Final exam in Digital Communications, Advanced Course on March 6, 2007, 8–13.

Department of Information Technology
Lund University

- During this final exam, You are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.
  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.
  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.
- If You want or if You do not want Your result to appear on the department's web site, please write so on the cover page of the exam.

Good Luck!

Problem 1: Consider the TCM scheme in Figure 8.6a in the compendium. This scheme consists of a rate 2/3 convolutional encoder in combination with 8-ary PSK.

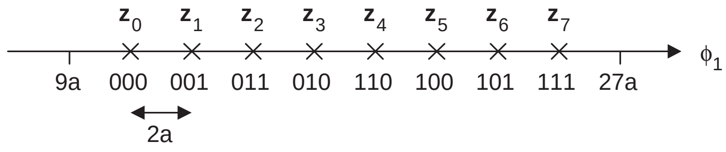
Determine for each of the five statements below if it is true or false.

Observe! As usual motivations to your answers should be given.

1. "The sequence s_2, s_5, s_0, s_1 is a possible sequence of sent signal alternatives."
2. "Within the time interval $8T_s \leq t \leq 12T_s$ there are 8^4 different possible output sequences."
3. "If it is known that the sequence $s_7, s_4, s_2, s_3, s_1, s_5, s_0$ is sent, then 17 information bits are known, of which 13 equals "1"."
4. "If the scheme is replaced with uncoded QPSK using the same bit rate, then the communication bandwidth will be the same."
5. Assume that the bit rate used in Figure 8.6a is R_b = 1Mbps. "If this scheme now is replaced with a rate 1/4 convolutional encoder in combination with BPSK, then the bit rate must be reduced to 250 kbps to maintain the same communication bandwith."

(10 points)

Problem 2: Assume 8 equally likely signal alternatives, AWGN channel with $R_N(f) = N_0/2$, and ML symbol receiver. The signal alternatives are shown in signal space below together with the 3-tuple of information bits that is mapped to each signal.



a) Assume that the three bits 110 are sent and that the noise component w_1 in the correlator output r_1 equals $w_1 = -3.25a$. Will the bits be erroneously decoded?

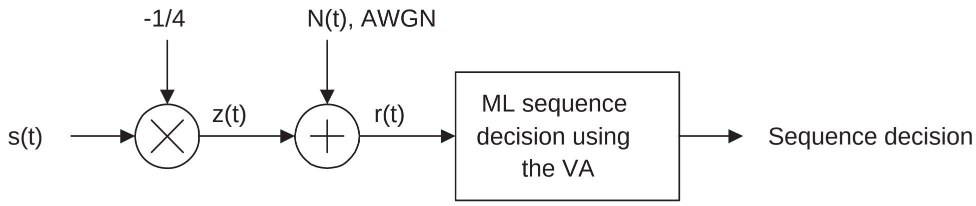
b) Calculate the symbol error probability exactly if it is known that E_2/N_0 is 34.83 dB (E_2 is the signal energy in the signal alternative $z_2(t)$).

c) Calculate $\mathcal{E}_b/N_0$ for the case considered in b). Conclusions?

(10 points)

Problem 3:

a) Consider the communication system below,



The signal $s(t)$ is the output signal shown in Figure 8.6 in the compendium. It is known that after processing the received noisy signal point $(r_1[i], r_2[i])$ the Viterbi algorithm has saved the following accumulated squared Euclidean distances:

State	Saved value
0	$28.4a^2$
1	$27.9a^2$
2	$28.8a^2$
3	$28.0a^2$
4	$28.3a^2$
5	$27.8a^2$
6	$28.5a^2$
7	$27.7a^2$

It is known that the next received noisy signal point is $(r_1[i+1] = 0, r_2[i+1] = a/8)$. Calculate the value of the accumulated squared Euclidean distance that will be stored by the Viterbi algorithm in state number 2 after processing $(r_1[i+1], r_2[i+1])$.

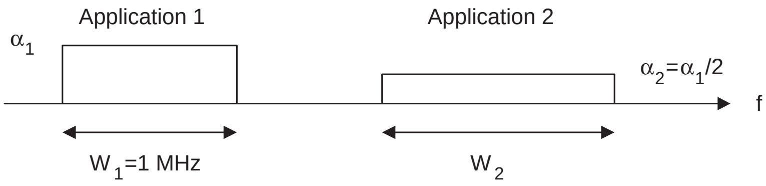
b) Assume that QAM signal alternatives are sent from a transmitter.

Give examples of situations when the received signal alternative can be considered to be an attenuated and rotated version of the sent QAM signal alternative.

How does attenuation and rotation affect the receiver design, and the symbol error probability?

(10 points)

Problem 4: Assume two communication applications located in separate frequency bands, see below.



The transmitted signal power for application i is denoted $P_{sent,i}$, $i = 1, 2$.

The transfer function $H(f)$ is such that $H(f) = \alpha_i$ within the frequency band W_i, $i = 1, 2$.

AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$ disturbs the received signal.

The capacity C_i, within the frequency band W_i, is

$$C_i = W_i \log_2 \left(1 + \frac{\alpha_i^2 P_{sent,i}}{N_0 W_i}\right), \quad i = 1, 2$$

a) Assume that $W_2 = 4W_1$. Determine the parameters C_{tot} and $\mathcal{E}_b/N_0$, defined as

- $C_{tot} = C_1 + C_2$
- $\frac{\mathcal{E}_b}{N_0} = \frac{\alpha_1^2 P_{sent,1} + \alpha_2^2 P_{sent,2}}{C_{tot} N_0}$

for the two cases below:

i) $P_{sent,1} = \frac{N_0 W_1}{\alpha_1^2} \cdot 3, \quad P_{sent,2} = 0$

ii) $P_{sent,1} = \frac{N_0 W_1}{\alpha_1^2} \cdot 4 = P_{sent,2}$

The parameters C_{tot} and $\mathcal{E}_b/N_0$ indicate the joint potential of the two applications.

What are your comments to your answers in i) and ii)?

When is it efficient to send all signal power in only one channel (as in i))?

b) From a standardization organization point of view a reasonable strategi, to decide how much bandwidth W_2 that should be allocated to application 2, is to require that both communication links should have the same potential in terms of capacity, i.e., $C_1 = C_2$, provided that $P_{sent,1} = P_{sent,2}$.

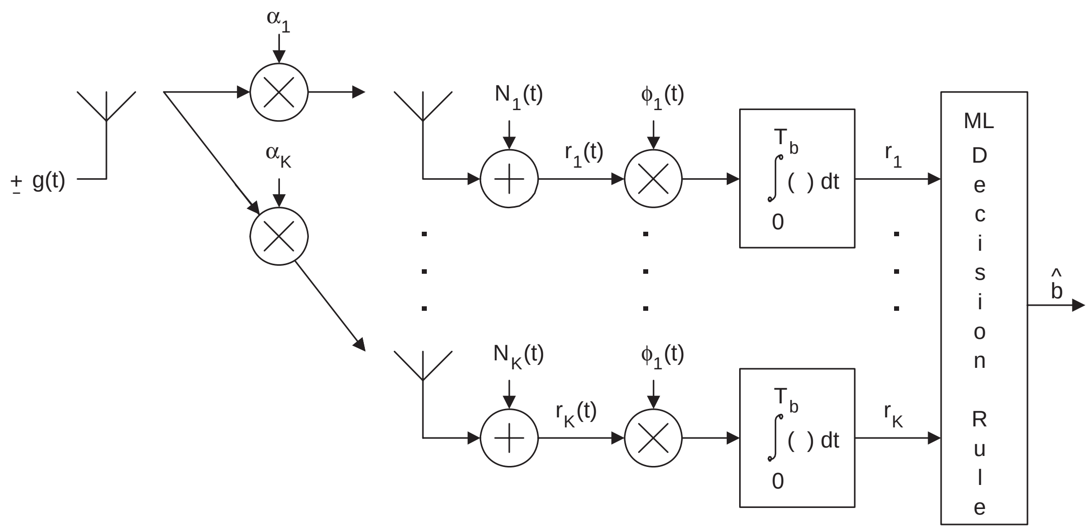
Determine the bandwidth W_2 that should be allocated to application 2, given that

- $P_{sent,1} = P_{sent,2}$
- $C_1 = C_2$
- $\frac{\mathcal{E}_{b,1}}{N_0} = \frac{\alpha_1^2 P_{sent,1}}{C_1 N_0} = 100$

The last condition implies that the quality of communication link 1 is assumed to be quite high.

(10 points)

Problem 5: Assume the communication link below where 1 transmitting antenna and K receiving antennas are used. The transmitter uses uncoded equally likely binary antipodal signals with $s_1(t) = g(t)$, and the energy in $g(t)$ is denoted E_g.



The noise processes $N_i(t)$ and $N_j(t)$ $(i \neq j)$ are independent AWGN processes, each with power spectral density $N_0/2$.

If $g(t)$ is sent then the received noisy vector $\boldsymbol{r}$ is,

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_K \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_K \end{pmatrix} \sqrt{E_g} + \begin{pmatrix} w_1 \\ \vdots \\ w_K \end{pmatrix} = \boldsymbol{z}_1 + \boldsymbol{w}$$

Hence, the received signal point $\boldsymbol{z}_1$ is K-dimensional.

a) Explain how the receiver above makes a decision. Determine the squared Euclidean distance between $\boldsymbol{z}_0$ and $\boldsymbol{z}_1$, and also the bit error probability if all α_i are deterministic (and known), and if $K = 8$.

b) Explain and motivate the use of K antennas as above if all α_i are independent zero-mean Gaussian random variables.

c) Generalize the communication link above by introducing one more transmitting antenna, that also sends uncoded binary antipodal signals. Let the corresponding additional channel coefficients be $\beta_1, \beta_2, \ldots, \beta_K$.
Explain how the receiver's signal points are located in signal space.

(10 points)