$$ISI = \sum_{\substack{n=-\infty \\ n\neq i}}^{\infty} A[n]x[i-n] = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} A[i-n]x[n] \tag{6.19}$$

$$ISI_{wc}^{+} = \max(ISI) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \max(A[i-n]x[n]) = \tag{6.20}$$

$$= (M-1) \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} |x[n]|$$

In the same way it is found that,

$$ISI_{wc}^{-} = -(M-1) \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} |x[n]| \tag{6.21}$$

Hence, the worst case ISI occurs when the information sequence $A[n]$ consists of a specific pattern of $\pm(M-1)$-values.

6.2 Nyquist Condition for ISI-Free Reception

$$\boxed{\boldsymbol{x[i] = x(\mathcal{T} + iT_s) = x_0\delta[i] = \begin{cases} x_0 & , \quad i = 0 \\ 0 & , \quad i \neq 0 \end{cases}}} \tag{6.23}$$

$$x(t) = g(t) * h(t) * v(t) \tag{6.24}$$

Hence, in (6.31) we have arrived at **Nyquist** condition for zero ISI formulated in the frequency domain,

$$\boxed{\sum_{n=-\infty}^{\infty} X_{nc}(f - nR_s) = \frac{x_0}{R_s}} \qquad (6.33)$$

ISI will be present if the symbol rate satisfies,

$$\boldsymbol{R_s > 2W_{\ell p}} \qquad (6.36)$$

On the other hand, if the symbol rate satisfies,

$$\boxed{\boldsymbol{R_s \leq 2W_{\ell p}}} \qquad (6.37)$$

then **ISI-free reception is possible** *provided that* $X_{nc}(f)$ *has a proper shape.*

Some comments related to Problem 4.12b.

$z0(t)=g(t)$, $z1(t)=-g(t)$

$z2(t)=g1(t)$, $z3(t)=-g1(t)$

The pulses $g(t)$ and $g1(t)$ are orthogonal.

The union bound=........

D^2: 01,02,03,12,13,23

dmin^2=...........

1970. Breakthrough for the production of optical fibers. Glass with an attenuation of 20 dB/km was produced.

1982. G. Ungerboeck: Developed trellis-coded modulation, a (signal) power and bandwidth efficient technique.

1988. A fiber optical transatlantic cable between the United States and Europe, TAT-8.

1992. The European GSM system, for mobile digital telephony, started.

1993. C Berrou, A. Glavieux, and P. Thitimajshima: The development of so-called "turbo" codes.

1998. G.J. Foschini, M.J. Gans: Pioneering work within the concept of so-called MIMO systems (multiple-input-multiple-output systems).

⋮

Chapter 7

Optical Fiber Communications

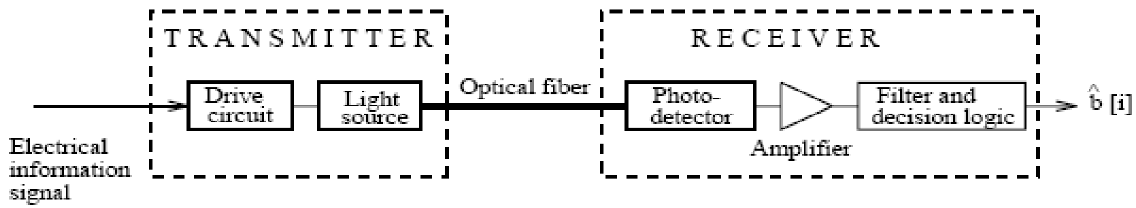


Figure 7.1: Basic elements in an optical fiber transmission link.

A simplified model of the optical signal $x(t)$ produced by the light source is

$$x(t) = \sqrt{2P(t)}\cos(\omega_c t + \theta(t)), \quad -\infty \leq t \leq \infty \tag{7.1}$$

$$P(t) = \sum_{i=-\infty}^{\infty} A[i]g(t - iT_b), \quad A[i] \in \{A_0, A_1\}, \quad -\infty \leq t \leq \infty \qquad (7.2)$$

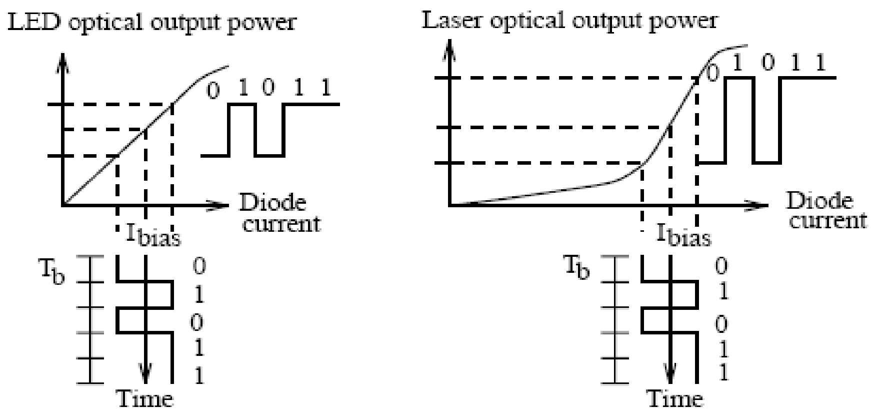


Figure 7.2: Optical output power versus diode current for LEDs and for laser diodes.

suitable for transmission. In the early 1970s the **first window** (800-900 [nm]) were used. Later on, in the 1980s, two windows were defined in the 1100-1600 [nm] region. The **second window** is centered around 1300 [nm] and the **third window** around 1550 [nm]. The corresponding optical carrier frequencies are approximately, according to (7.3), $3.53 \cdot 10^{14}$ [Hz], $2.31 \cdot 10^{14}$ [Hz] and $1.94 \cdot 10^{14}$ [Hz]. Furthermore, the bandwidth in Hz of each transmission window is roughly 20-40 [THz].

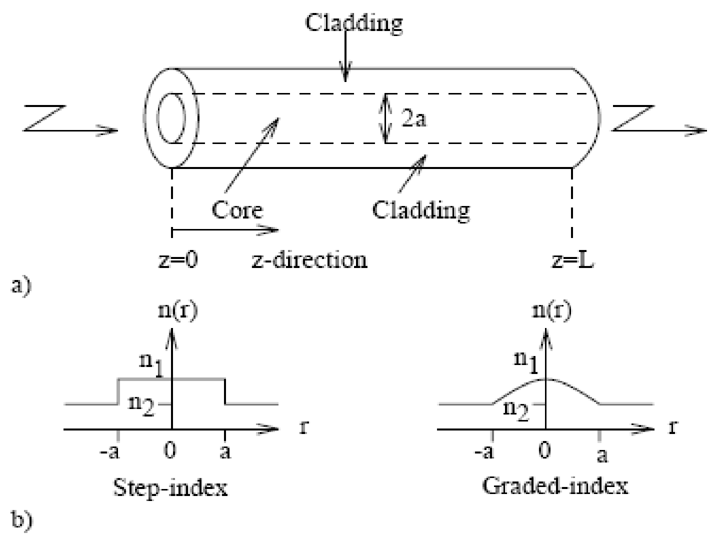


Figure 7.3: a) Structure of an optical fiber; b) Refractive index profiles.

“single mode”, “multi mode”, “chromatic dispersion”

$$\lambda = \frac{c}{f_c} = \frac{3 \cdot 10^8}{f_c} \ , [m] \qquad (7.3)$$

$$\alpha L = 10 \log_{10} \left(\frac{P_{tra}}{P_{rec}} \right) \tag{7.5}$$

EXAMPLE 7.1

Assume that the receiver in Figure 7.1 require a minimum received energy, E_{rec}, corresponding to 10 photons, to be able to detect a received optical pulse. The energy in a photon is $hf = hc/\lambda$, where h is Planck's constant, $h = 6.626 \cdot 10^{-34}$ [Ws^2]. Calculate the required transmitted energy E_{tra} if $\lambda = 850$ [nm], $L = 1$ [km] and if

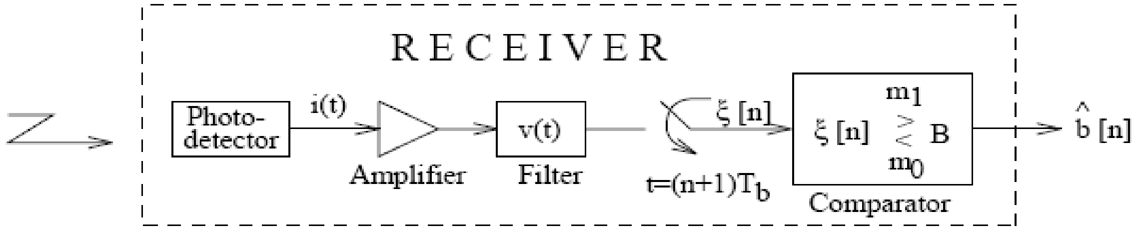
i) $\alpha = 1000$ *[dB/km]* *ii)* $\alpha = 20$ *[dB/km]* *iii)* $\alpha = 0.2$ *[dB/km]*

Solution:

$$\frac{E_{tra}}{E_{rec}} = 10^{\alpha/10} \Rightarrow E_{tra} = 10hf \cdot 10^{\alpha/10} = 10 \cdot \frac{6.626 \cdot 10^{-34} \cdot 3 \cdot 10^8}{850 \cdot 10^{-9}} \cdot 10^{\alpha/10} =$$

$$= \begin{cases} i) & 10^{101} \cdot hf = 2.34 \cdot 10^{82} \ [Ws] \ \textit{(unrealistic!)} \\ ii) & 1000 \cdot hf = 2.34 \cdot 10^{-16} \ [Ws] \\ iii) & 10.47 \cdot hf = 2.45 \cdot 10^{-18} \ [Ws] \end{cases}$$

since $hf = 2.34 \cdot 10^{-19}$ *[Ws].* □



$$\mathcal{I}_{ph}(t) = \frac{\mathcal{P}_{rec}(t)}{hf} \text{ [photons/s]} \tag{7.7}$$

$$\mathcal{I}_{e}(t) = \eta \cdot \mathcal{M} \cdot \mathcal{I}_{ph}(t) + \mathcal{I}_{d} = \eta \cdot \mathcal{M} \cdot \frac{\mathcal{P}_{rec}(t)}{hf} + \mathcal{I}_{d} \text{ [electrons/s]} \tag{7.8}$$

How can we get better Pb than binary antipodal signals?
PAM, PSK, QAM, PWM, PPM, FSK?

Uncoded:
memoryless, i.e. no dependency between sent signal alternatives.

Coding:
In a clever way introduce memory (dependency, redundancy) between the sent signal alternatives!

The memory can be used by the receiver to significantly reduce Pb!

Chapter 8

Trellis-coded Signals

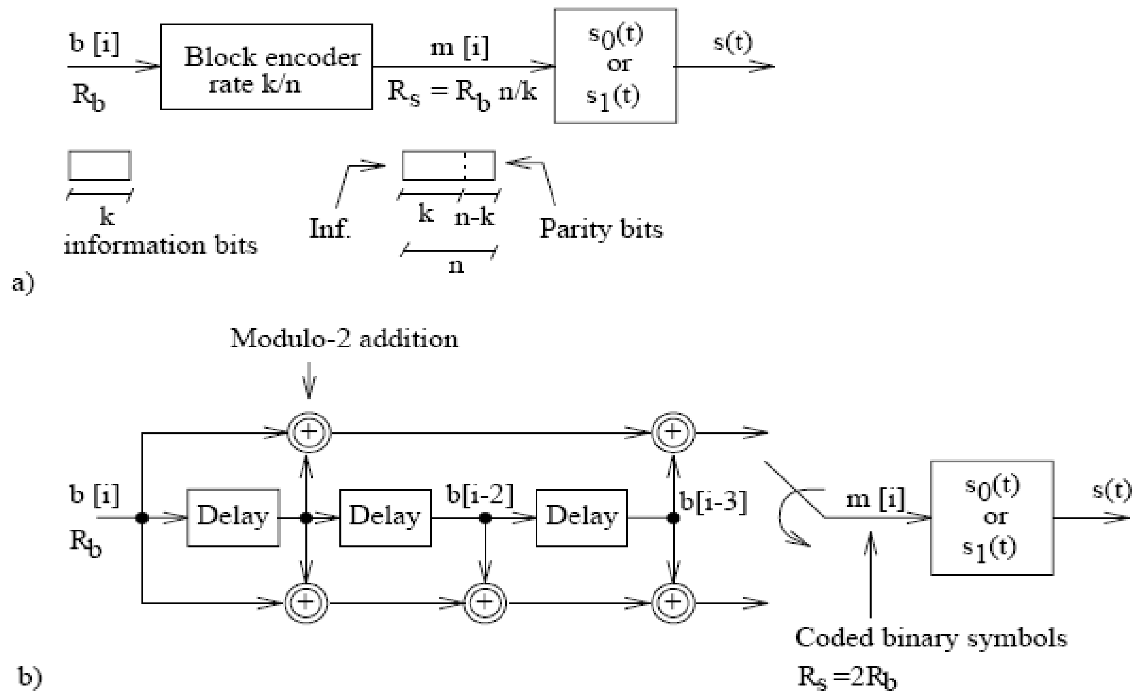


Figure 8.1: a) Block coding, $r_c = k/n$. b) Convolutional coding, $r_c = 1/2$.

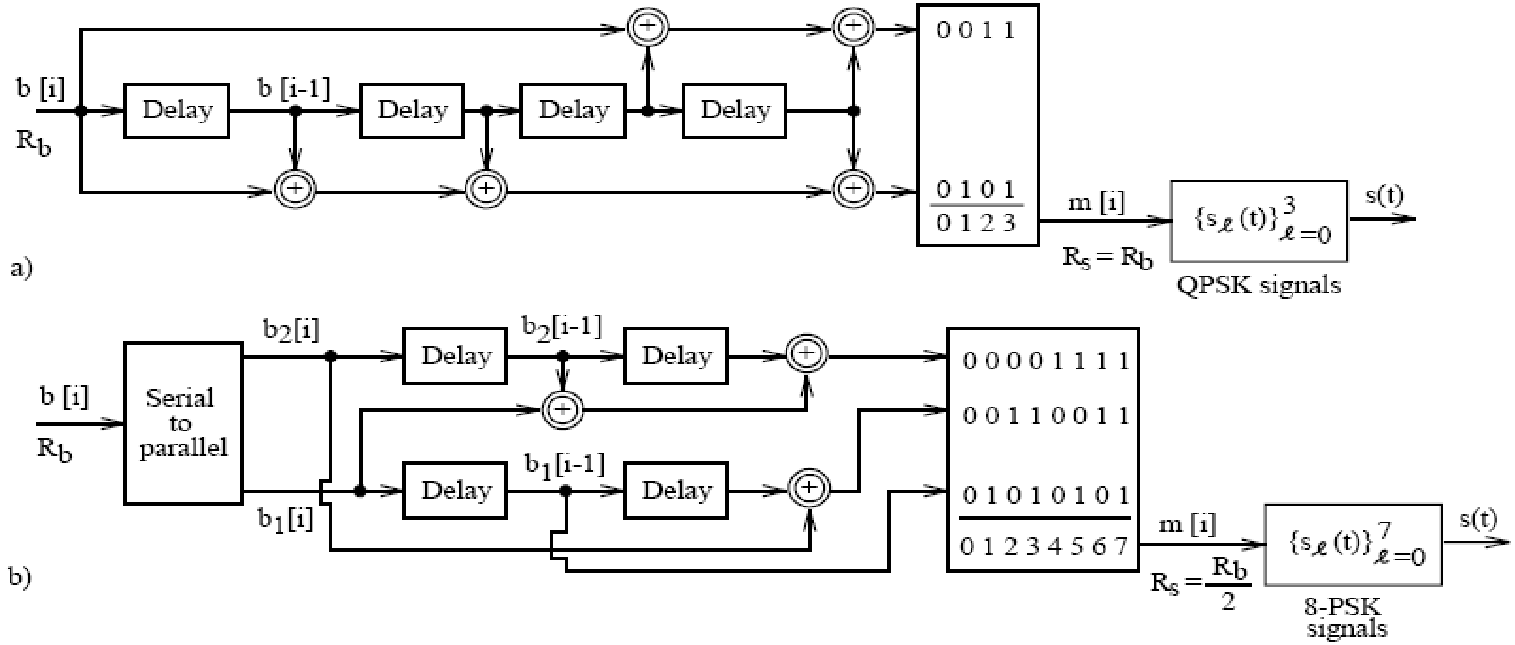
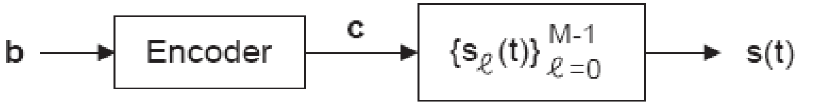


Figure 8.2: a) Rate $r_c = 1/2$ convolutional encoder combined with QPSK; b) Rate $r_c = 2/3$ convolutional encoder combined with 8-PSK, from [63], [64].

content in the shift-register (the three delay units). ***This implies that there is a dependency in the sequence of transmitted waveforms.*** As will be

Adaptive coding and modulation!

2.32 Let us here study adaptive coding and modulation according to the block diagram below.



b is the sequence of information bits, and the information bit rate is R_b [bps]. The encoder introduces redundancy, and its output sequence c here consists of coded bits ("0", or "1"). Assume that B information bits are encoded into C coded bits. Then we define the rate of the encoder, denoted r_c, as

$$r_c = B/C \text{ [inf.bit/coded bit]}$$