

A "standard" way to calculate the bandwidth W .

1. Identify the underlying pulse shape $g(t)$.
2. Identify the duration T of the pulse.
3. Use Table 2.1 on page 86.
4. Identify if you have a base-band (low-frequency) situation, or a band-pass (high-frequency) situation.
4. Calculate the bandwidth W .

Remember!

Thinking in the frequency domain is necessary to be able to understand communication methods and systems.

Problem 2.26a (M-ary PSK)!!

Problem 2.26b (M-ary QAM)!!

Problem 2.30 (Peak value of $R(f)$; Radiation aspects (EMC), standards)!!

How good can we do concerning bit rate & bit error probability?

$C=W*\log_2(1+SNR)$, Claude Shannon 1948

C is the largest bit rate (the capacity) for which error-free (!!) communication **is possible**.

W is the bandwidth.

SNR is a signal-to-noise ratio at the input to the receiver.

Practical consequences?

Chapter 3

Information Transmission with Carrier Modulation Techniques

This chapter deals with bandpass signals carrying digital or analog information. The characteristic feature of a bandpass signal $x(t)$ is that its frequency content is “concentrated” around a **carrier frequency** f_c [Hz], see Figure 3.1. Bandpass signals occur frequently in practice: mobile telephony, television, radio, satellite communication, wireless local area networks, navigation, optical fiber communication, etc.

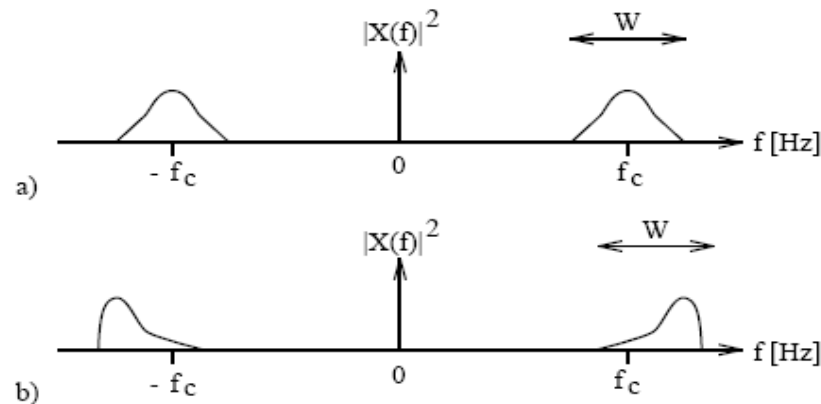


Figure 3.1: Examples of the frequency content in a bandpass signal $x(t)$. a) Symmetric spectrum around f_c . b) Non-symmetric spectrum around f_c .

General bandpass:
$$x(t) = x_I(t) \cos(2\pi f_c t) - x_Q(t) \sin(2\pi f_c t), \quad -\infty \leq t \leq \infty \quad (3.1)$$

A practical implementation is therefore:

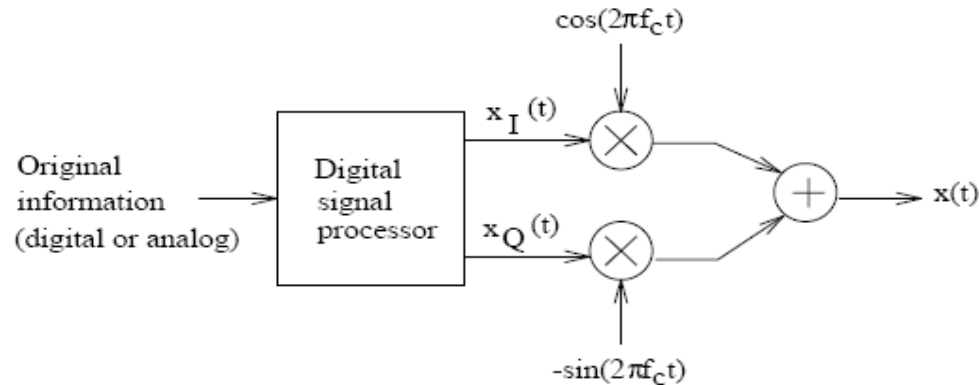


Figure 3.2: A transmitter structure for information transmission via carrier modulation. First the information is placed in the I - and Q -components $x_I(t)$ and $x_Q(t)$. After that a frequency conversion is made from baseband to the carrier frequency f_c .

$$x_{dsb-sc}(t) = x_I(t) \cos(2\pi f_c t) \quad (3.3)$$

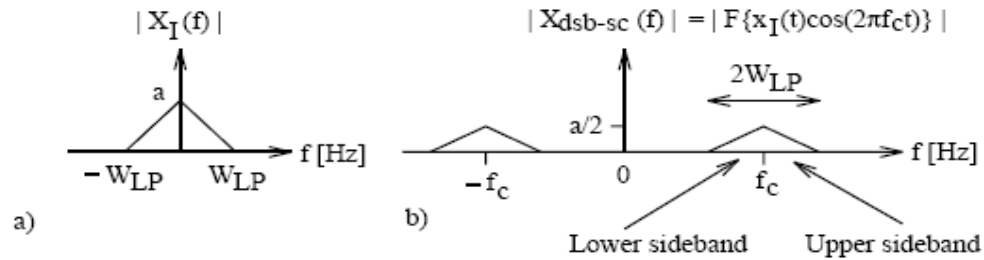
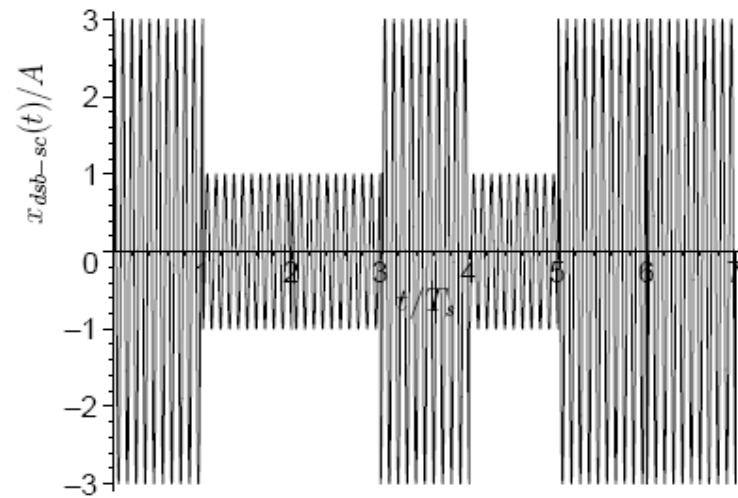
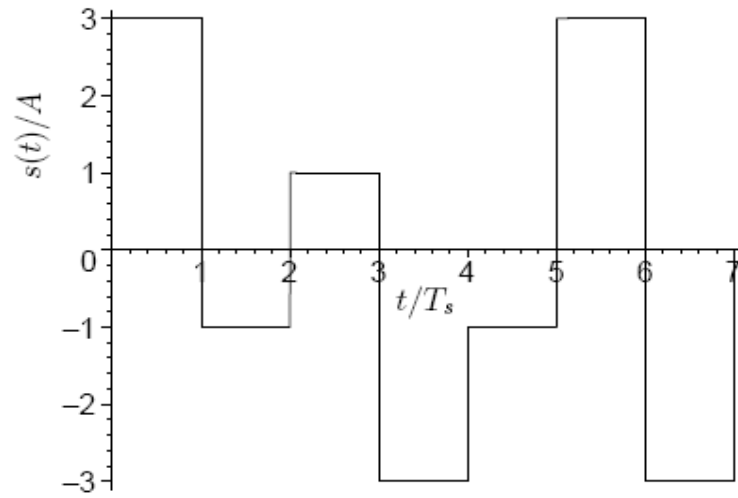
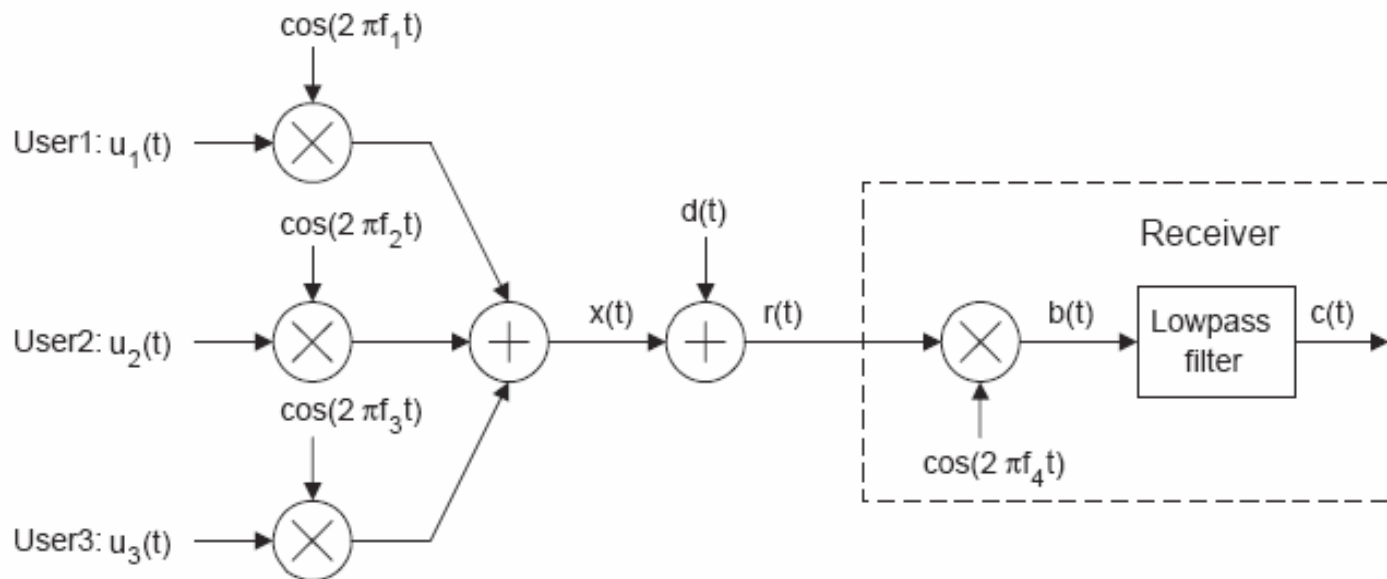
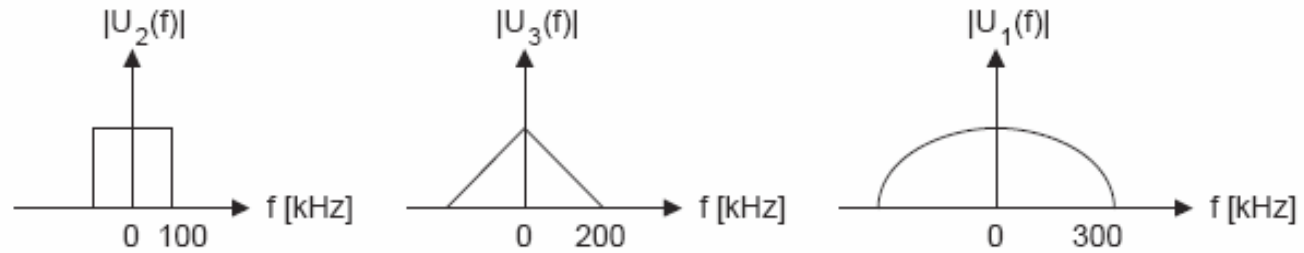


Figure 3.3: Illustrating the frequency content in $x_I(t)$ and $x_{dsb-sc}(t)$. a) $|X_I(f)|$; b) $|X_{dsb-sc}(f)|$.

Example 3.1:



3.9 In the three-user (digital) communication system below, the frequency content in the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are,



It is known that the individual carrier frequencies are: $f_1 = 3.5$ MHz, $f_2 = 4.0$ MHz, $f_3 = 3$ MHz. The disturbance $d(t)$ is $d(t) = \cos(2\pi 2f_d t)$ where $f_d = 1.7$ MHz.

Only frequencies up to 100 kHz pass the lowpass filter.

3.1.2 Envelope and Phase

Let us rewrite $x(t)$ in (3.1) in the following way ($\omega_c = 2\pi f_c$ and $Re\{ \}$ denotes “real part of”),

$$\begin{aligned}x(t) &= x_I(t) \cos(\omega_c t) - x_Q(t) \sin(\omega_c t) = \\&= Re\{(x_I(t) + jx_Q(t))e^{j\omega_c t}\} = \\&= Re\{|x_I(t) + jx_Q(t)|e^{j\theta(t)}e^{j\omega_c t}\} = \\&= Re\left\{\sqrt{x_I^2(t) + x_Q^2(t)} e^{j(\omega_c t + \theta(t))}\right\} = \\&= \underbrace{\sqrt{x_I^2(t) + x_Q^2(t)}}_{e(t) \geq 0} \cos(\omega_c t + \theta(t))\end{aligned}\tag{3.12}$$

where

$$\begin{array}{l}e(t) = \sqrt{x_I^2(t) + x_Q^2(t)} \geq 0 \\x_I(t) = e(t) \cos(\theta(t)) \\x_Q(t) = e(t) \sin(\theta(t))\end{array}\tag{3.13}$$

Hence, an important alternative description of a general bandpass signal $x(t)$ is,

$$\boxed{x(t) = e(t) \cos(2\pi f_c t + \theta(t))}, \quad -\infty \leq t \leq \infty\tag{3.14}$$

where the so-called **envelope** $e(t)$ (or amplitude) and the **instantaneous phase** $\theta(t)$ are introduced. Note that the envelope $e(t)$ is non-negative by definition.

$$x(t) = x_I(t) \cos(\omega_c t) - x_Q(t) \sin(\omega_c t)$$

$$x(t) = e(t) \cos(2\pi f_c t + \theta(t))$$

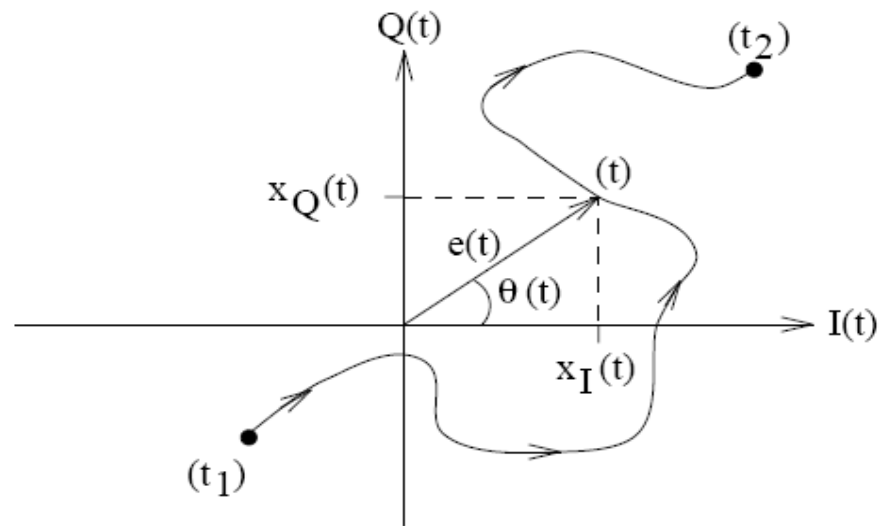
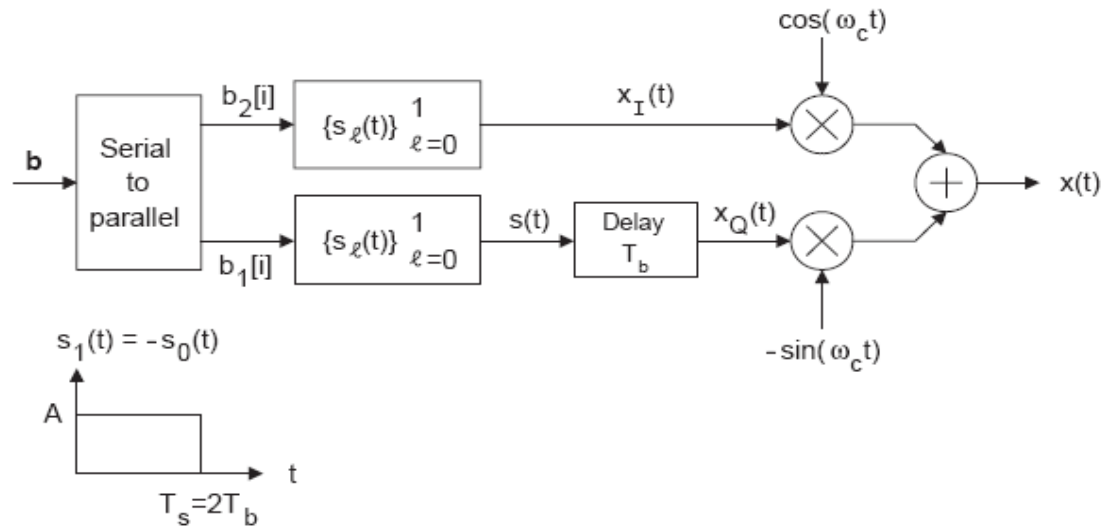


Figure 3.4: The I-Q diagram.

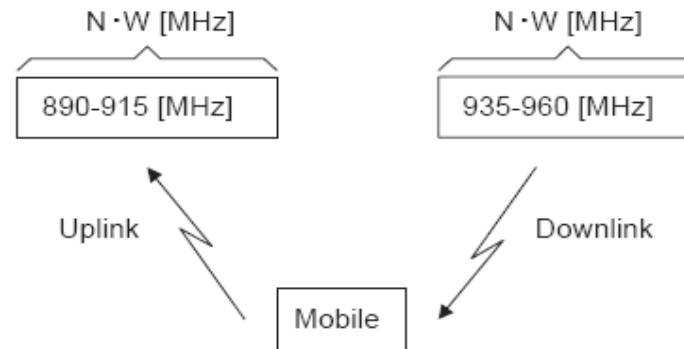
EXAMPLE 3.5

Below, two information carrying baseband signals $x_I(t)$ and $s(t)$ are first generated. Binary antipodal signaling with a rectangular pulse shape is used for both $x_I(t)$ and $s(t)$. The signal $x_Q(t)$ is a delayed version of $s(t)$, $x_Q(t) = s(t - T_b)$.



EXAMPLE 3.7

In GSM (Global System for Mobile Communications) information is sent from the mobiles to the basestation (so-called reverse link or uplink) in the frequency band 890–915 [MHz], while information from the basestation to the mobiles (so-called forward link or downlink) is sent in the frequency band 935–960 [MHz]. These 25 [MHz] bands are each divided in N sub-bands with bandwidth W [Hz] (hence, $N \cdot W = 25$ [MHz]). The figure below illustrates the situation.



Each sub-band of W [Hz] carries information from X users, which are time-multiplexed using X time-slots. The total number of speech-channels (or data-channels) in the uplink (and in the downlink) is $N \cdot X$.

A specific user is allocated one of the N sub-bands, and one of the X time-slots. A time-slot has duration 576.92 [μ s], and 148 binary symbols are transmitted within this



In 3G mobile telephony systems all users use the same time-interval and the same frequency-interval simultaneously!

CDMA is used instead of TDMA and FDMA.

Each user has a unique code.

In 4G, OFDM is used and users are assigned different sub-channels.

$$v(t) = \underbrace{a(t)B \cos(2\pi f_c t + \varphi)}_{\text{DSB-SC}} + \underbrace{C \cos(2\pi f_c t + \varphi)}_{\substack{\text{explicit carrier} \\ \text{frequency signal}}} \quad (3.46)$$

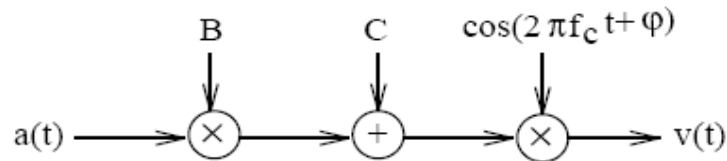


Figure 3.5: Generation of an AM signal $v(t)$.

$$\boxed{v(t) = (1 + ma_n(t))C \cos(2\pi f_c t + \varphi)}, \quad -\infty \leq t \leq \infty \quad (3.50)$$

AM: The information is in the envelope.

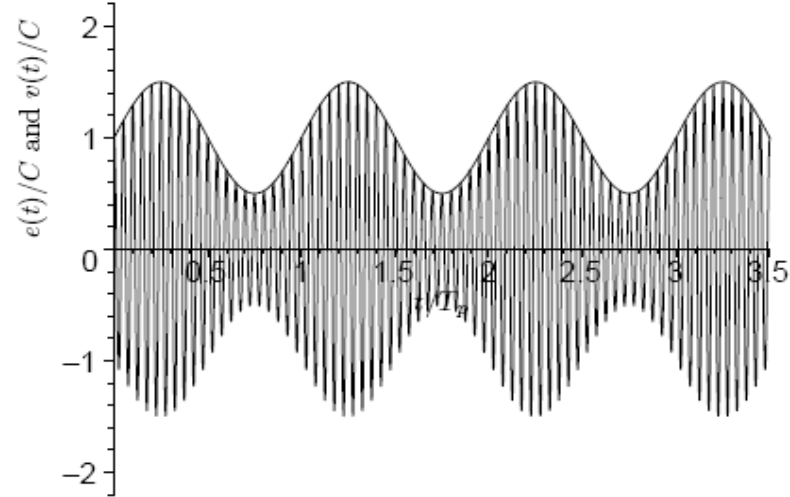


Figure 3.6: Shows $e(t)/C = 1 + \frac{1}{2} a_n(t)$, and the bandpass AM signal $v(t)/C$ for the case $m = 1/2$ and $a_n(t) = \sin(2\pi f_p t)$, $f_p = 1/T_p$.

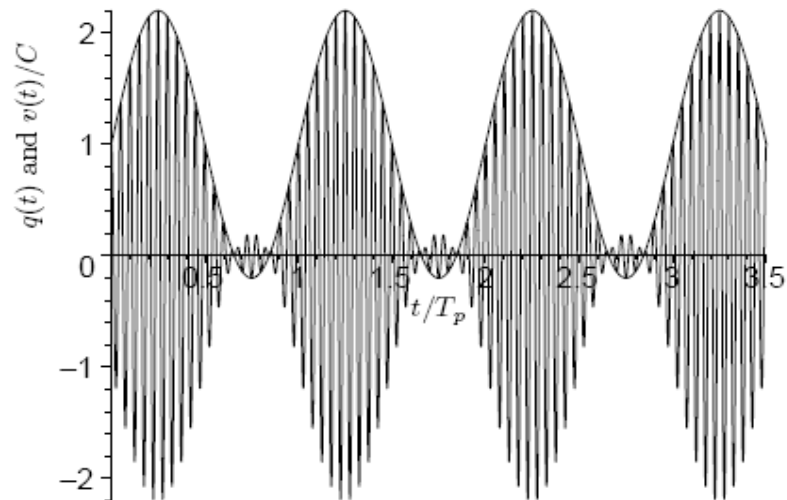


Figure 3.7: An example of an over-modulated AM signal $v(t)$. The expression for $v(t)$ is the same as in Figure 3.6, but the modulation index is here increased to $m = 1.2$. The baseband signal $q(t) = (1 + 1.2a_n(t))$ is also shown in this figure, and it is seen that $q(t)$ is **not** equal to the envelope.

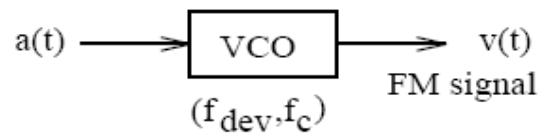


Figure 3.8: Conventional analog frequency modulation (FM).

$$\boxed{v(t) = \sqrt{2P} \cos(2\pi f_c t + \theta(t))}, \quad -\infty \leq t \leq \infty \quad (3.58)$$

A VCO is also useful to generate FSK and GSM signals!

The relationship between the information carrying input signal $a(t)$, and the instantaneous phase $\theta(t)$ is,

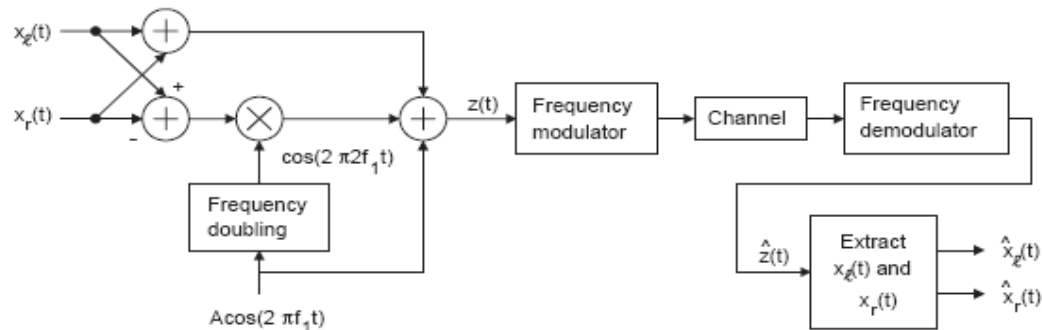
$$\frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_{dev} \cdot a(t) \quad (3.59)$$

$$\boxed{f_{ins}(t) = f_c + \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + f_{dev} a(t)} \quad (3.60)$$

So, the information signal $a(t)$ modulates the instantaneous frequency of the FM signal $v(t)$.

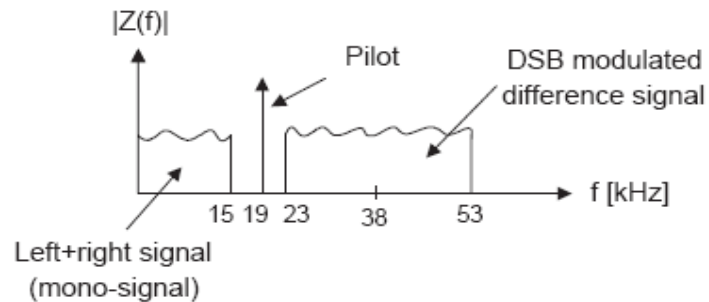
EXAMPLE 3.13

A possible block-diagram of conventional analog FM stereo is shown below.



$x_l(t)$ and $x_r(t)$ denotes the left and the right audio-channel, respectively, and they are both bandlimited to 15 [kHz]. The frequency $f_1 = 19$ [kHz] (often referred to as a so-called pilot-tone).

Interpretation in the frequency domain:



3.4.3 N-Ray Channel Model

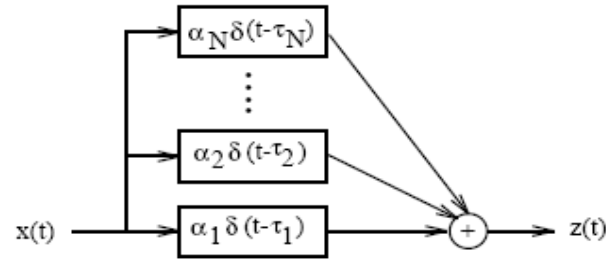


Figure 3.15: N -ray (multi-path) channel model.

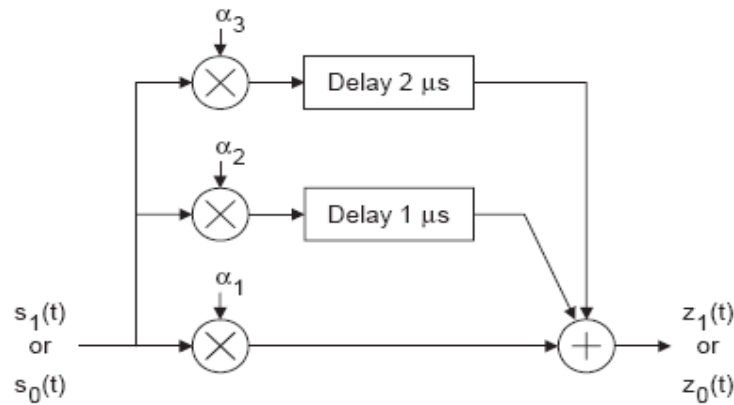
Common challenge in both Wireless and Wireline applications!

$$z(t) = x(t) * \underbrace{\left(\sum_{i=1}^N \alpha_i \delta(t - \tau_i) \right)}_{\text{Impulse response } h(t)} = \sum_{i=1}^N \alpha_i x(t - \tau_i) \quad (3.126)$$

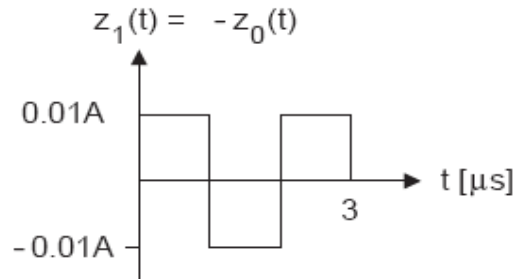
$$H(f) = \mathcal{F}\{h(t)\} = \sum_{i=1}^N \alpha_i e^{-j2\pi f \tau_i} \quad (3.128)$$

Remember the training bits in GSM!

EXAMPLE 3.19



$$s_1(t) = -s_0(t) = \begin{cases} A & , \quad 0 \leq t \leq 10^{-6} \\ 0 & , \quad \text{otherwise} \end{cases}$$



Observe that the signal alternatives are changed significantly by the channel (filtering), and that the duration of both signal alternatives is increased from 1 [μs] before the channel, to 3[μs] after the channel!

This will cause overlapping signals unless T_b is increased to 3 μs!

EXAMPLE 3.20

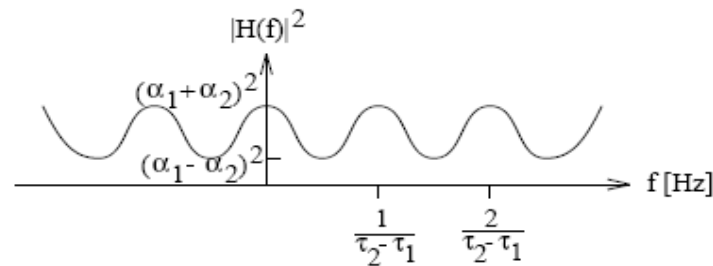
Calculate and sketch $|H(f)|^2$ for the 2-ray channel model.

Solution:

From (3.128) we obtain,

$$\begin{aligned} H(f) &= \alpha_1 e^{-j2\pi f\tau_1} + \alpha_2 e^{-j2\pi f\tau_2} = \\ &= e^{-j2\pi f\tau_1} \left(\alpha_1 + \alpha_2 e^{-j2\pi f(\tau_2 - \tau_1)} \right) \\ |H(f)|^2 &= \left(\alpha_1 + \alpha_2 e^{-j2\pi f(\tau_2 - \tau_1)} \right) \left(\alpha_1 + \alpha_2 e^{+j2\pi f(\tau_2 - \tau_1)} \right) = \\ &= \alpha_1^2 + \alpha_2^2 + \alpha_1 \alpha_2 \left(e^{j2\pi f(\tau_2 - \tau_1)} + e^{-j2\pi f(\tau_2 - \tau_1)} \right) = \\ &= \alpha_1^2 + \alpha_2^2 + 2\alpha_1 \alpha_2 \cos(2\pi f(\tau_2 - \tau_1)) \end{aligned}$$

Rough sketch:



It is seen in this figure that the two signal paths add constructively or destructively (fading) depending on the frequency. Furthermore, if $\alpha_1 \approx \alpha_2$ then $|H(f)|$ is very close to zero at certain frequencies (so-called deep fades)!

□

3.5.3 Noise

In almost all applications the input signal $r(t)$ to the receiver contains noise, implying a reduced communication performance (in terms of, e.g., bit error probability, and/or information bit rate). A typical situation is represented by,

$$r(t) = z(t) + N(t) \quad (3.143)$$

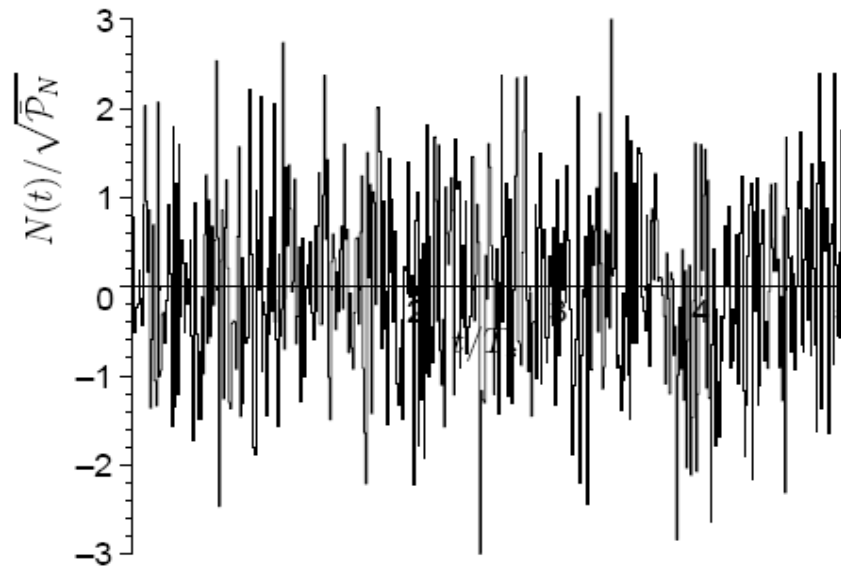
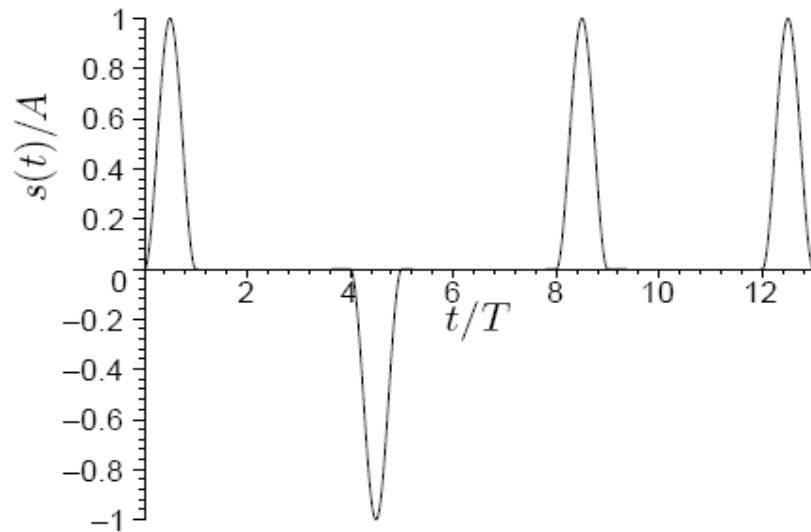
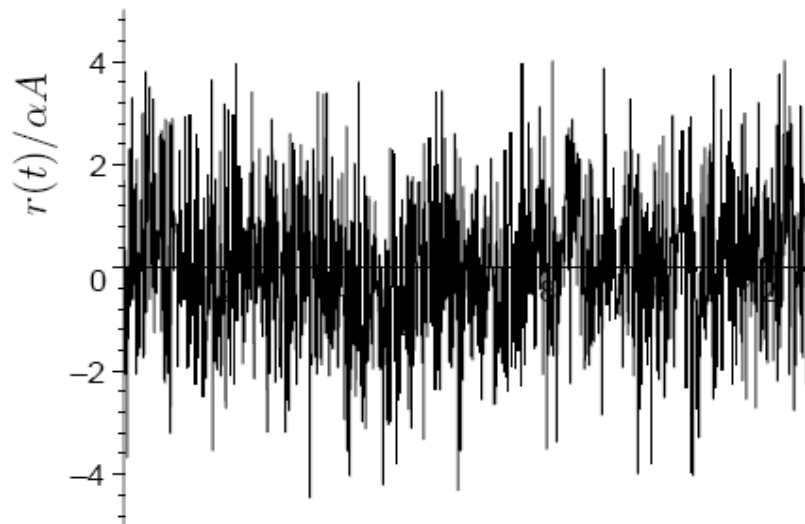


Figure 3.18: An example of noise $N(t)$ over the interval $0 \leq t \leq 5T_s$.

Sent:



Received:



How do we handle this situation?

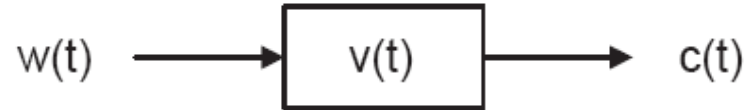
3.5.3.1 White Gaussian Noise

$$R_w(f) = \int_{-\infty}^{\infty} r_w(\tau) e^{-j2\pi f\tau} d\tau = \frac{N_0}{2}, \quad -\infty \leq f \leq \infty \quad (3.149)$$

White Gaussian noise is an abstract zero-mean random noise process with infinite average power, and infinite bandwidth.

All frequencies are equally disturbed.

What is the probability that the output noise is above a critical level A (“bit-error”)?



Gaussian probability distribution:

$$p(c) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(c-m)^2/2\sigma^2} \quad -\infty < c < \infty \quad (3.152)$$

We are often interested in the probability that the noise value $c(t_0)$ is larger than some given value A [Volt], i.e. $c(t_0) \geq A$, where A is a known value. This probability can be written as,

$$\Pr\{c(t_0) \geq A\} = \Pr\left\{\frac{c(t_0)-m}{\sigma} \geq \frac{A-m}{\sigma}\right\} = Q\left(\frac{A-m}{\sigma}\right) \quad (3.156)$$

Where the **$Q(x)$ -function** is defined as (see [68]),

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy = 1 - Q(-x) \quad (3.157)$$

$$Q(x) = \frac{1}{2} \operatorname{erfc}\left(x/\sqrt{2}\right) \quad (3.160)$$

Very useful
tables!

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.0	5.0000e-01	3.0	1.3499e-03	6.0	9.8659e-10	9.0	1.1286e-19
0.1	4.6017e-01	3.1	9.6760e-04	6.1	5.3034e-10	9.1	4.5166e-20
0.2	4.2074e-01	3.2	6.8714e-04	6.2	2.8232e-10	9.2	1.7897e-20
0.3	3.8209e-01	3.3	4.8342e-04	6.3	1.4882e-10	9.3	7.0223e-21
0.4	3.4458e-01	3.4	3.3693e-04	6.4	7.7688e-11	9.4	2.7282e-21
0.5	3.0854e-01	3.5	2.3263e-04	6.5	4.0160e-11	9.5	1.0495e-21
0.6	2.7425e-01	3.6	1.5911e-04	6.6	2.0558e-11	9.6	3.9972e-22
0.7	2.4196e-01	3.7	1.0780e-04	6.7	1.0421e-11	9.7	1.5075e-22
0.8	2.1186e-01	3.8	7.2348e-05	6.8	5.2310e-12	9.8	5.6293e-23
0.9	1.8406e-01	3.9	4.8096e-05	6.9	2.6001e-12	9.9	2.0814e-23
1.0	1.5866e-01	4.0	3.1671e-05	7.0	1.2798e-12	10.0	7.6199e-24
1.1	1.3567e-01	4.1	2.0658e-05	7.1	6.2378e-13		
1.2	1.1507e-01	4.2	1.3346e-05	7.2	3.0106e-13		
1.3	9.6800e-02	4.3	8.5399e-06	7.3	1.4388e-13		
1.4	8.0757e-02	4.4	5.4125e-06	7.4	6.8092e-14		
1.5	6.6807e-02	4.5	3.3977e-06	7.5	3.1909e-14		
1.6	5.4799e-02	4.6	2.1125e-06	7.6	1.4807e-14		
1.7	4.4565e-02	4.7	1.3008e-06	7.7	6.8033e-15		
1.8	3.5930e-02	4.8	7.9333e-07	7.8	3.0954e-15		
1.9	2.8717e-02	4.9	4.7918e-07	7.9	1.3945e-15		
2.0	2.2750e-02	5.0	2.8665e-07	8.0	6.2210e-16		
2.1	1.7864e-02	5.1	1.6983e-07	8.1	2.7480e-16		
2.2	1.3903e-02	5.2	9.9644e-08	8.2	1.2019e-16		
2.3	1.0724e-02	5.3	5.7901e-08	8.3	5.2056e-17		
2.4	8.1975e-03	5.4	3.3320e-08	8.4	2.2324e-17		
2.5	6.2097e-03	5.5	1.8990e-08	8.5	9.4795e-18		
2.6	4.6612e-03	5.6	1.0718e-08	8.6	3.9858e-18		
2.7	3.4670e-03	5.7	5.9904e-09	8.7	1.6594e-18		
2.8	2.5551e-03	5.8	3.3157e-09	8.8	6.8408e-19		
2.9	1.8658e-03	5.9	1.8175e-09	8.9	2.7923e-19		

Table 3.1: $Q(x)$ for some x in the interval $0 \leq x \leq 10$. In this table e-n means 10^{-n} . See also Figure 3.23 on page 183.

$Q(1.2816) \approx 10^{-1}$	$Q(5.1993) \approx 10^{-7}$
$Q(2.3263) \approx 10^{-2}$	$Q(5.6120) \approx 10^{-8}$
$Q(3.0902) \approx 10^{-3}$	$Q(5.9978) \approx 10^{-9}$
$Q(3.7190) \approx 10^{-4}$	$Q(6.3613) \approx 10^{-10}$
$Q(4.2649) \approx 10^{-5}$	$Q(6.7060) \approx 10^{-11}$
$Q(4.7534) \approx 10^{-6}$	$Q(7.0345) \approx 10^{-12}$

Table 3.2: Some specific $Q(x)$ values.

Q(x) versus x

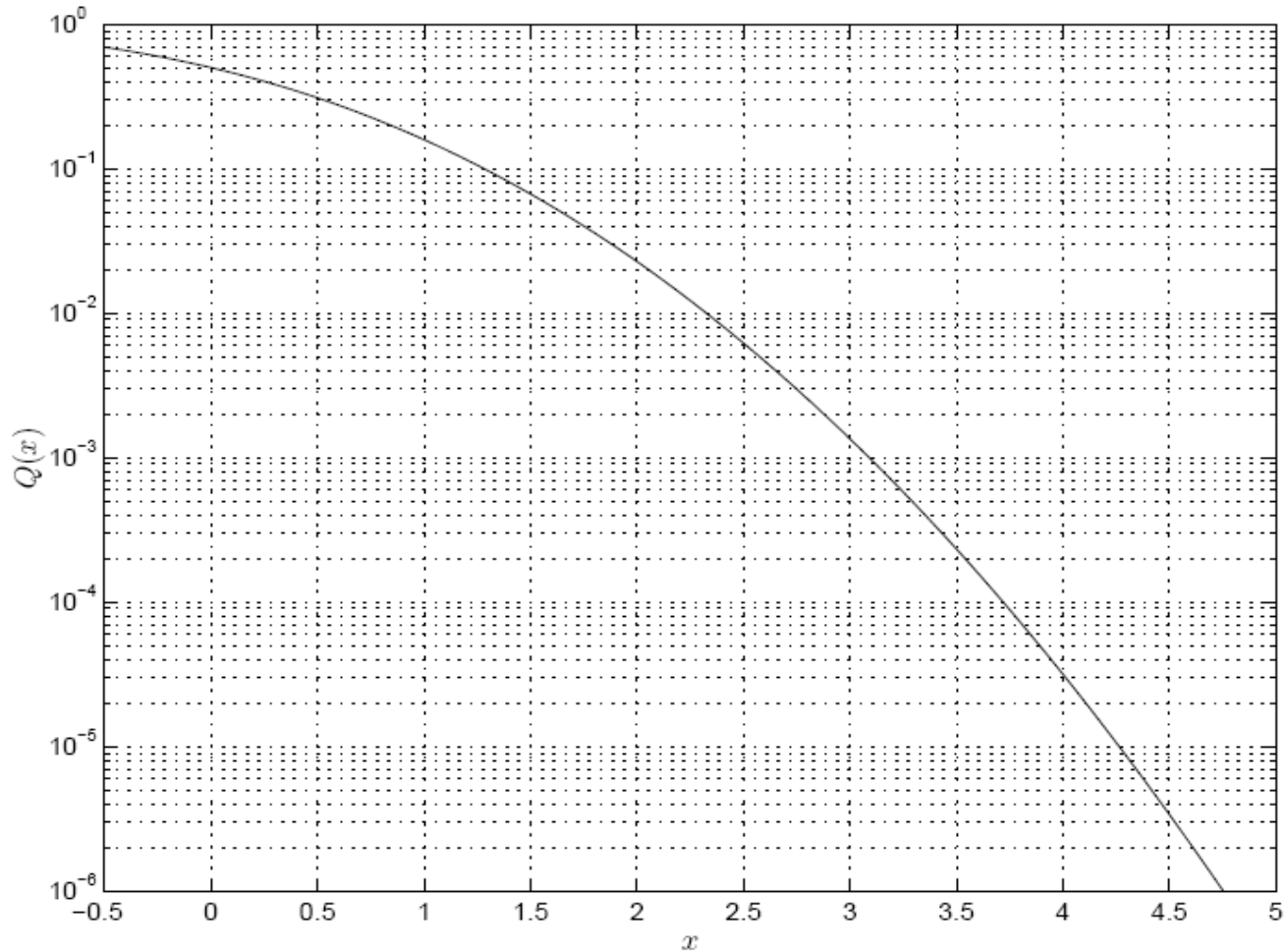
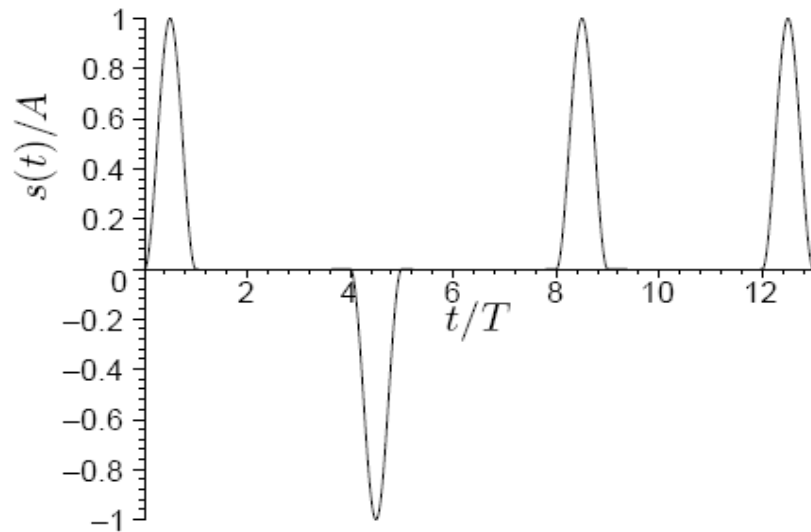
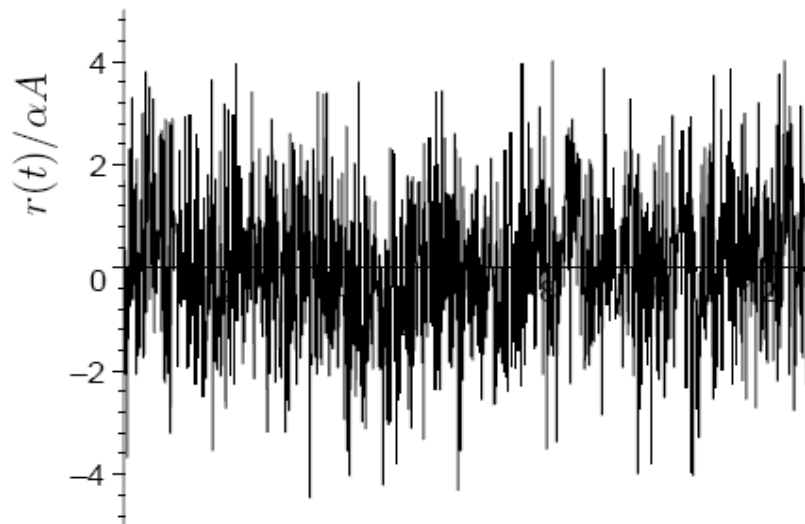


Figure 3.23: $Q(x)$ versus x . $Q(x)$ is defined in (3.157). See also (3.160). $Q(x)$ is tabulated in Tables 3.1–3.2 on page 182.

Sent:



Received:



How can we find the sent information bits?

$P_b = ?$

Chapter 4

Receivers in Digital Communication Systems – Part I

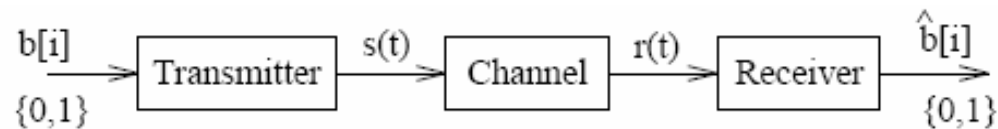


Figure 4.1: A digital communication system.

When the receiver tries to extract these B information bits from the received noisy signal $r(t)$, it will occasionally produce some bits which are in error. Denote by B_{err} the total number of bit errors made by the receiver,

$$B_{err} = d_H(\mathbf{b}, \hat{\mathbf{b}}) \quad (4.2)$$

$d_H(\mathbf{b}, \hat{\mathbf{b}})$ denotes the so-called Hamming distance ([43], [26]) between the two binary sequences $b[i]$ and $\hat{b}[i]$. The Hamming distance equals the total number of positions where the two binary sequences are **unequal**.

The **bit error probability** is defined as the average number of information bit errors per detected information bit,

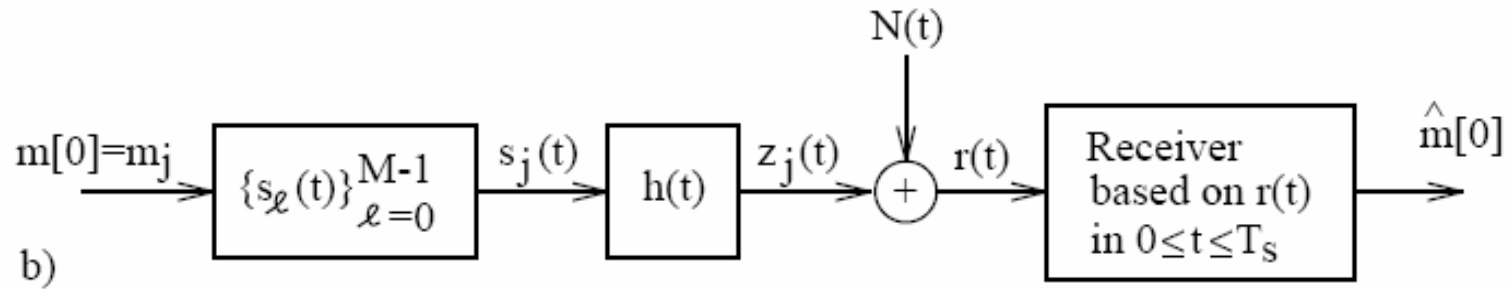
$$\boxed{P_b = \frac{E\{d_H(\mathbf{b}, \hat{\mathbf{b}})\}}{B}} \quad (4.3)$$

where $E\{ \}$ denotes the expected value.

(The same definition as in (2.12))

Sent message (k bits):

Decided
message:



- *How are decisions made in the receiver?*
- *How large is the bit error probability of the receiver?*

How is your decision strategy?

The disturbed image on the black-board: Is it a house or a boat?

How is your decision strategy?

The disturbed image on the black-board: Is it a house or a boat?

Optimal decision strategy!

of an erroneous decision. Consequently, the receiver should be designed such that the probability of a correct decision,

$$\boxed{\Pr\{m = \hat{m}(r(t))|r(t)\}} \quad (4.13)$$

is maximized. The received signal $r(t)$ is observed over the symbol interval

Since the decided message must be $0, 1, 2, 3, \dots, (M-2)$ or $(M-1)$ we test all cases in equation (4.13).

Hence, the decision rule that minimizes the symbol error probability P_s for the receiver in Figure 4.2b on page 229 is,

$$\hat{m}(r(t)) = m_\ell \Leftrightarrow \max_{\{i\}} \Pr\{m=m_i|r(t)\} = \Pr\{m=m_\ell|r(t)\} \quad (4.14)$$

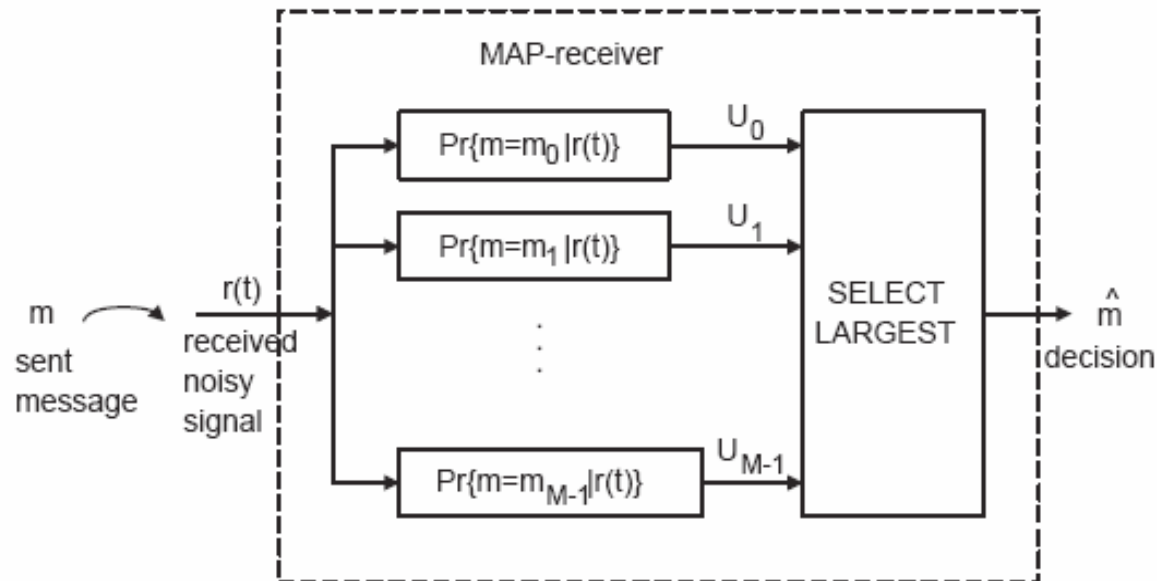
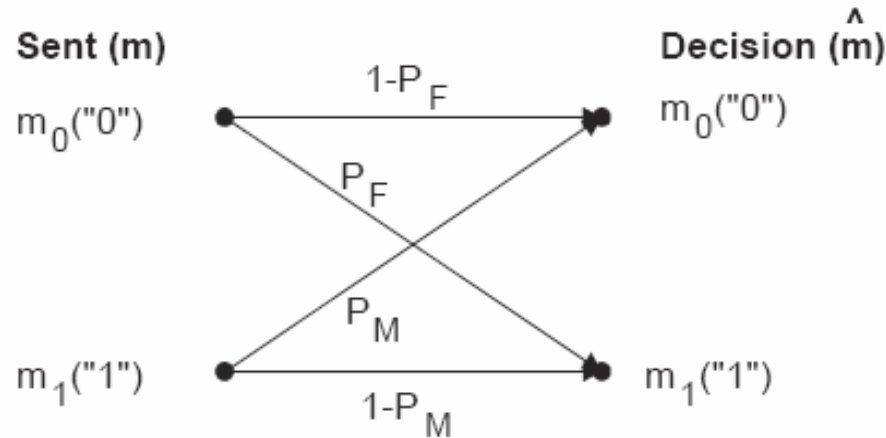


Figure 4.3: The structure of the general MAP receiver, compare with (4.14).

Note the importance of the MAP receiver; it shows how to implement a receiver that achieves the lowest possible symbol error probability. Consequently, it is impossible, by any means, to obtain a smaller symbol error probability. This knowledge is of great practical and theoretical importance.

Furthermore, we can quite easy implement the optimal receiver!

4.2.3.1 Binary Signaling ($M = 2$)



Probability of “false alarm”: “0” is sent but the receiver decides “1”.
Probability of a “miss” : “1” is sent but the receiver decides “0”.

$$\boxed{P_b = P_0 P_F + P_1 P_M} \quad (4.31)$$

4.3 The Minimum Euclidean Distance Receiver

This receiver is optimal if the signal alternatives are equally likely. The received signal is compared with all noise-free signal alternatives. *That is why the channel must be known to the receiver!*

$$\begin{aligned} D_{r,i}^2 &= \int_0^{T_s} (r(t) - z_i(t))^2 dt = \int_0^{T_s} (r^2(t) - 2r(t)z_i(t) + z_i^2(t)) dt = \\ &= E_r - 2 \int_0^{T_s} r(t)z_i(t) dt + E_i, \quad i = 0, 1, \dots, M - 1 \end{aligned} \quad (4.32)$$

The minimum Euclidean distance receiver is defined by the decision rule:

$$\begin{aligned} \text{Decision } \hat{m} = m_\ell \quad \Leftrightarrow \quad \min_{\{i\}} D_{r,i}^2 = D_{r,\ell}^2 \end{aligned} \quad (4.33)$$

\Downarrow
 \Uparrow

$$\max_{\{i\}} \left\{ \underbrace{\int_0^{T_s} r(t)z_i(t) dt - E_i/2}_{\xi_i} \right\} = \int_0^{T_s} r(t)z_\ell(t) dt - \frac{E_\ell}{2}$$

$$\max_{\{i\}} \left\{ \underbrace{\int_0^{T_s} r(t)z_i(t)dt - E_i/2}_{\xi_i} \right\} = \int_0^{T_s} r(t)z_\ell(t)dt - \frac{E_\ell}{2}$$

“Correlation receiver”:

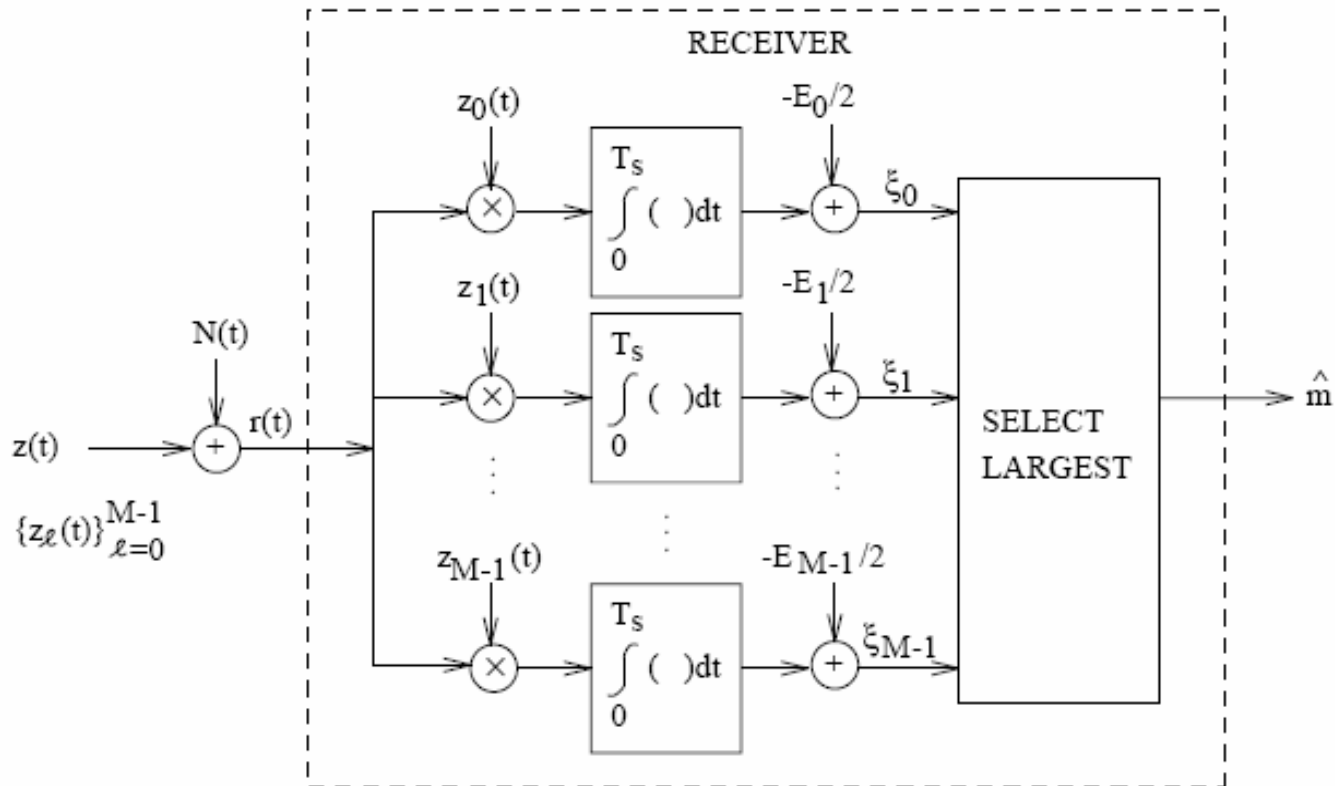


Figure 4.8: The minimum Euclidean distance receiver, see (4.33).

EXAMPLE 4.4

Assume that $\{z_\ell(t)\}_{\ell=0}^{M-1}$ is a 64 -ary QAM signal constellation. Draw a block-diagram of a minimum Euclidean distance receiver that uses only two integrators.

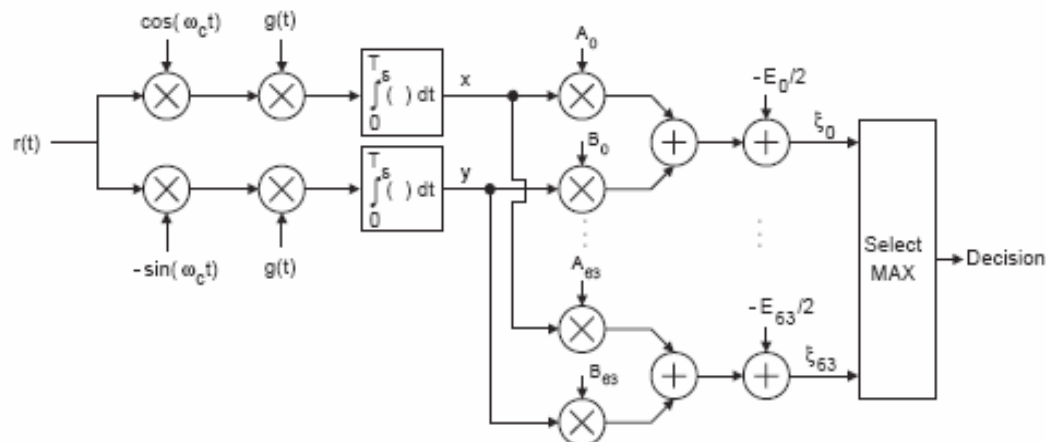
Solution:

A QAM signal alternative can be written as $z_i(t) = A_i g(t) \cos(\omega_c t) - B_i g(t) \sin(\omega_c t)$, where $g(t)$ is a baseband pulse. The output value from the i :th correlator in Figure 4.8 is,

$$\begin{aligned} \int_0^{T_s} r(t) z_i(t) dt &= A_i \underbrace{\int_0^{T_s} r(t) g(t) \cos(\omega_c t) dt}_x - B_i \underbrace{\int_0^{T_s} r(t) g(t) \sin(\omega_c t) dt}_{-y} = \\ &= A_i x + B_i y \end{aligned}$$

Observe that x and y do not depend on the index i .

Hence, a possible implementation of the receiver is to **first** generate x and y , and then calculate the M correlations $A_i x + B_i y$, $i = 0, 1, \dots, M-1$. By subtracting the value $E_i/2$ from the i :th correlation, the decision variables ξ_0, \dots, ξ_{M-1} are finally obtained. The implementation of this receiver is shown below:



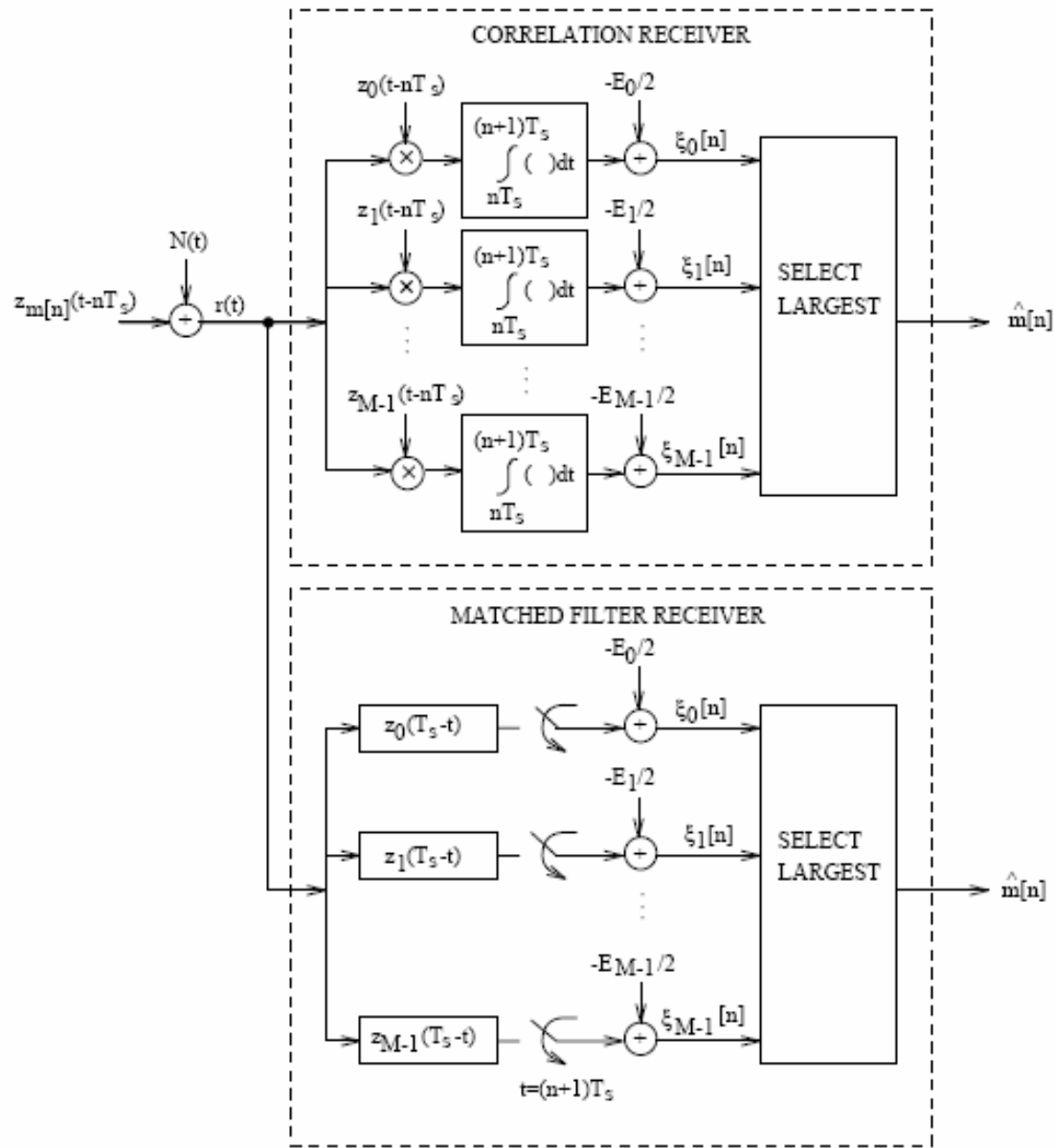


Figure 4.9: Two implementations of the minimum Euclidean distance symbol-by-symbol receiver: the correlation receiver (upper), and the matched filter receiver (lower). Note that units for synchronization are not included in this figure.

A short summary of some important parameters/concepts in this course during **study-week 3:**

Chapter 3:

Study the implementation of bandpass signals shown in Figure 3.2 on page 119.

I & Q – description, and envelope & phase – description is found on page 125.

It is instructive to take a closer look at Example 3.7 on pages 135-136.

Analog AM is described on pages 139-142.

Analog FM is described on pages 147-148. Example 3.13 illustrates analog FM stereo.

The N-ray (or multi-path) channel model is defined and described on pages 167-170. This channel model is very important, so study this channel model in detail.

Additive White Gaussian Noise (AWGN) is defined on page 178. As we will see in chapter 4, this noise implies that the bit error probability follows the Q()-function in a special way.

The Q(x)-function is defined in equ. (3.157) on page 180, and it is tabulated on page 182.

Chapter 4:

Figure 4.5 illustrates “false alarm” and “miss” in the binary signaling case, and the corresponding bit error probability is given in equ. (4.31).

The minimum Euclidean distance receiver is defined in sub-section 4.3, and the correlator-based implementation is shown in Figure 4.8 on page 241. This receiver is also referred to as the ML receiver since it can be shown that the minimum Euclidean distance receiver is the optimal receiver in case of equally likely signal alternatives.