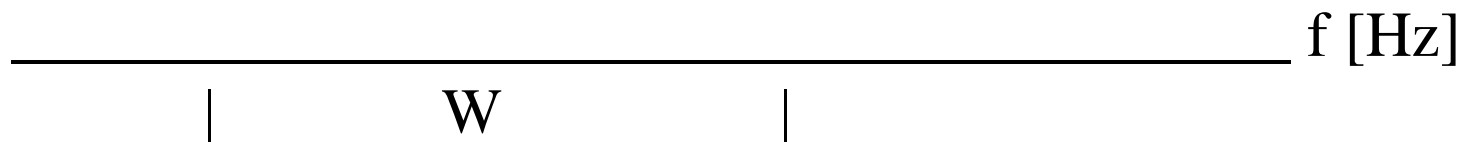


2.5 The Bandwidth of the Transmitted Signal

The bandwidth W of a signal is the width of the frequency interval where most of the signal energy (or power) is located.

The bandwidth W is measured on the positive frequency axis.

We must have control over the bandwidth consumption of the transmitted signal!



**How large is the bandwidth W [Hz]
for a given information bit rate R_b [bps]?**

The Fourier transform of a signal alternative shows the frequency content of the signal alternative.

Hence, the Fourier transform is the tool we need to obtain practical bandwidth results.

Parseval's relation (see e.g. [4], [18], [22]) states that,

$$\boxed{E_x = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} |X(f)|^2 df} \quad (2.150)$$

$$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt = X_{Re}(f) + jX_{Im}(f) = |X(f)|e^{j\varphi(f)}$$

—

(2.143)

$$x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft} df \quad (2.147)$$

$$x_{rec}(t) = \begin{cases} A & , \quad -T/2 \leq t \leq T/2 \\ 0 & , \quad \text{otherwise} \end{cases} \quad (2.151)$$

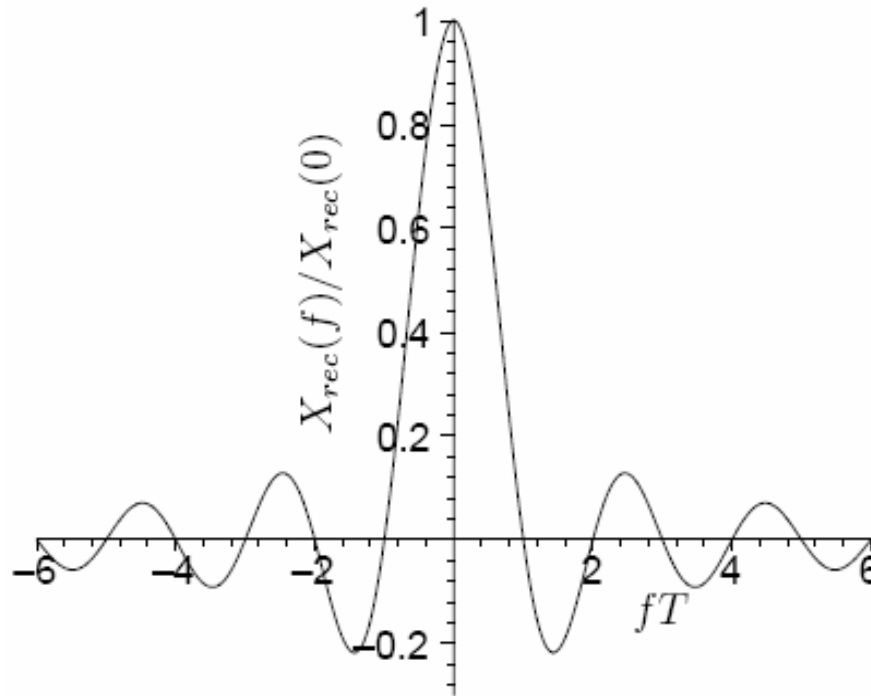
Straight-forward calculations, according to (2.143), give the Fourier transform, denoted $X_{rec}(f)$,

$$\begin{aligned} X_{rec}(f) &= \mathcal{F}\{x_{rec}(t)\} = \int_{-\infty}^{\infty} x_{rec}(t)e^{-j2\pi ft} dt = \\ &= \int_{-T/2}^{T/2} Ae^{-j2\pi ft} dt = \left[-\frac{Ae^{-j2\pi ft}}{j2\pi f} \right]_{-T/2}^{T/2} = \frac{A}{\pi f} \cdot \frac{e^{j\pi fT} - e^{-j\pi fT}}{2j} = \\ &= AT \frac{\sin(\pi fT)}{\pi fT} \end{aligned} \quad (2.152)$$

Hence, we have found that,

$$\boxed{x_{rec}(t) \longleftrightarrow X_{rec}(f) = \mathcal{F}\{x_{rec}(t)\} = AT \frac{\sin(\pi fT)}{\pi fT} = AT \text{sinc}(fT)} \quad (2.153)$$

EXAMPLE 2.17



Baseband (low-frequency) spectrum.

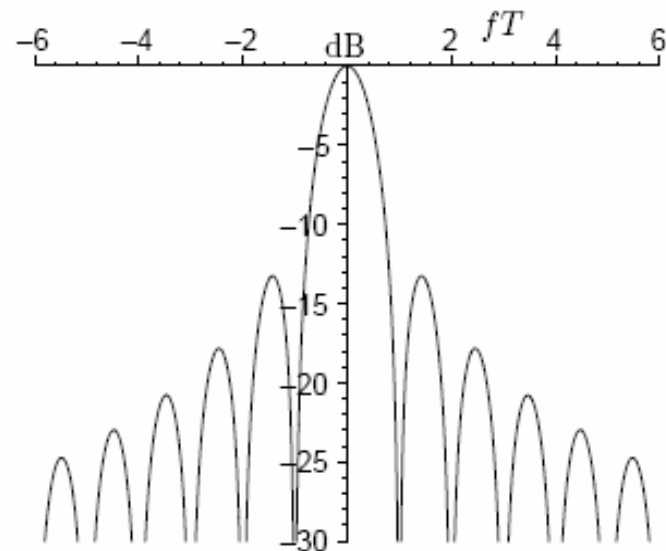
Main-lobe + several **side-lobes**.

Most of the signal energy is contained within the main-lobe!!

Normalized frequency $fT=2$ means that the true frequency $f=2/T$ [Hz].

$$|X_{rec}(f)|^2 = A^2 T^2 \left(\frac{\sin(\pi f T)}{\pi f T} \right)^2 = E_x T \left(\frac{\sin(\pi f T)}{\pi f T} \right)^2$$

$$10 \log_{10}(|X_{rec}(f)|^2 / E_x T) = 10 \log_{10} \left(\left(\frac{\sin(\pi f T)}{\pi f T} \right)^2 \right)$$



Main-lobe
Side-lobes

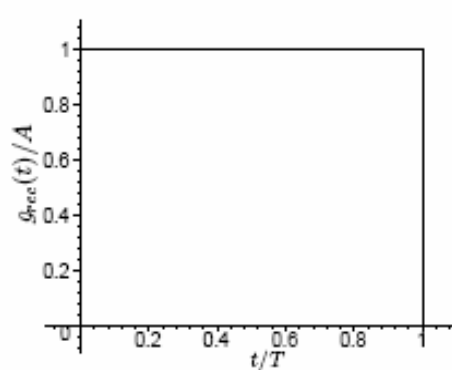


Figure D.1: $g_{rec}(t)/A$.

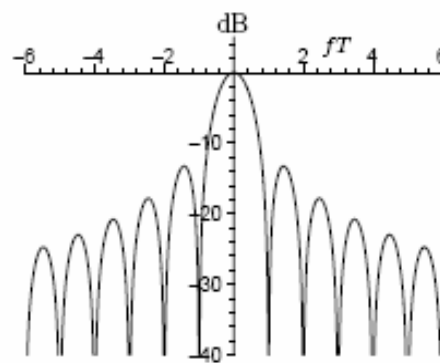


Figure D.2: $\frac{|G_{rec}(f)|^2}{E_g T}$ in dB.

1. The rectangular pulse:

$$g_{rec}(t) = \begin{cases} A & , \quad 0 \leq t \leq T \\ 0 & , \quad \text{otherwise} \end{cases} \quad (\text{D.1})$$

$$E_g = \int_0^T g_{rec}^2(t) dt = \int_{-\infty}^{\infty} |G_{rec}(f)|^2 df = A^2 T \quad (\text{D.2})$$

The Fourier transform of $g_{rec}(t)$ is denoted $G_{rec}(f)$,

$$G_{rec}(f) = \mathcal{F}\{g_{rec}(t)\} = AT \frac{\sin(\pi f T)}{\pi f T} e^{-j\pi f T} \quad (\text{D.3})$$

Observe that $G_{rec}(n/T) = 0$ if $n = \pm 1, \pm 2, \pm 3, \dots$

$$|G_{rec}(f)|^2 = A^2 T^2 \left(\frac{\sin(\pi f T)}{\pi f T} \right)^2 \quad (\text{D.4})$$

Figure D.1 shows $g_{rec}(t)/A$ versus t . Note the labeling on the time axis: 0.6, 0.8 and 1 means that $t = 0.6T$, $t = 0.8T$ and $t = T$, respectively.

Figure D.2 shows $10 \log_{10}(|G_{rec}(f)|^2/E_g T)$ versus f . Note the labeling on the frequency axis: ± 2 , ± 4 , etc. means the frequencies $f = \pm 2/T$, $f = \pm 4/T$, etc.

In communication applications we differ between non-return-to-zero format (NRZ-format) and return-to-zero format (RZ-format) depending on whether the pulse occupies the whole symbol interval ($T = T_s$) or not ($T < T_s$).

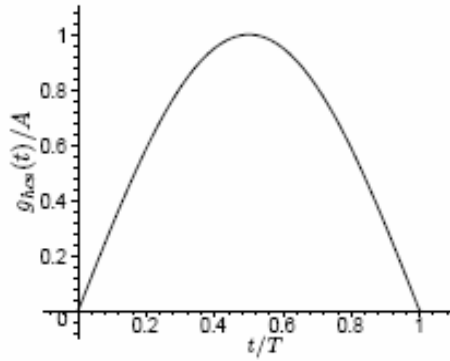


Figure D.7: $g_{hcs}(t)/A$.

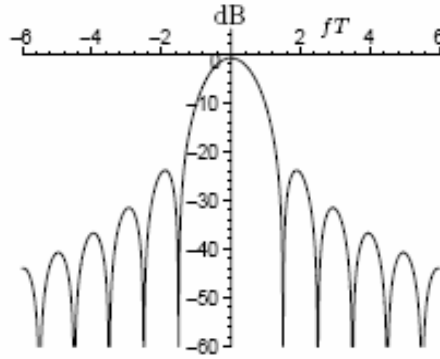


Figure D.8: $\frac{|G_{hcs}(f)|^2}{E_g^2}$ in dB.

4. The half cycle sinusoidal pulse:

$$g_{hcs}(t) = \begin{cases} A \sin(\pi t/T) & , \quad 0 \leq t \leq T \\ 0 & , \quad \text{otherwise} \end{cases} \quad (\text{D.13})$$

$$E_g = A^2 T / 2 \quad (\text{D.14})$$

The Fourier transform of $g_{hcs}(t)$ is denoted $G_{hcs}(f)$,

$$G_{hcs}(f) = \mathcal{F}\{g_{hcs}(t)\} = \frac{2AT}{\pi} \frac{\cos(\pi fT)}{1 - (2fT)^2} e^{-j\pi fT} \quad (\text{D.15})$$

$$G_{hcs}(f = \pm 1/2T) = \mp jAT/2 \quad (\text{D.16})$$

Observe that $G_{hcs}(n/T) = 0$ if $n = \pm 3/2, \pm 5/2, \pm 7/2, \dots$

$$|G_{hcs}(f)|^2 = \frac{4A^2 T^2}{\pi^2} \left(\frac{\cos(\pi fT)}{1 - (2fT)^2} \right)^2 \quad (\text{D.17})$$

Figure D.7 shows $g_{hcs}(t)/A$ versus t . Note the labeling on the time axis: 0.6, 0.8 and 1 means that $t = 0.6T$, $t = 0.8T$ and $t = T$, respectively.

Figure D.8 shows $10 \log_{10}(|G_{hcs}(f)|^2/E_g^2)$ versus f . Note the labeling on the frequency axis: $\pm 2, \pm 4$, etc. means the frequencies $f = \pm 2/T$, $f = \pm 4/T$, etc.

2.5.1.1 Frequency Shift Operations

$$\begin{aligned} g(t) \cos(2\pi f_c t) &\longleftrightarrow \frac{1}{2} (G(f + f_c) + G(f - f_c)) \\ g(t) \sin(2\pi f_c t) &\longleftrightarrow j\frac{1}{2} (G(f + f_c) - G(f - f_c)) \\ g(t)e^{j2\pi f_c t} &\longleftrightarrow G(f - f_c) \end{aligned} \quad (2.157)$$

$G(f+f_c)$ is left shift of $G(f)$.

$G(f-f_c)$ is right shift of $G(f)$.

This means that we can move our signals on the frequency axis to a suitable location, by changing the carrier frequency only!

$$x(t) = \underbrace{g_{rc}(t + T/2)}_{g(t)} \cdot \cos(2\pi f_c t) \quad (2.161)$$

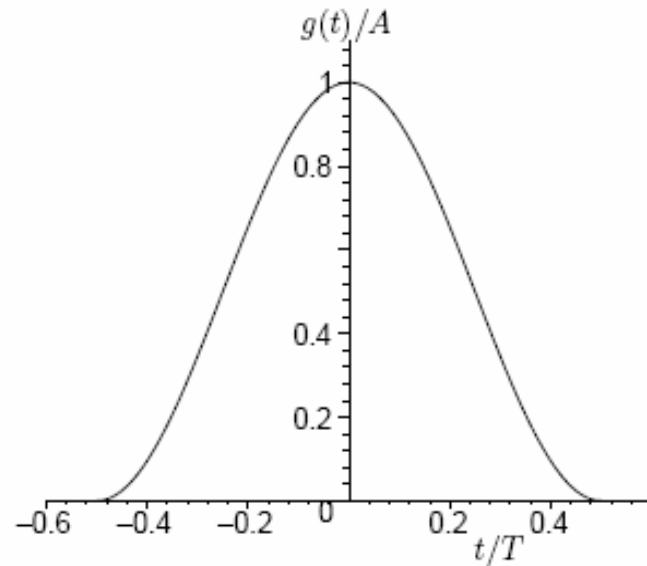


Figure 2.15: The baseband pulse $g(t)/A$ in (2.161).

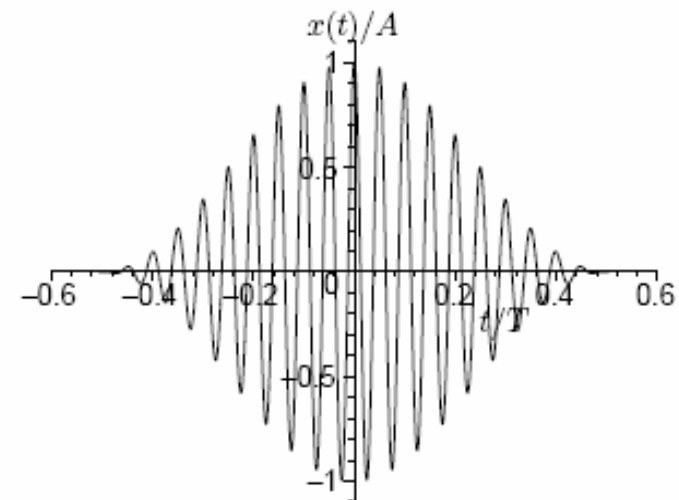


Figure 2.16: The bandpass signal $x(t)/A$ for the case $f_c = 20/T$, see (2.161).

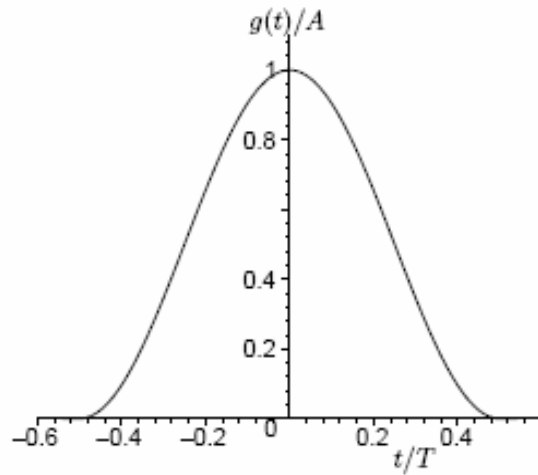


Figure 2.15: The baseband pulse $g(t)/A$ in (2.161).

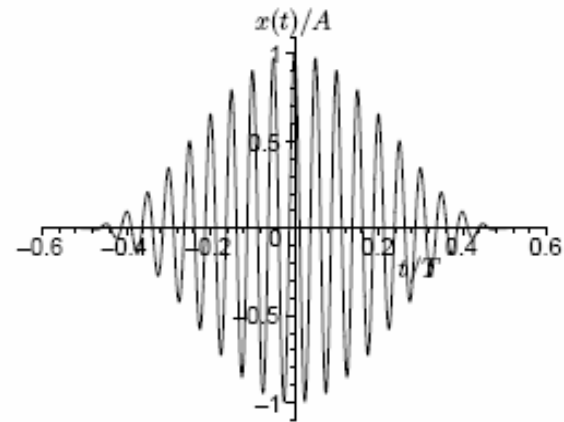


Figure 2.16: The bandpass signal $x(t)/A$ for the case $f_c = 20/T$, see (2.161).

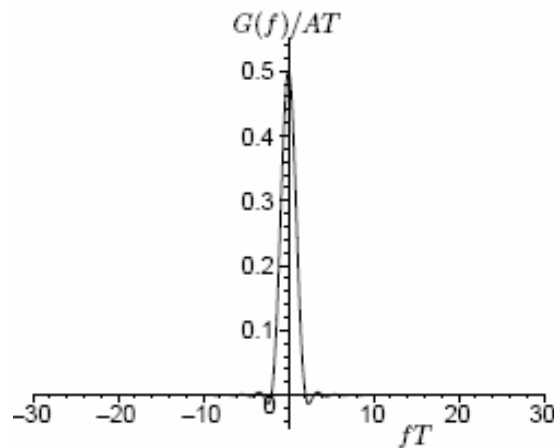


Figure 2.17: $\frac{G(f)}{AT}$, see (2.162).

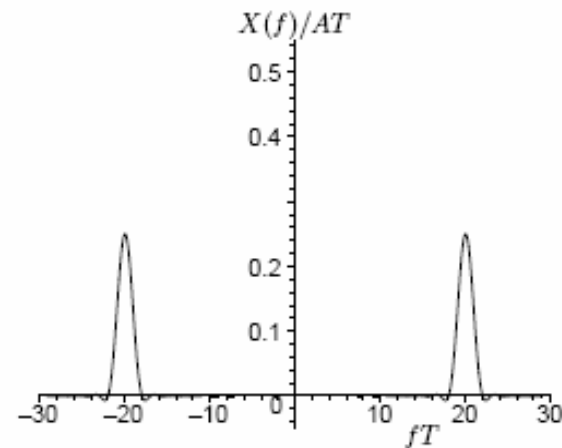


Figure 2.18: $\frac{X(f)}{AT}$ for the case $f_c = 20/T$, see (2.157).

A complication:

The transmitted signal consists of *a sequence of random signal alternatives*, since the underlying information bit stream is random.

What is the bandwidth W [Hz] of the transmitted signal?

2.5.2 The Power Spectral Density Function $R(f)$: M-ary Digital Transmission

The average transmitted signal power is \bar{P} [V^2], and an extremely important property of the power spectral density function $R(f)$ [V^2/Hz] is that,

$$\boxed{\bar{P} = \bar{E}_b R_b = \int_{-\infty}^{\infty} R(f) df} \quad (2.168)$$

Hence, the power spectral density function $R(f)$ shows how the average signal power \bar{P} is distributed along the frequency axis on the average.

Hence, once $R(f)$ is found, the bandwidth can be calculated.

Consequently, we need to find $R(f)$ for a given M and a given set of M signal alternatives.

Most of the average signal power P will be contained within the main-lobe of R(f)!!

2.5.2.2 Summary of results

The result is,

$$\boxed{R(f) = R_c(f) + R_d(f)} \quad (2.202)$$

where,

$$\begin{aligned} R_c(f) &= \frac{1}{T_s} \sum_{n=0}^{M-1} P_n |S_n(f) - A(f)|^2 = \frac{1}{T_s} \sum_{n=0}^{M-1} P_n |S_n(f)|^2 - \frac{|A(f)|^2}{T_s} = \\ &= \mathcal{F} \left\{ \frac{1}{T_s} \sum_{n=0}^{M-1} P_n [s_n(t) * s_n(-t)] - \frac{a(t) * a(-t)}{T_s} \right\} \\ R_d(f) &= \frac{|A(f)|^2}{T_s^2} \sum_{n=-\infty}^{\infty} \delta(f - n/T_s) \end{aligned} \quad (2.203)$$

With these **general results** we can also analyze the consequences in the frequency domain of technical errors in the transmitter.

Observe from (2.203) that **if** the average signal $a(t) = 0$ for all t , then

$$R(f) = R_c(f) = R_s \sum_{n=0}^{M-1} P_n |S_n(f)|^2 = R_s E\{|S_{m[n]}(f)|^2\} \quad (2.205)$$

However, in this course we will study special cases that are used in practice, instead of the general result.

2.5.3 $R(f)$: Binary Signaling

With only two signal alternatives ($M = 2$), the expression for the power spectral density $R(f)$, in (2.202–2.203), is significantly simplified. The result is,

$$R(f) = \underbrace{\frac{P_0 P_1}{T_b} |S_0(f) - S_1(f)|^2}_{R_c(f)} + \underbrace{\frac{|P_0 S_0(f) + P_1 S_1(f)|^2}{T_b^2} \sum_{n=-\infty}^{\infty} \delta(f - n/T_b)}_{R_d(f)} \quad (2.206)$$

and this is shown in Example 2.20 below. *The reader is strongly recommended to carefully study the basic and important special cases in Examples 2.21, 2.23 and in Example 2.24 below!*

Note!

With general formulas we can also investigate the consequences of malfunctions at the transmitter!

EXAMPLE 2.21

Assume equally likely antipodal signal alternatives, such that

$$s_1(t) = -s_0(t) = g(t)$$

where $g(t) = g_{rec}(t)$, and $g_{rec}(t)$ is given in (D.1). Assume also that $T \leq T_b$.

- i) Calculate the power spectral density $R(f)$.
- ii) Calculate the bandwidth W defined as the one-sided width of the mainlobe of $R(f)$, if the information bit rate is 10 [kbps], and if $T = T_b/2$.
Calculate also the bandwidth efficiency ρ .
- iii) Estimate the attenuation in dB of the first sidelobe of $R(f)$ compared to $R(0)$.

Solution:

From (2.206) we obtain for equally likely antipodal signals that ($P_0 = P_1 = 1/2$, $S_1(f) = -S_0(f) = G(f)$),

$$R(f) = R_b |S_1(f)|^2 = R_b |S_0(f)|^2 = R_b |G(f)|^2$$

Useful for: M=2 and equally likely antipodal signals!

Remember Appendix D! Main-lobe and side-lobes!

EXAMPLE 2.23

Assume equally likely antipodal signal alternatives below. Assume that $s_1(t) = -s_0(t) = g_{rc}(t)$, where the time raised cosine pulse $g_{rc}(t)$ is defined in (D.18). Assume also that $T = T_b$.

Find an expression for the power spectral density $R(f)$. Calculate the bandwidth W , defined as the one-sided width of the mainlobe of $R(f)$, if R_b is 10 [kbps]. Calculate also the bandwidth efficiency ρ .

Solution:

In the same way as in Example 2.21 we obtain from (2.206) that,

$$R(f) = R_b |G_{rc}(f)|^2 = R_b E_g T \cdot \underbrace{\frac{|G_{rc}(f)|^2}{E_g T}}_{\text{Figure D.10}}$$

From (D.22), or from Figure D.10 we find that the one-sided width of the mainlobe is,

$$\boxed{W = 2/T \text{ [Hz]}}$$

Hence, $W = 2/T = 2/T_b = 2 \cdot 10^4 = 20$ [kHz].

EXAMPLE 2.24**Bandpass case.**

Assume $P_0 = P_1$ and that,

$$s_1(t) = -s_0(t) = g_{rc}(t) \cos(2\pi f_c t)$$

with $T = T_b$, and $f_c \gg 1/T$. Hence, a version of binary PSK signaling is considered here (alternatively binary antipodal bandpass PAM). Calculate the bandwidth W , defined as the double-sided width of the mainlobe around the carrier frequency f_c . Assume that the information bit rate is 10 [kbps]. Calculate also the bandwidth

Solution:

We may here consider the high-frequency pulse $g_{hf}(t)$,

$$g_{hf}(t) = g_{rc}(t) \cos(2\pi f_c t)$$

Then $s_1(t) = -s_0(t) = g_{hf}(t)$, and

$$R(f) = R_b |G_{hf}(f)|^2$$

So, the problem is to find the Fouriertransform $G_{hf}(f) = \mathcal{F}\{g_{hf}(t)\}$. This is treated in Subsection 2.5.1.1 (frequency shift operations), see especially (2.157) and also compare with Figures 2.17–2.18 on page 69. From (2.157) we obtain,

$$R(f) = R_b \left| \frac{G_{rc}(f + f_c)}{2} + \frac{G_{rc}(f - f_c)}{2} \right|^2$$

and the bandwidth W around the carrier frequency f_c is in this bandpass case therefore twice as large as in Example 2.23 (the low-pass case),

$$\boxed{W = 4/T \text{ [Hz]}}$$

VERY USEFUL!

Pulse shape	W_{lobe}	% power in W_{lobe}	W_{90}	W_{99}	$W_{99.9}$	Asymptotic decay
rec	$2/T$	90.3	$1.70/T$	$20.6/T$	$204/T$	f^{-2}
tri	$4/T$	99.7	$1.70/T$	$2.60/T$	$6.24/T$	f^{-4}
hcs	$3/T$	99.5	$1.56/T$	$2.36/T$	$5.48/T$	f^{-4}
rc	$4/T$	99.95	$1.90/T$	$2.82/T$	$3.46/T$	f^{-6}
Nyquist	R_s	100	$0.9R_s$	$0.99R_s$	$0.999R_s$	ideal

Table 2.1: Double-sided bandwidth results for power spectral densities according to (2.212). The $g_{rec}(t)$, $g_{tri}(t)$, $g_{hcs}(t)$ and $g_{rc}(t)$ pulse shapes are defined in Appendix D, and T denotes the duration of the pulse. The Nyquist pulse shape is not limited in time and it is defined in (D.49) with parameters $\beta = 0$ and $T = T_s$.

2.5.5 $R(f)$: M-ary PAM Signals

With M-ary PAM signaling, the signal alternatives $s_\ell(t)$ are, see (2.38),

$$s_\ell(t) = A_\ell g(t), \quad \ell = 0, 1, \dots, M - 1 \quad (2.215)$$

$$R(f) = \underbrace{\frac{\sigma_A^2}{T_s} |G(f)|^2}_{R_c(f)} + \underbrace{\frac{m_A^2}{T_s^2} |G(f)|^2 \sum_{n=-\infty}^{\infty} \delta(f - n/T_s)}_{R_d(f)} \quad (2.216)$$

If the average amplitude $m_A = 0$, then we obtain from (2.216) and (2.218) that $\bar{P} = \sigma_A^2 E_g R_s$ and,

$$R(f) = R_c(f) = \frac{\sigma_A^2}{T_s} |G(f)|^2 = \bar{P} \frac{|G(f)|^2}{E_g} \quad (2.219)$$

Consequently, Table 2.1 can be used also in this M-ary PAM case!

Saving Bandwidth!

EXAMPLE 2.28

Assume the bit rate $R_b = 9600$ [bps], M -ary PAM transmission and that $m_A = 0$. Determine the (baseband) bandwidth W , defined as the one-sided width of the mainlobe of the power spectral density $R(f)$, if $M = 2$, $M = 4$ and $M = 8$, respectively. Furthermore, assume a rectangular pulse shape with amplitude A_g , and duration $T = T_s$. Calculate also the bandwidth efficiency ρ .

Solution:

It is given that $T = T_s$. From (2.219) and Example 2.21 (or (D.4)) we obtain $R(f)$ as,

$$R(f) = \frac{\sigma_A^2}{T_s} |G(f)|^2 = \frac{\sigma_A^2}{T_s} \cdot \underbrace{A_g^2 T_s^2 \left(\frac{\sin(\pi f T_s)}{\pi f T_s} \right)^2}_{\text{Figure D.2}}$$

and $R(1/T_s) = 0$. So, the width of the one-sided mainlobe equals,

$$\boxed{W = \frac{1}{T_s} = \frac{1}{kT_b} = \frac{R_b}{k} \text{ [Hz]}}$$

This gives the following bandwidth results:

$$\begin{aligned} k = 1 \ (M = 2) &\Rightarrow W = 9600 \text{ [Hz]} \\ k = 2 \ (M = 4) &\Rightarrow W = 4800 \text{ [Hz]} \\ k = 3 \ (M = 8) &\Rightarrow W = 3200 \text{ [Hz]} \end{aligned}$$

The bandwidth efficiency ρ is obtained as,

$$\rho = R_b/W = k \text{ [bps/Hz]}$$

□

Example 2.28: $T = T_s = k * T_b$.

T is increased if M is increased.

$W = R_b/k$ is then decreased (*bandwidth saving!*)

Assume instead in Example 2.28 that T_s is fixed, i.e. the bandwidth W is fixed!

Then, since $R_b = k * W$, *higher bit rates* can be communicated if M is increased.

If $W = 1 \text{ Mhz}$, then

$R_b = 1 \text{ Mbps}$ if $M = 2$ ($k = 1$)

$R_b = 10 \text{ Mbps}$ if $M = 1024$ ($k = 10$)

2.5.6 $R(f)$: M-ary QAM Signals

Hence, if the average signal $a(t)$ in (2.36) equals zero, i.e. if $a(t) = 0$ for all t , and if $f_c T \gg 1$, then the power spectral density $R(f)$ for M-ary QAM is,

$$R(f) = R_c(f) = \bar{P} \left(\frac{|G(f+f_c)|^2}{2E_g} + \frac{|G(f-f_c)|^2}{2E_g} \right) \quad (2.227)$$

Observe that this result can be used for a number of important special cases, such as: M-ary PSK, and M-ary bandpass PAM ($m_A = E\{A_\ell\} = 0$, and $B_\ell = 0$ for all ℓ).

Hence, if the same pulse $g(t)$ is used then M-ary QAM, M-ary bandpass PAM, and M-ary PSK have the same bandwidth W . Furthermore,

Consequently, Table 2.1 can also be used for:

M-ary QAM

M-ary PSK

M-ary bandpass PAM

Ex.: Bandwidth consumption for BPSK, QPSK, 16-QAM if the bit rate is the same. $f_c=100\times R_b$.

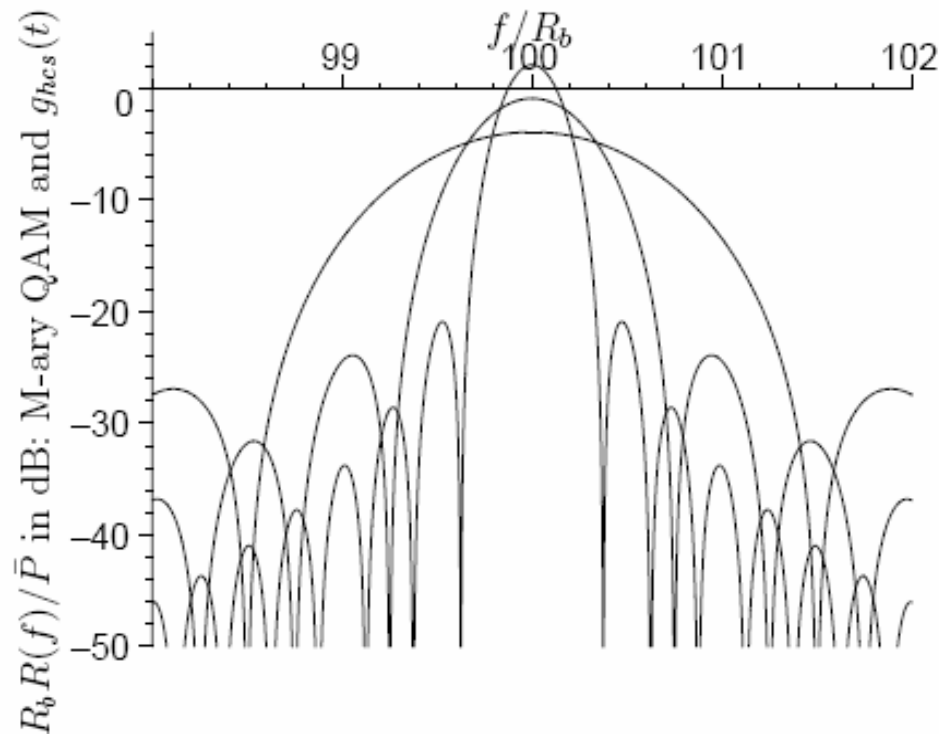
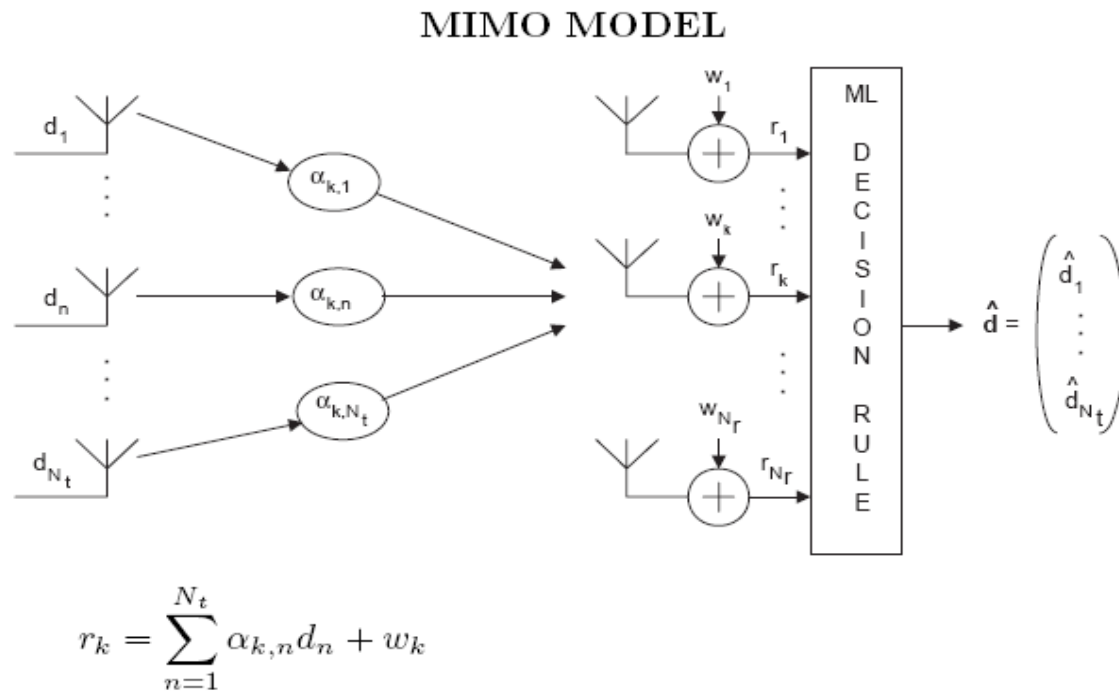


Figure 2.20: The power spectral density for binary QAM (BPSK, widest mainlobe), 4-ary QAM (QPSK), and 16-ary QAM (smallest mainlobe). The figure shows $10 \log_{10}(R_b R(f)/\bar{P})$ [dB] in the frequency interval $98R_b \leq f \leq 102R_b$. The carrier frequency is $f_c = 100R_b$ [Hz], and a $T_s = kT_b$ long $g_{hcs}(t)$ pulse is assumed. See also (2.227) and (2.230).

Problem 5.34 page 421 (not in this course)



The same frequency interval is used by all transmitted signals!

2.5.7 $R(f)$: OFDM-type of Signals

As in (2.106) we here model an OFDM symbol (signal alternative) $x(t)$ by,

$$x(t) = \sum_{n=0}^{N-1} s_{n,QAM}(t) \quad (2.231)$$

$$R(f) = R_c(f) = \sum_{n=0}^{N-1} \bar{P}_n \left(\frac{|G(f + f_n)|^2}{2E_g} + \frac{|G(f - f_n)|^2}{2E_g} \right) \quad (2.233)$$

Ex.: OFDM with $N=16$.

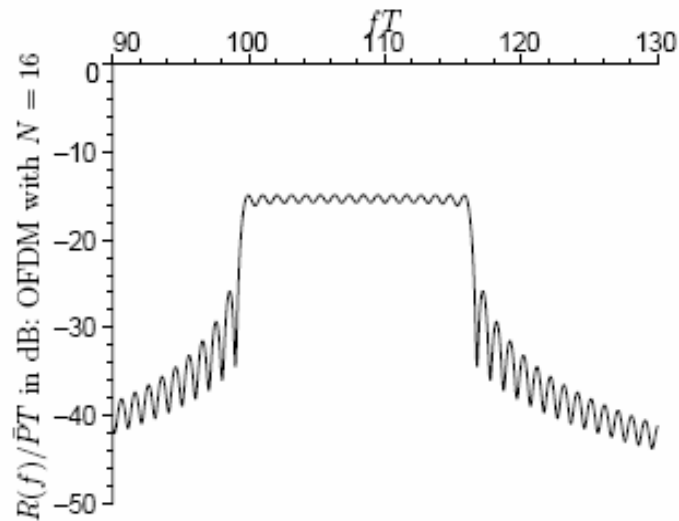


Figure 2.21: $10 \log_{10}(R(f)/\bar{P}T)$ [dB] for an example of OFDM with $N = 16$ subchannels. $R(f)$ is given in (2.237), and $R(f)/\bar{P}T$ is plotted in Figure 2.22.

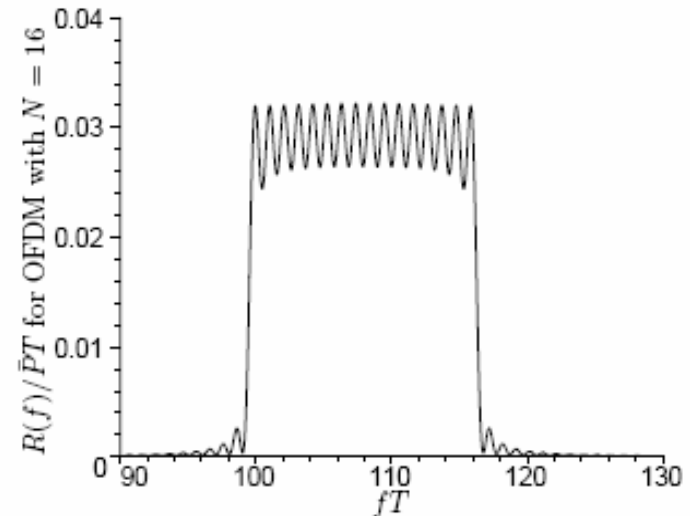


Figure 2.22: $R(f)/\bar{P}T$ for an example of OFDM with $N = 16$ subchannels. $R(f)$ is given in (2.237), and $R(f)/\bar{P}T$ in [dB] is plotted in Figure 2.21.

EXAMPLE 2.34

Let us here continue to investigate the WLAN application considered in Example 2.15 on page 52.

Estimate the subchannel carrier spacing, and the total bandwidth if $T_s = 4$ [μs], and if $\Delta h = 0.8$ [μs].

Solution:

The subchannel carrier spacing is $f_{\Delta} = 1/T_{obs} = 10^6/3.2 = 312.5$ kHz.

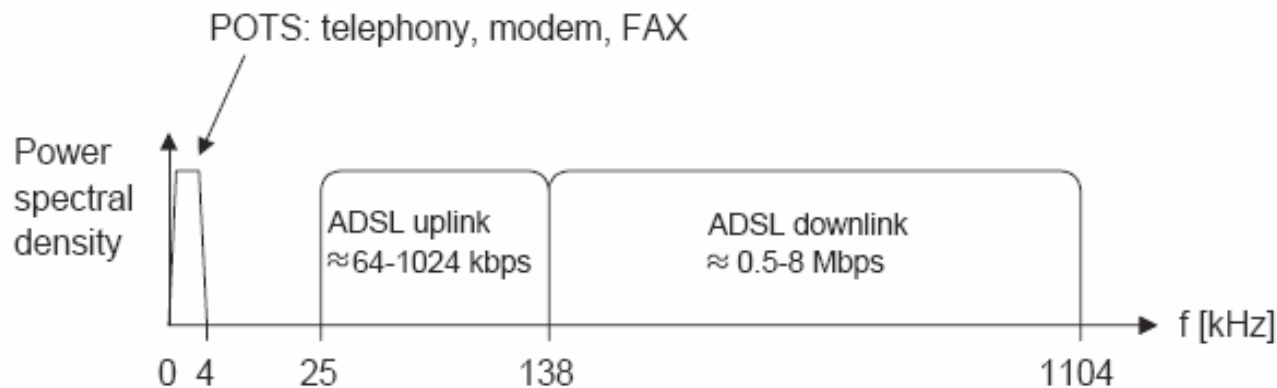
The total bandwidth W is estimated by (see (2.235)),

$$W \approx (N + 1)f_{\Delta} = 49 \cdot 10^6/3.2 = 15.3 \text{ [MHz]}$$

□

EXAMPLE 2.35

In ADSL technology (Asymmetric Digital Subscriber Line) the ADSL signals use the same twisted-wire (copper) pair as the plain old telephone system (POTS) does, [48], [24]. ADSL signal transmission from the network (communication node) to the end user (approximately 0.1–5 [km]) is referred to as the downlink (or forward channel). ADSL signal transmission from the end user to the network is referred to as the uplink (or reverse channel). In order to facilitate simultaneous operation of POTS, ADSL downlink and uplink applications, a frequency division duplex arrangement may be used. The figure below illustrates this method.



In ADSL, a coded OFDM technique is used. The level of the power spectral density in the downstream is roughly -73 dB. As a basic example, let us here assume that the OFDM symbol rate in the downlink is 4000 [symbol/s], and that the subchannel carrier spacing is 5 kHz. Furthermore, it is here also assumed that uncoded 16-ary QAM is used in each subchannel (assumes a very “good” communication link).

For the ADSL downlink above, determine the bit rate in each subchannel, the total bit rate, and the bandwidth efficiency.

2.5.8 $R(f)$: M-ary FSK Signals

With M-ary FSK signaling the signal alternatives $s_\ell(t)$ are given by (see (2.68)),

$$s_\ell(t) = A \cos(2\pi f_\ell t + \nu), \quad \ell = 0, 1, \dots, M - 1 \quad (2.239)$$

$$W_{M-FSK} \approx R_s + \underbrace{(M - 1)f_\Delta}_{f_{M-1} - f_0} + R_s = (M - 1)f_\Delta + 2R_s \quad (2.243)$$

$$f_\Delta = I \cdot R_s / 2 \quad (2.244)$$

$$\rho_{M-FSK} = \frac{R_b}{W_{M-FSK}} \approx \frac{R_b}{(M-1)f_\Delta + 2R_s} = \frac{R_b}{((M-1)I/2 + 2)R_s} = \frac{\log_2(M)}{(M-1)I/2 + 2}$$

(2.245)

Observe that for orthogonal M-ary FSK, the bandwidth efficiency in (2.245) is small if M is large, in contrast to M-ary PAM, M-ary PSK and M-ary QAM signaling.

Consequently, for M-FSK the bandwidth efficiency is bad for large M!

EXAMPLE 2.36

Assume that orthogonal M -ary FSK is used to communicate digital information in the frequency band $1.1 \leq f \leq 1.2$ [MHz].

For each M below, find the largest bit rate that can be used (use bandwidth approximations):

i) $M = 2$ ii) $M = 4$ iii) $M = 8$ iv) $M = 16$ v) $M = 32$

Which of the M -values above give a higher bit rate than the $M = 2$ case?

Solution:

It is given that $W_{M\text{-FSK}} = 100$ [kHz]. From (2.245), the largest bit rate is obtained with $I = 1$:

$$R_b \approx 10^5 \cdot \frac{\log_2(M)}{(M-1)/2 + 2}$$

M	$\frac{\log_2(M)}{(M-1)/2+2}$	R_b
2	$\frac{1}{5/2} = 0.4$	40 kbps
4	$\frac{2}{7/2} = \frac{4}{7} \approx 0.5714$	≈ 57 kbps
8	$\frac{3}{11/2} = \frac{6}{11} \approx 0.5455$	≈ 55 kbps
16	$\frac{4}{19/2} = \frac{8}{19} \approx 0.4211$	≈ 42 kbps
32	$\frac{5}{35/2} = \frac{10}{35} \approx 0.2857$	≈ 29 kbps

From this table it is seen that $M = 4, 8, 16$ give a higher bit rate than $M = 2$. □

Example: Also discrete frequencies in $R(f)$!!

Assume: $M=2$

$$s_0(t)=0$$

$$s_1(t)=5, \text{ pulse duration } T=T_b/2$$

Average signal= $a(t)=(s_0(t)+s_1(t))/2=2.5$ within T .

Then we can write (within T):

$$s_0(t)=-2.5 + a(t)$$

$$s_1(t)=+2.5 + a(t)$$

1. This method is a waste of signal energy since $a(t)$ does not carry any information!
2. Repetition of $a(t)$ in every symbol interval gives a **periodic signal component** in the time domain which gives us the discrete frequencies in the frequency domain.

A short summary of some important parameters/concepts in this course during **study-week 2:**

Chapters 2:

Study the concepts of main-lobe and side-lobes on pages 64-65.

Frequency-shift operations are defined and illustrated pages 67-69.

Definition and purpose of the power spectral density function is found on page 71.

Basic examples for binary signaling: Example 2.21, Example 2.23, Example 2.24.

Table 2.1 on page 86 is useful.

The power spectral density and bandwidth efficiency for M-PAM, M-QAM, OFDM, M-FSK are investigated on pages 88-102.