

Answers and non-complete solutions to:

Exam in Digital Communications, Advanced course
(TT055), MARCH 7, 2008, 8-13.

1,

a) The decision is that bit 5 since w_6 was sent since r is closest to z_0 .

b) It is found that $d_{min} = 1$.

$$P_e = 2 Q(\sqrt{E_b/N_0}) - Q^2(\sqrt{E_b/N_0}) = 2Q(3.02) - Q^2(3.02) \approx 2.7 \cdot 10^{-3}$$

c) See the comp medium.

a),

a)

$d[2]$

0	1	2	3	4	5	6	7
---	---	---	---	---	---	---	---

0: 0/0 0/1 1/1 1/0 2/3 2/2 3/2 3/3

$d[2]$
1: 4/1 4/0 5/0 5/1 6/2 6/3 7/3 7/2

$d[2+1]/m[2]$

b) $R_0 = r \log_2(M) \frac{W}{c} = \begin{cases} W/c, & \text{current system} \\ 7W/c, & \text{new system} \end{cases}$

So, a factor 7 in increased bit rate.

c)

The same S , reduced R_b , increased receiver complexity (VH).

3.

1) TRUE: $C = \log_2(1 + 20 \text{SNR}) \approx \log_2(20 \text{SNR}) = 1 + \log_2(\text{SNR})$

2) FALSE: $Z(f) = a e^{j\omega t} \cos(\omega_c t + \theta_c(t) + \phi)$

3) FALSE, e.g., if frequency diversity is desired then frequency-selectivity is an advantage.

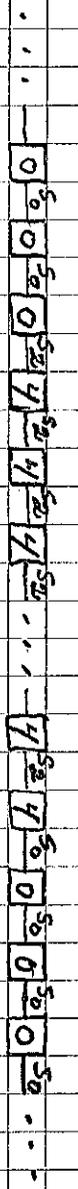
4) FALSE: $T_s \ll T_{\text{coh}}$

5) FALSE: $\rho = 10 \text{ requires } (E_b/N_0)_{\text{min}} = \frac{2^{\rho}-1}{10} = \frac{1023}{10}$
 Hence, the requirements are impossible to realize.

4.

a) Only the signals s_0, s_1, s_4, s_5 are sent, and this is a non-efficient signal constellation.

New signal sequences are generated, e.g.,



The "encoder" has only 4 effective states, and two effective outputs.

b) Assume that the sequence in a) is the true sent sequence. Also assume no AWGN ($N_0=0$).

The VR will then assign $D_{\text{me}} = |a|^2$ to the branch $[1|-1]$ since the VR compares with s_0 .

This means that the VR will consider the sequence in a) to be highly unlikely.

to be sent even in the noiseless case!

Furthermore, when s_0 is sent, the VQ assigns

$$D_{no} = 2\alpha^2 \text{ to the branch } [0, 0]. \text{ This means}$$

that the all-zero path in the trellis will (by the VQ) be considered to be more likely than the true path (in the noise-less case)!

Hence, the error in the transmitter give very serious consequences in the receiver, since bit errors can be made in the noise-less case.

5.

a) Frequency-diversity: $W =$ sub-channel bandwidth

$$g = \frac{R_b}{W_{tot}} = \frac{R_b}{KW} \text{ decreases with } K.$$

K receive antennas: g independent of K .

b) ML: $\min_S (D_{r,z}^2) = \min_S (r^T r - 2r^T \alpha s + K \alpha^2 s^2) =$

$$= \min_S (-2r^T \alpha s + K \alpha^2 s^2)$$

Identifying gives $y = r^T \alpha = \sum_{n=1}^K r_n \alpha_n$

c) $Z_n = \alpha s_n, Z_f = \alpha s_f \Rightarrow Z_0 - Z_f = \alpha (s_0 - s_f)$

$$D_{z_0, z_f}^2 = (s_0 - s_f)^2 \sum_{n=1}^K \alpha_n^2 \text{ (diversity } g: 1)$$

$$D_{min} = 4\alpha^2 \sum_{n=1}^K \alpha_n^2$$

any $Z_f = \alpha \cdot s_f$ (K -dimensional signal space)

$$P_s = \frac{2}{M} (M-1) \alpha \left(\frac{D_{min}^2}{2D_0} \right)$$