1948!

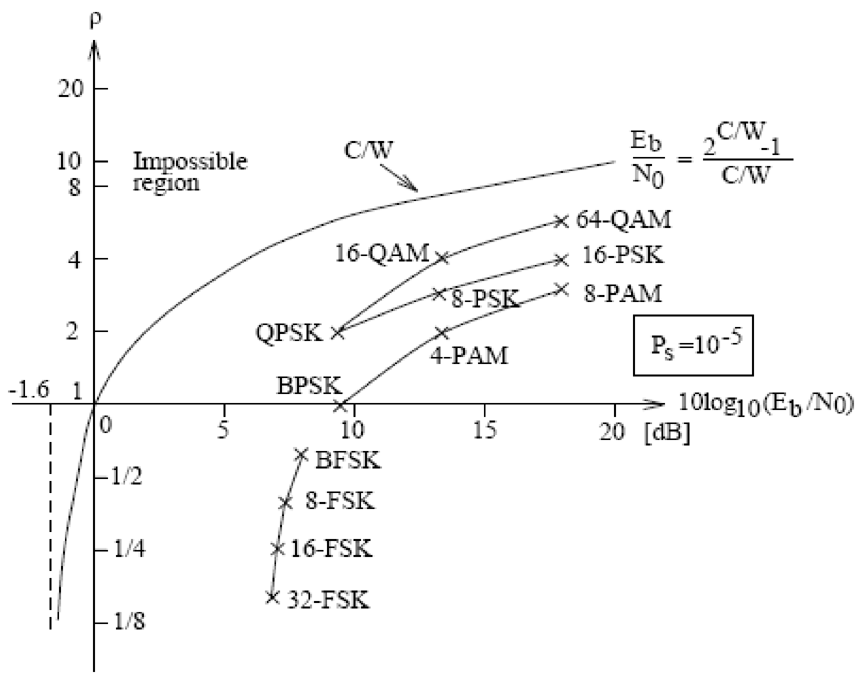


Figure 5.17: Sketch of the ρ versus $\mathcal{E}_b/N_0$ performance for some of the schemes studied in this section. Reliable communication is not possible above the capacity curve (see (5.64).

5.4.3 A Simplified Model of Multiuser Communication

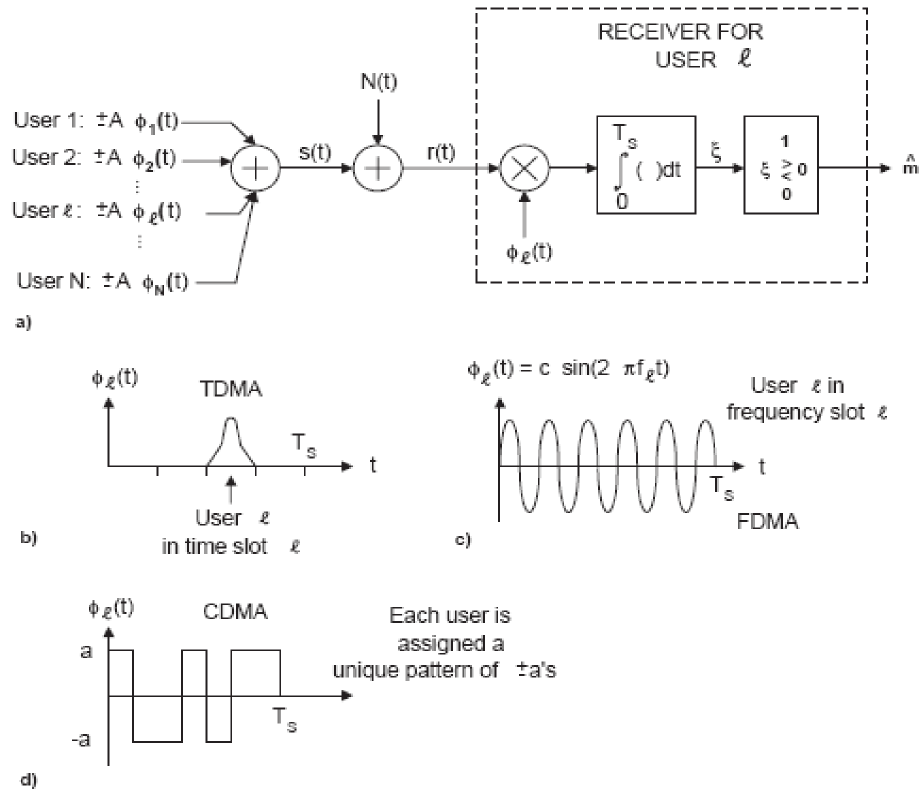


Figure 5.27: a) An example of multiuser communication; b,c,d) Examples of signals illustrating TDMA, FDMA and CDMA.

$$
\begin{aligned}
\xi &= \int_0^{T_s} \phi_\ell(t) r(t) dt = \int_0^{T_s} \phi_\ell(t) \left(\sum_{n=1}^{N} A_n \phi_n(t) + N(t) \right) dt \\
&= A_\ell + \int_0^{T_s} N(t) \phi_\ell(t) dt \qquad (5.102)
\end{aligned}
$$

5.4.5 Differential Phase-Shift-Keying

$$s(t) = g(t)\sqrt{2E}\cos(\omega_c t + \theta_n),\ nT_s \le t \le (n+1)T_s \tag{5.123}$$

where,

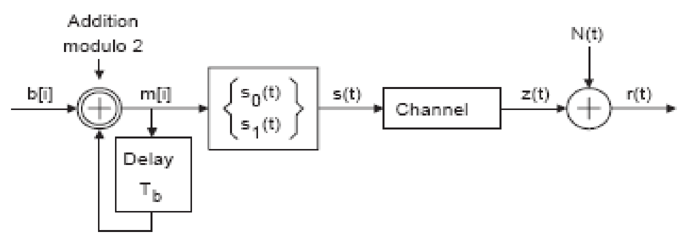
$$\theta_n = \theta_{n-1} + 2\pi i/M,\ i = 0, 1, \ldots, M-1 \tag{5.124}$$

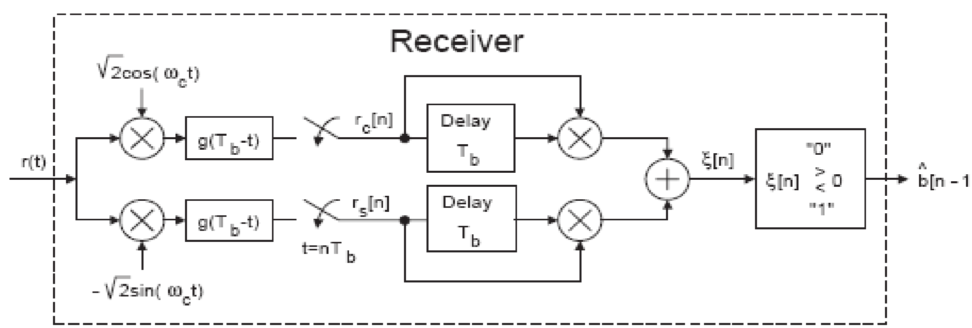
The received signal is here assumed to be,

$$r(t) = \alpha g(t)\sqrt{2E}\cos(\omega_c t + \theta_n + \nu) + N(t),\ nT_s \le t \le (n+1)T_s \tag{5.125}$$

Example 5.25: Receiver implementation can be improved by differential coding.

”1” changes signal





Chapter 6

Intersymbol Interference

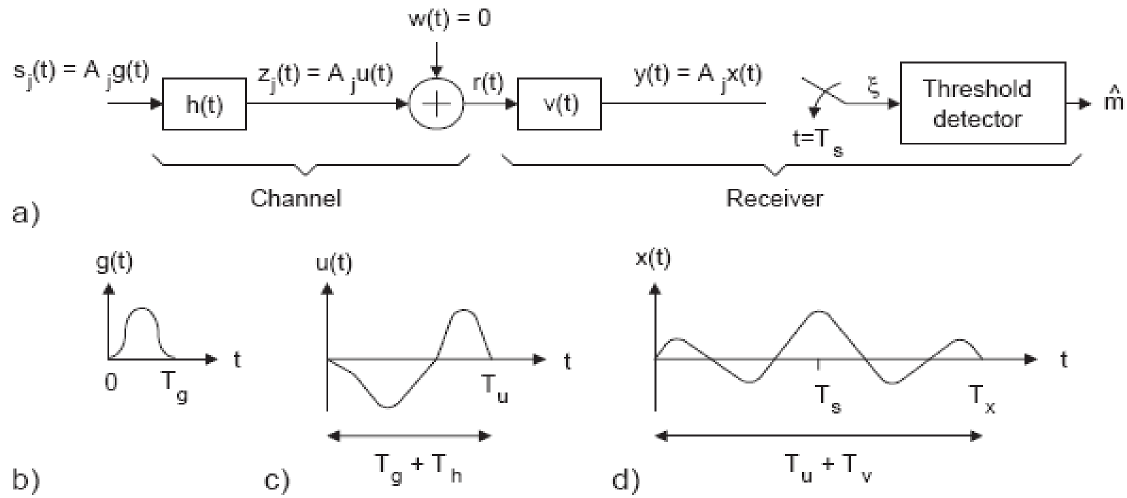


Figure 6.1: a) Transmission and reception of a single M-ary PAM signal alternative. b-d) Examples of pulse shapes at different points in the communication system.

$$x(t) = u(t) * v(t) = g(t) * h(t) * v(t) \tag{6.3}$$

see Figure 6.1d. Hence, **if the signal $A_j g(t)$ is sent from the transmitter, then the signal $A_j x(t)$ occurs at the output of the receiver filter $v(t)$.**

How should we choose the signaling time Ts (Ts=kTb, Rb=kRs)?

If Ts is chosen to be long (Ts=Tu or Ts > Tu) then the bit rate becomes small.

If Ts is chosen to be short (Ts<Tu) then *overlapping signals* may appear after the multi-path channel which might cause problems for the receiver resulting in a large error probability.

When possible: we want to use a short Ts and we can accept only a minor increase in error probability.

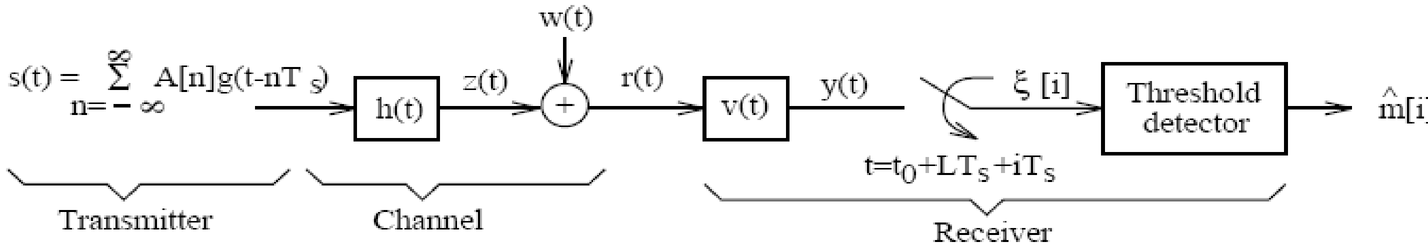


Figure 6.2: Sequential transmission, compare with Figure 6.1.

Our aim in this chapter is to investigate how to obtain good symbol error probability performance, with the receiver in Figure 6.2, when the symbol rate R_s is increased above $1/T_u$ symbols per second.

$$y(t) = \sum_{n=-\infty}^{\infty} A[n]x(t - nT_s) + w_c(t) \tag{6.10}$$

$$\mathcal{T} = t_0 + LT_s \tag{6.11}$$

$$\xi[i] = y(\mathcal{T} + iT_s) = \sum_{n=-\infty}^{\infty} A[n]x(\mathcal{T} + iT_s - nT_s) + w_c(\mathcal{T} + iT_s) \tag{6.12}$$

Will the symbol error probability be decreased if a sampling rate faster than 1/Ts is used in the receiver?

Is the choice of sampling rate 1/Ts related to the sampling theorem?

No, since the ML receiver structure is a consequence of decision theory (see chapter 4), and it can not be improved!

$$\xi[i] = y(\mathcal{T} + iT_s) = \sum_{n=-\infty}^{\infty} A[n]x(\mathcal{T} + iT_s - nT_s) + w_c(\mathcal{T} + iT_s) \qquad (6.12)$$

Now define,

$$x[i] = x(\mathcal{T} + iT_s) \qquad (6.13)$$

$$w_c[i] = w_c(\mathcal{T} + iT_s) \qquad (6.14)$$

Then the decision variable $\xi[i]$ in (6.12) can be written as,

$$\boxed{\xi[i] = A[i] * x[i] + w_c[i] = \sum_{n=-\infty}^{\infty} A[n]x[i-n] + w_c[i]} \qquad (6.15)$$

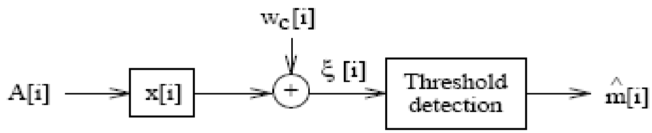


Figure 6.3: Discrete-time model of the M-ary PAM communication system in Figure 6.2.

Example of $x(t)=g(t)*h(t)*v(t)$ and $x[i]$:

How large is ISI?

Is it too large?

Can we make ISI=0?

$$\xi[i] = A[i] * x[i] + w_c[i] = \sum_{n=-\infty}^{\infty} A[n]x[i-n] + w_c[i] \tag{6.15}$$

$$\boldsymbol{\xi[i]} = \underbrace{\boldsymbol{A[i]x[0]}}_{\text{Message term}} + \underbrace{\sum_{\substack{\boldsymbol{n=-\infty} \\ \boldsymbol{n \neq i}}}^{\boldsymbol{\infty}} \boldsymbol{A[n]x[i-n]}}_{\text{ISI}} + \underbrace{\boldsymbol{w_c[i]}}_{\text{noise}} \tag{6.17}$$

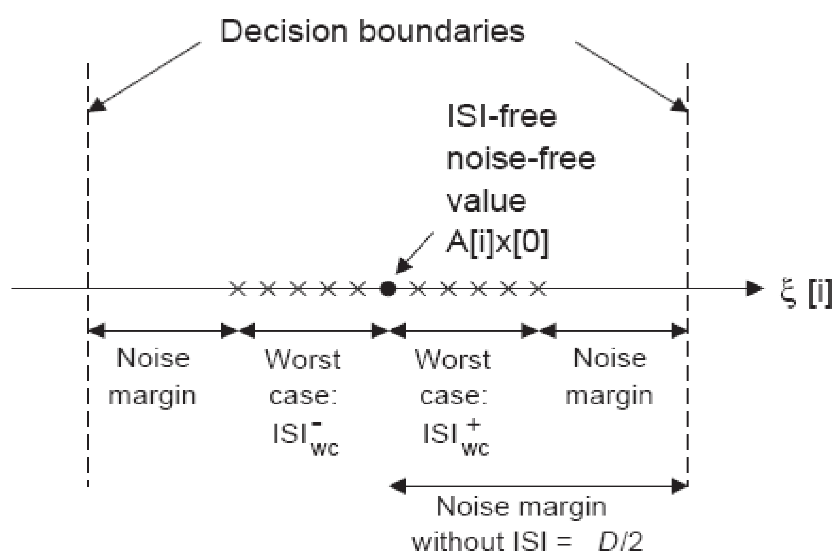


Figure 6.5: Illustrating the influence of ISI on the decision variable $\xi[i]$ in the noise-free case.

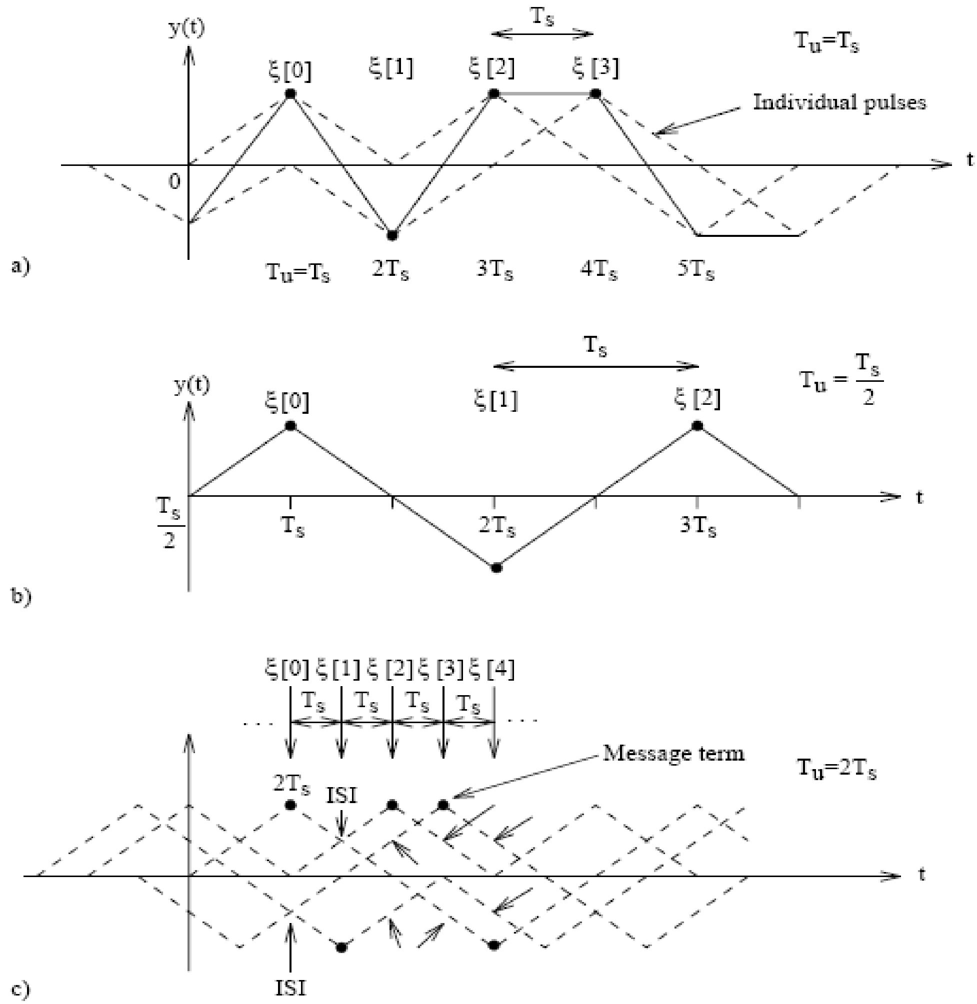


Figure 6.4: Illustrating intersymbol interference (ISI) in the receiver. a) no ISI, $R_s = 1/T_u$; b) no ISI, $R_s = 1/2T_u$; c) ISI, $R_s = 2/T_u$.

$$ISI = \sum_{\substack{n=-\infty \\ n \neq i}}^{\infty} A[n]x[i-n] = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} A[i-n]x[n] \tag{6.19}$$

$$ISI_{wc}^{+} = \max(ISI) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \max(A[i-n]x[n]) = \tag{6.20}$$

$$= (M-1) \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} |x[n]|$$

In the same way it is found that,

$$ISI_{wc}^{-} = -(M-1) \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} |x[n]| \tag{6.21}$$

Hence, the worst case ISI occurs when the information sequence $A[n]$ consists of a specific pattern of $\pm(M-1)$-values.

6.2 Nyquist Condition for ISI-Free Reception

$$\boxed{\boldsymbol{x[i] = x(\mathcal{T} + iT_s) = x_0\delta[i] = \begin{cases} x_0 & , \quad i = 0 \\ 0 & , \quad i \neq 0 \end{cases}}} \tag{6.23}$$

$$x(t) = g(t) * h(t) * v(t) \tag{6.24}$$

Example: ISI=0 by choosing Ts correct.

6.2.1 Equivalent Condition in the Frequency Domain

Further insight can be gained by formulating the condition for ISI-free reception in the frequency domain. To do this we need the following basic result from signal theory:

Suppose $a(t)$ is a continuous-time signal with Fourier transform $A(f)$,

$$A(f) = \int_{-\infty}^{\infty} a(t) e^{-j2\pi ft} dt \tag{6.26}$$

If the signal $a(t)$ is sampled at the time instants $t = iT_{samp}$, $i = ..-2, -1, 0, 1, 2, ..$ then a discrete-time signal $a[i] = a(iT_{samp})$ is obtained. Denote the Fourier transform of $a[i]$ by $\mathcal{A}(\nu)$,

$$\mathcal{A}(\nu) = \sum_{n=-\infty}^{\infty} a[n] e^{-j2\pi\nu n} \tag{6.27}$$

It can be shown that,

$$\mathcal{A}(\nu) = \frac{1}{T_{samp}} \sum_{n=-\infty}^{\infty} A\left(\frac{\nu - n}{T_{samp}}\right) \tag{6.28}$$

or alternatively,

$$\boxed{\mathcal{A}(fT_{samp}) = \frac{1}{T_{samp}} \sum_{n=-\infty}^{\infty} A\left(f - \frac{n}{T_{samp}}\right)} \tag{6.29}$$

The condition for ISI-free reception, formulated in the frequency domain, is obtained by first Fourier transforming the sequence $x[i]$ in (6.23),

$$\mathcal{F}\{x[i]\} = \mathcal{X}(\nu) = x_0 \tag{6.30}$$

Using this result, and (6.29), it is found that,

$$\mathcal{X}(fT_s) = x_0 = R_s \sum_{n=-\infty}^{\infty} X_{nc}(f - nR_s) \tag{6.31}$$

Hence, in (6.31) we have arrived at **Nyquist** condition for zero ISI formulated in the frequency domain,

$$\boxed{\sum_{n=-\infty}^{\infty} X_{nc}(f - nR_s) = \frac{x_0}{R_s}} \tag{6.33}$$

ISI will be present if the symbol rate satisfies,

$$\boldsymbol{R_s > 2W_{\ell p}} \tag{6.36}$$

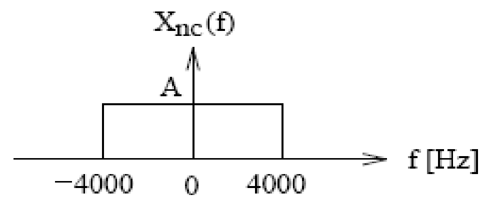
On the other hand, if the symbol rate satisfies,

$$\boxed{\boldsymbol{R_s \leq 2W_{\ell p}}} \tag{6.37}$$

then **ISI-free reception is possible** *provided that* $X_{nc}(f)$ *has a proper shape.*

EXAMPLE 6.8

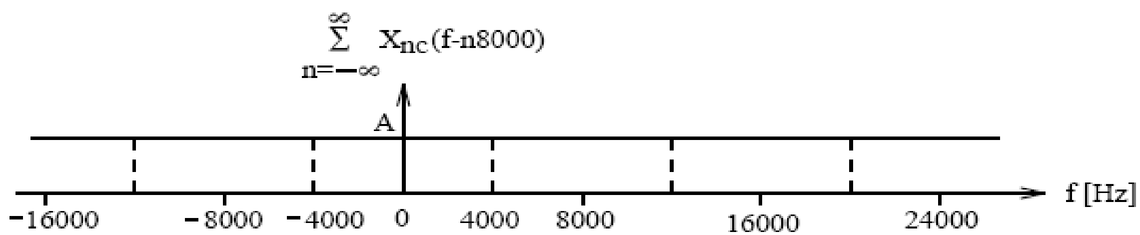
Assume that $X_{nc}(f)$ is,



$A = x_0 T_s$.

Show that there is no ISI if the symbol rate is $R_s = 8000$ [symbol/s].

Solution:



Since $\sum_{n=-\infty}^{\infty} X_{nc}(f - n8000) = x_0/R_s$, *for all f, there is no ISI in the receiver.* □

6.2.2 Spectral Raised Cosine Spectrum

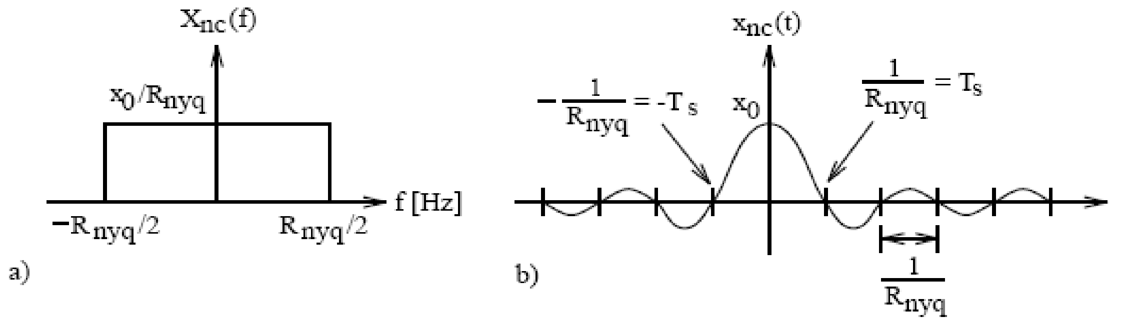


Figure 6.6: a) Ideal Nyquist spectrum; b) Ideal Nyquist pulse.

With ideal Nyquist signaling the bandwidth efficiency is,

$$\boxed{\rho_{nyq} = \frac{R_b}{W_{\ell p}} = \frac{R_{nyq}\log_2(M)}{R_{nyq}/2} = 2\log_2(M) = 2k \quad \left[\frac{b/s}{Hz}\right]} \qquad (6.41)$$

$$
\begin{aligned}
x_{nc}(t) &= x_0 \, \frac{\sin(\pi t/T_s)}{\pi t/T_s} \cdot \frac{\cos(\pi \beta t/T_s)}{1-(2\beta t/T_s)^2} \quad , -\infty \le t \le \infty \\
X_{nc}(f) &= \begin{cases} x_0 T_s & , \quad 0 \le |f| \le \frac{1-\beta}{2T_s} \\ \frac{x_0 T_s}{2}\left[1+\cos\left(\frac{\pi |f| T_s}{\beta} - \frac{\pi}{2}\cdot\frac{1-\beta}{\beta}\right)\right] & , \quad \frac{1-\beta}{2T_s} \le |f| \le W_{\ell p} \\ 0 & , \quad |f| > W_{\ell p} \end{cases} \\
W_{\ell p} &= \frac{1+\beta}{2T_s} = (1+\beta)\,\frac{R_s}{2}, \qquad 0 \le \beta \le 1
\end{aligned}
\tag{6.42}
$$

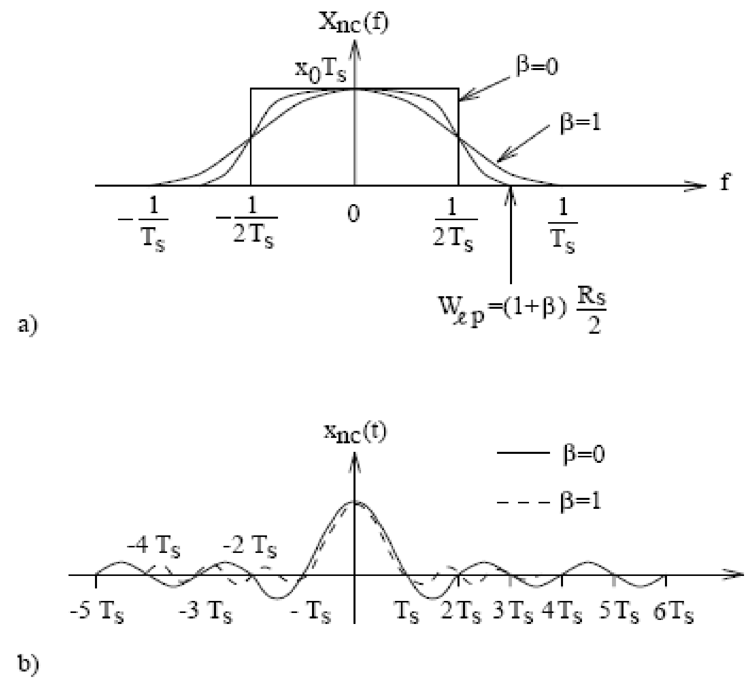
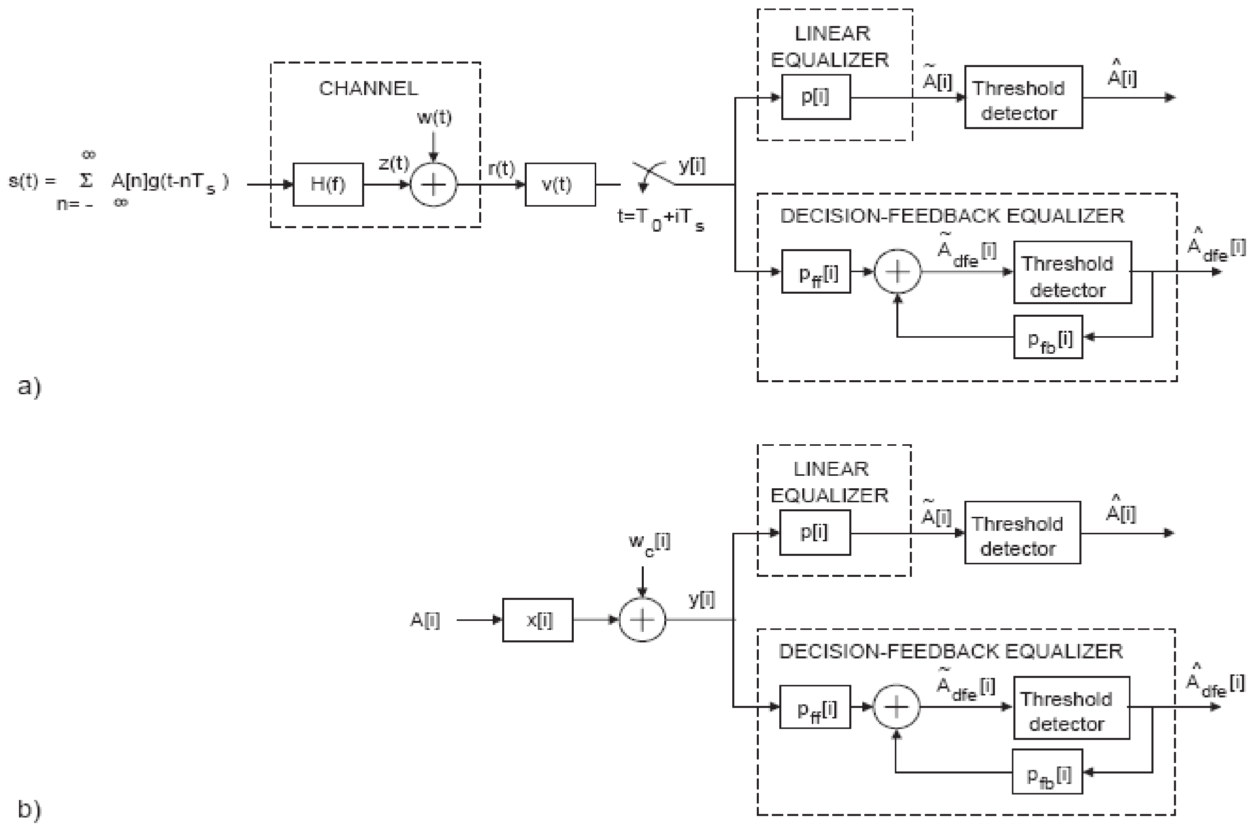


Figure 6.7: The spectral raised cosine family of pulses, $0 \le \beta \le 1$. a) In the frequency domain for different β; b) Pulse shapes for $\beta = 0$ and $\beta = 1$.

6.2.4 An Introduction to Equalizers



Some comments related to Problem 4.12b.

$z0(t)=g(t)$, $z1(t)=-g(t)$

$z2(t)=g1(t)$, $z3(t)=-g1(t)$

The pulses $g(t)$ and $g1(t)$ are orthogonal.

The union bound=........

D^2: 01,02,03,12,13,23

dmin^2=...........

A short summary of some important parameters/concepts in this course during **study-week 5:**

Chapter 5 :

The importance of the concept of orthogonality in the context of multi-user communications is illustrated in Equ. (5.102) and in Figure 5.27 on page 396.

Chapter 6 :

Note that only M-PAM is considered in this chapter.

The analog signal x(t) on page 436 is especially important in this chapter.

The symbol T in, e.g., Equ. (6.13) on page 438, denotes the precise time when the signal x(t) has its maximal value.

The discrete-time model in Figure 6.3 is important, and it describes the decision variable exactly.

ISI is an extra disturbance that appear in the receiver, and it is defined in Equ.(6.17).

The worst case ISI can be calculated, see page 442.

Zero ISI is obtained if the Nyquist condition (Equ. (6.23) is satisfied.

The Nyquist condition can also be formulated in the frequency domain (Equ. (6.33)

The family of so-called spectral raised cosine pulses x(t) (page 452) is a special kind of signals designed such that ISI=0.