Answers/short solutions to Examination in Digital Communications – advanced course, March 7, 2006

Problem 1:

a) z_{13} sent and z_3 decided $\Rightarrow$ 3 bit errors are made.

b)

$$\frac{D_{\min}^2}{2N_0} = \frac{2a^2}{N_0}$$

$$\frac{E_6}{N_0} = 100 = \frac{9a^2 + a^2}{N_0} = \frac{10a^2}{N_0}$$

$$\frac{a^2}{N_0} = 10$$

$$\frac{D_{\min}^2}{2N_0} = 20$$

$$Q(\sqrt{20}) = Q(4.47)$$

$$P_s = 3Q(4.47) - \frac{9}{4}\, Q^2(4.47) \approx 10^{-5}$$

c) 0 bit errors if message 8 (i.e. $\boldsymbol{z}_8$) is sent.
1 bit error if: 0, 4, 12, 9, 10 or 11 is sent
2 bit errors if: 1, 2, 3, 5, 6, 7, 13, 14 or 15 is sent

Problem 2:

The two pairs ($s_a = s_2,\ s_b = s_2$) and ($s_a = s_0,\ s_b = s_6$) are possible.

Problem 3:

i)

$$P_b \le \frac{1}{2} \cdot \frac{e^{-\frac{K\mathcal{E}_b/N_0}{K+1+\mathcal{E}_b/N_0}}}{1 + \frac{\mathcal{E}_b/N_0}{K+1}} \le 10^{-5}$$

$$\frac{\mathcal{E}_b}{N_0} = 31.62$$

$K = 10$:

$$\frac{e^{-\frac{316.2}{42.62}}}{1 + \frac{31.52}{11}} = 1.55 \cdot 10^{-4}$$

$K = 15$:

$$\frac{e^{-\frac{474.3}{47.62}}}{1 + \frac{31.62}{16}} = 1.59 \cdot 10^{-5} < 2 \cdot 10^{-5} \quad \text{(OK)}$$

if $K \ge 14.5$ then it is OK.

ii) K "very small" approaches Rayleigh fading case,

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\mathcal{E}_b/N_0}{1+\mathcal{E}_b/N_0}}\right) \leq \frac{1}{2}\cdot\frac{1}{1+\mathcal{E}_b/N_0}$$

K "very large" approaches AWGN case,

$$P_b = Q\left(\sqrt{2\frac{\mathcal{E}_b}{N_0}}\right) \leq \frac{1}{2}\, e^{-\mathcal{E}_b/N_0}$$

Large improvements as K increases compared with the $K = 0$ case!

iii) Rayleigh distributed attenuation + equally distributed phase-contribution.

Problem 4:

i)

$$\rho = \frac{R_b}{W} = \frac{R_b}{cR_s} = \frac{1}{c}\cdot\frac{k}{n}\ \log_2(M)$$

Uncoded: $k = n = 1$ and $M = 16 \Rightarrow \rho = \frac{1}{c}\cdot\frac{k}{n}\ \log_2(M) = \frac{4}{c}$ bps/Hz

So,

$$r_c = \frac{4}{5} \text{ and 32-ary PSK or}$$

$$r_c = \frac{4}{6} \text{ and 64-ary QAM}$$

give the desired bandwidth efficiency.

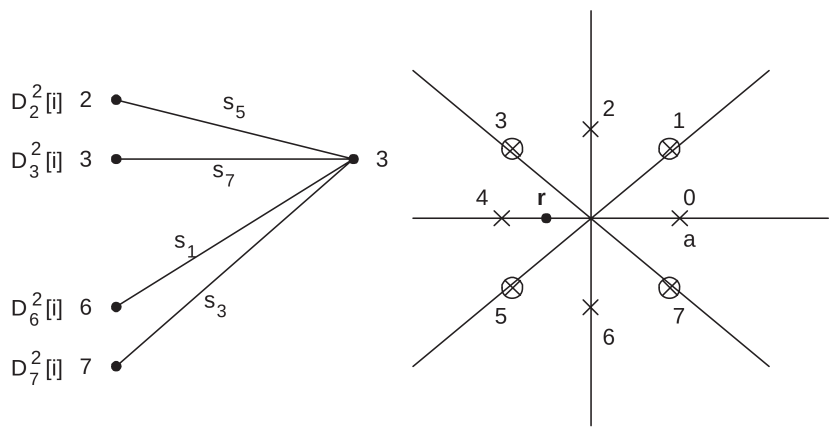
ii)

$$\rho = \frac{R_b}{W} = \frac{R_b}{cR_s} = \frac{1}{c}\cdot r_c \leq \frac{1}{c}$$

This is a too small value of ρ when the channel is good (then should a high R_b be communicated).

iii)

$$D^2_{acc}[i+1] = D^2[i] + D^2_{inc}$$



The branch corresponding to the smallest $D^2_{acc}[i+1]$ will be saved for further processing.

Problem 5:

$$C_1 + C_2 = W_1 \log_2 \left(1 + \frac{P}{\left(1 + \frac{W_2}{W_1}\right) N_0 W_1} \right) +$$

$$+ W_2 \log_2 \left(1 + \frac{P}{\left(1 + \frac{W_1}{W_2}\right) N_0 W_2} \right) =$$

$$= W_1 \log_2 \left(1 + \frac{P}{(W_1 + W_2) N_0} \right) +$$

$$+ W_2 \log_2 \left(1 + \frac{P}{(W_1 + W_2) N_0} \right) =$$

$$= (W_1 + W_2) \log_2 \left(1 + \frac{P}{(W_1 + W_2) N_0} \right)$$

If the signal power $P_i = W_i \frac{P}{W_1 + W_2}$ for the i:th link is evenly spread over its bandwidth W_i, $i = 1, 2$, then the total signal power $P = (P_1 + P_2)$ is evenly spread over the entire bandwidth $(W_1 + W_2)$, and $C_1 + C_2$ then represents the capacity of the over-all link. Hence, the current power distribution is the best in this case (evenly spread). See also the compendium.