Answers and non-complete solutions to:

Exam in Digital Communications, Advanced Course (ETT055), MARCH 7, 2008, 8-13.

1.

a) The decision is that message m_0 was sent since $\underline{r}$ is closest to $\underline{z}_0$.

b) It is found that $d^2_{min} = 1$.

$$P_s = 2\,Q\left(\sqrt{E_b/N_0}\right) - Q^2\left(\sqrt{E_b/N_0}\right) = 2Q(3.02) - Q^2(3.02) \approx 2.7\cdot 10^{-3}$$

c) See the compendium.

2.

a)

$b[i]$ \ $\sigma[i]$	0	1	2	3	4	5	6	7
0:	0/0	0/1	1/1	1/0	2/3	2/2	3/2	3/3
1:	4/1	4/0	5/0	5/1	6/2	6/3	7/3	7/2

$\sigma[i+1]/m[i]$

b)

$$R_b = r_c \log_2(M)\,\frac{W}{c} = \begin{cases} W/c, & \text{current system} \\ 7W/c, & \text{new system} \end{cases}$$

So, a factor 7 in increased bit rate.

c)

The same S, reduced P_b, increased receiver complexity (VA).

3.

1) TRUE: $\frac{C}{W} = \log_2(1 + 2\,SNR) \approx \log_2(2\,SNR) = 1 + \log_2(SNR)$.

2) FALSE: $z(t) = a\, e_s(t) \cos(\omega_c t + \theta_s(t) + \phi)$

!   !

3) FALSE, e.g., if frequency diversity is desired then frequency-selectivity is an advantage.

4) FALSE: $T_s \ll t_{coh}$

5) FALSE: $\rho = 10$ requires $(\mathcal{E}_b/N_0)_{min} = \frac{2^{10}-1}{10}$ (20.1 dB). Hence, the requirements are impossible to realize.

4.

a) Only the signals s_0, s_1, s_2, s_3 are sent, and this is a non-efficient signal constellation. New signal sequences are generated, e.g.,

$$\cdots -\boxed{0}\overset{s_0}{-}\boxed{0}\overset{s_0}{-}\boxed{0}\overset{s_2}{-}\boxed{4}\overset{s_2}{-}\boxed{4}\overset{s_2}{-}\boxed{4}\overset{s_2}{-}\cdots-\boxed{4}\overset{s_2}{-}\boxed{4}\overset{s_0}{-}\boxed{0}\overset{s_0}{-}\boxed{0}\overset{s_0}{-}\boxed{0}\overset{s_0}{-}\cdots$$

The "encoder" has only 4 effective states, and two effective outputs.

b) Assume that the sequence in a) is the true sent sequence. Also assume no AWGN ($N_0=0$). The VA will then assign $D^2_{inc} = 4a^2$ to the branch $\boxed{4}-\boxed{4}$ since the VA compares with s_6! This means that the VA will consider the sequence in a) to be highly unlikely to be sent even in the noisless case!

Furthermore, when s_2 is sent, the VA assigns $D^2_{inc} = 2a^2$ to the branch $\boxed{0}-\boxed{0}$. This means that the all-zero path in the trellis will (by the VA) be considered to be more likely than the true path (in the noise-less case)!

Hence, the error in the transmitter give very serious concequences in the receiver, since bit errors can be made in the noise-less case.

5.

a) Frequency-diversity: W = sub-channel bandwidth

$$\rho = \frac{R_b}{W_{tot}} = \frac{R_b}{KW} \quad \text{decreases with } K.$$

K receive antennas: ρ independent of K.

b)

$$ML: \min_{s}\left(D^2_{r,z}\right) = \min_{s}\left(\underline{r}^{tr}\underline{r} - 2\underline{r}^{tr}\underline{\alpha}\, s + \underline{\alpha}^{tr}\underline{\alpha}\, s^2\right) =$$

$$= \min_{s}\left(-2\underline{r}^{tr}\underline{\alpha}\, s + \underline{\alpha}^{tr}\underline{\alpha}\, s^2\right)$$

Identifying gives $y = \underline{r}^{tr}\underline{\alpha} = \sum_{i=1}^{K} r_i \alpha_i$

c)

i) $\underline{z}_i = \underline{\alpha} s_i$, $\underline{z}_j = \underline{\alpha} s_j \Rightarrow \underline{z}_i - \underline{z}_j = \underline{\alpha}(s_i - s_j)$

$$D^2_{z_i, z_j} = (s_i - s_j)^2 \sum_{i=1}^{K} \alpha_i^2 \quad (\text{diversity!!})$$

$$D^2_{min} = 4a^2 \sum_{i=1}^{K} \alpha_i^2$$

ii) $\underline{z}_j = \underline{\alpha} \cdot s_j$ (K-dimensional signal space)

$$P_s = \frac{2}{M}(M-1)\, Q\left(\sqrt{\frac{D^2_{min}}{2N_0}}\right)$$