Final exam in

Digital Communications, Advanced Course

on March 7, 2006, 8–13.

Department of Information Technology
Lund University

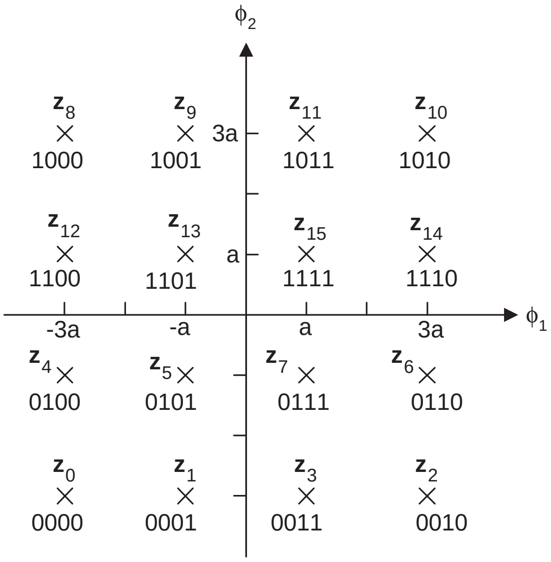
- During this final exam, You are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.

  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.

  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.
- If You want or if You do not want Your result to appear on the department's web site, please write so on the cover page of the exam.

Good Luck!

Problem 1: Assume 16 equally likely signal alternatives, AWGN channel with $R_N(f) = N_0/2$, and ML symbol receiver. The signal alternatives are shown in signal space below together with the 4-tuple of information bits that is mapped to each signal.



a) Assume that the four bits 1101 are sent, and that the correlator output $\boldsymbol{r}$ in the receiver is

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} 0.75a \\ -2.5a \end{pmatrix}$$

Determine the number of information bit errors then made by the ML receiver.

b) Calculate the symbol error probability exactly if it is known that E_6/N_0 is 20 dB (E_6 is the energy in the signal alternative $z_6(t)$).

c) How many information bit errors are made by the ML receiver if the noise components w_1 and w_2 (in each dimension, respectively) are $w_1 = -1.4a$ and $w_2 = 1.2a$?

(10 points)

Problem 2: Consider the system given in Figure 8.6a in the compendium. This system consists of a rate 2/3 convolutional encoder in combination with 8-ary PSK modulation. It is known that a part of the sequence of sent signal alternatives is:

$$s_3, s_4, s_5, s_a, s_b, s_0, s_5, s_3$$

Determine the signal alternatives s_a and s_b that are possible in the given sequence.

(10 points)

Problem 3: In a BPSK application over a so-called Ricean fading channel the received signal $r(t)$ is

$$r(t) = z_1(t) + N(t) \tag{1}$$

if $s_1(t) = -s_0(t)$ is sent. $N(t)$ is AWGN with $R_N(f) = N_0/2$, and $z_0(t) = -z_1(t)$. For this channel $z_1(t)$ consists of two signal parts: one deterministic (los-part, **l**ine-**o**f-**s**ight) part, and one random (Rayleigh distributed, n-los part, **n**on-**l**ine-**o**f-**s**ight) part;

$$z_1(t) = z_{1,los}(t) + z_{1,n-los}(t)$$

The average signal energy in $z_{1,los}(t)$ and in $z_{1,n-los}(t)$ is denoted E_{los} and E_{n-los}, respectively. Furthermore, the so-called Ricean K-factor is defined by

$$K = E_{los}/E_{n-los} \tag{2}$$

For the ML receiver, P_b can in this case be upper bounded by the expression below,

$$P_b < \frac{1}{2} \cdot \frac{e^{-\frac{K\mathcal{E}_b/N_0}{K+1+\mathcal{E}_b/N_0}}}{1 + \frac{\mathcal{E}_b/N_0}{K+1}} \tag{3}$$

i) Use (3) to estimate requirements on K if it is required that $P_b < 10^{-5}$ when $\mathcal{E}_b/N_0$ is 15 dB.

ii) What happens with P_b when K is "very small" and "very large", respectively? Conclusions?

iii) What happens with signals when they are communicated over so-called Rayleigh fading channels?

(10 points)

Problem 4: In a specific application adaptive coding and modulation schemes are used.

i) Assume that the same bandwidth efficiency as uncoded 16-ary QAM is desired. Suggest two coded schemes, with different code and modulation parameters, that give the desired bandwidth efficiency.

ii) Assume a scheme using adaptive coding, but BPSK as a fix modulation method. What is the main disadvantage with this scheme?

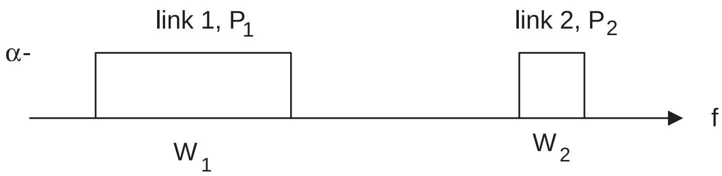
iii) Assume the system in Figure 8.6a in the compendium, and an AWGN channel. Explain in detail the calculations that are made by the Viterbi algorithm at state 3 if the received noisy signalpoint is

$$\boldsymbol{r} = \begin{pmatrix} -a/2 \\ 0 \end{pmatrix}$$

(10 points)

Problem 5: Two communication links are available (with AWGN), each with bandwidth W_1 and W_2, respectively.

$H(f)$:



The average received signal power used in link 1 is P_1 (and P_2 in link 2). Hence, the capacity of link 1 is $C_1 = W_1 \log_2 \left(1 + \frac{P_1}{N_0 W_1}\right)$ (and C_2 in link 2).

Assume that $P = P_1 + P_2$ and that the total received signal power P is distributed such that

$$P_1 = \frac{P}{1 + W_2/W_1} \tag{4}$$

$$P_2 = \frac{P}{1 + W_1/W_2} \tag{5}$$

Calculate the sum $C_1 + C_2$, and determine if it is true that this sum equals the total capacity for the two links when viewed as one channel.

Hence, is the distribution in (4)–(5) good with respect to total capacity?

Also explain the practical significance of the basic capacity curve.

(10 points)