Answers/short solutions to Examination in Digital Communications – advanced course, March 6, 2007

Problem 1:

1. False
2. False (see 1)
3. True, since the state sequence is

   6 — 5 — 4 — 6 — 7 — 7 — 5 — 6

4. True since $R_s = R_b/2$ in both cases.
5. False since different R_s is obtained ($10^6 \neq 10^6/2$).

Problem 2:

a) The decision will be $\boldsymbol{z}_2$, i.e., (0,1,1). So the first and last bit are wrong.

b)

$$P_s = \frac{7}{4}\, Q\left(\sqrt{\frac{2a^2}{N_0}}\right)$$

$$\frac{E_2}{N_0} = \frac{225a^2}{N_0} = 3041.2, \quad \frac{2a^2}{N_0} = 5.1993^2$$

$$P_s = \frac{7}{4}\, Q(5.2) = 1.75 \cdot 10^{-7}$$

c)

$$\frac{\mathcal{E}_b}{N_0} = \frac{1}{3}\,\frac{\bar{E}_z}{N_0} = \frac{115a^2}{N_0} \approx 1554 \text{ (31.9 dB)}$$

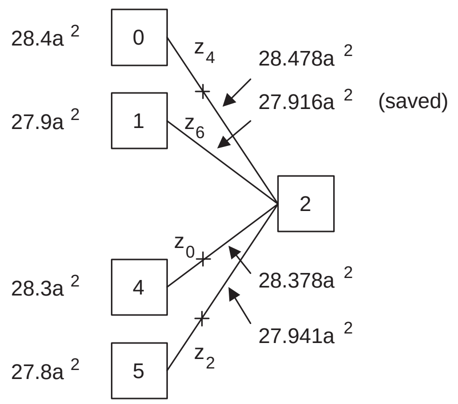
The energy efficiency is very bad.

Problem 3:

a)

$$D^2_{r,z_4} = D^2_{r,z_0} = \frac{a^2}{64} + \frac{a^2}{16} = \frac{5a^2}{64}$$

$$D^2_{r,z_6} = a^2/64, \quad D^2_{r,z_2} = 9a^2/64$$



b) See the compendium.

Problem 4:

a) i)

$$C_1 = W_1 \log_2\left(1 + \frac{\alpha_1^2}{N_0 W_1} \cdot \frac{N_0 W_1}{\alpha_1^2}\, 3\right) = 2W_1 = C_{tot}$$

$$\frac{\mathcal{E}_b}{N_0} = \frac{\alpha_1^2}{2W_1 N_0} \cdot \frac{N_0 W_1}{\alpha_1^2} \cdot 3 = 3/2$$

ii)

$$C_1 = W_1 \log_2\left(1 + \frac{\alpha_1^2}{N_0 W_1}\, \frac{N_0 W_1}{\alpha_1^2}\, 4\right) = W_1 \log_2(5) = 2.3219 W_1$$

$$C_2 = W_2 \log_2\left(1 + \frac{\alpha_2^2}{N_0 W_2}\, \frac{N_0 W_1}{\alpha_1^2}\, 4\right) =$$

$$= 4W_1 \log_2\left(1 + \frac{\alpha_1^2\, N_0 W_1 4}{4 N_0 4 W_1 \alpha_1^2}\right) = 4W_1 \log_2(5/4) = 1.2877 W_1$$

$$C_{tot} = 3.61 W_1$$

$$\frac{\mathcal{E}_b}{N_0} = \frac{N_0 W_1 4 + N_0 W_1}{3.61 W_1 N_0} = \frac{5}{3.61} = 1.385$$

Low $P_{sent} \Rightarrow$ only the best channel is used (waterfilling).

b)

$$\frac{\mathcal{E}_{b,1}}{N_0} = \frac{\alpha_1^2 P_{sent,1}}{C_1 N_0} = 100 = \frac{2^{C_1/W_1} - 1}{C_1/W_1}$$

gives $\dfrac{C_1}{W_1} \approx 9.962$

$$\frac{\mathcal{E}_{b,2}}{N_0} = \frac{\alpha_2^2 P_{sent,2}}{C_2 N_0} = \frac{\alpha_1^2 P_{sent,1}}{4 C_1 N_0} = 25 = \frac{2^{C_2/W_2} - 1}{C_2/W_2}$$

gives $\dfrac{C_2}{W_2} \approx 7.572$

$$\frac{C_1/W_1}{C_2/W_2} = \frac{W_2}{W_1} = \frac{9.962}{7.572} = 1.316, \; W_2 = 1.316 \text{ MHz}$$

Problem 5:

a) The receiver decides that $\hat{b} = 0$ if $\boldsymbol{r}$ is closer to $\boldsymbol{z}_0$ than to $\boldsymbol{z}_1$.

$$D^2 = 4 \cdot E_{z_1} = 4 E_g \sum_{k=1}^{K} \alpha_k^2$$

$$P_b = Q\left(\sqrt{\frac{D^2}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_g}{N_0} \sum_{k=1}^{K} \alpha_k^2}\right)$$

b) The purpose is to obtain K-fold diversity.

c)

$$\boldsymbol{r} = \underbrace{\pm \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_K \end{pmatrix} \sqrt{E_g} \pm \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_K \end{pmatrix} \sqrt{E_g}}_{\boldsymbol{z}_0, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{z}_3} + \boldsymbol{w}$$