K \end{pmatrix} s + \begin{pmatrix} w_1 \\ \vdots \\ w_K \end{pmatrix} = \boldsymbol{\alpha} s + \boldsymbol{w} = \boldsymbol{z} + w$$

where s is the sent M-ary PAM amplitude, and $s \in \{\pm a, \pm 3a, \ldots, \pm(M-1)a\}$.
$\{\alpha_i\}_{i=1}^{K}$ are deterministic channel parameters, and $\{w_i\}_{i=1}^{K}$ are independent zero-mean Gaussian random variables each with variance $N_0/2$. The decision rule in the ML receiver above is: "Choose as the decision the value of s that minimizes the value $(-2ys + s^2 \sum_{i=1}^{K} \alpha_i^2)$".

Since the receiver above is ML it is equivalent with the minimum Euclidean distance receiver which minimizes $D^2_{\boldsymbol{r},\boldsymbol{z}} = (\boldsymbol{r}^{tr} - \boldsymbol{\alpha}^{tr} s)(\boldsymbol{r} - \boldsymbol{\alpha} s)$, where $\boldsymbol{r}^{tr} = (r_1 \ldots r_K)$.

a) Explain in detail two techniques that provide the receiver with K distorted versions of the sent signal as above.

Will the bandwidth efficiency for the two techniques depend on K?

b) Determine the value y if:
i) $K = 1$ ii) $K = 6$

c) i) Calculate the minimum squared Euclidean distance, $D^2_{\min}$, in the receiver's signal space, and also explain how we in $D^2_{\min}$ can see the effect of diversity.

ii) Determine the symbol error probability.

(10 points)