

Final exam in  
Digital Communications  
(ETT051)  
on October 28, 2015, 14–19.



Department of Electrical and Information Technology  
Lund University

- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper. Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course. If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.

**Good Luck!**

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**Problem 1:** Determine for each of the five statements below if it is true or false.  
*Observe! As usual, motivations to your answers should be given.*

- Assume a conventional 1024-ary QAM system. Also assume that  $g(t) = g_{rc}(t)$  with duration  $T_s/2$ . “If the width of the main lobe is 2 MHz then the bit rate is 2.5 Mbps.”
- “16-QAM has twice as high bandwidth efficiency than 8-PSK.”
- Assume a conventional 16-ary PSK system that uses  $g(t) = g_{rc}(t)$  with duration  $T_s/2$ , a conventional AWGN channel, and ML receiver. “If  $\mathcal{E}_b/N_0$  is 19.65 dB then  $P_s \approx 1.85 \cdot 10^{-7}$ .”
- “The specific mapping of the  $k$  information bits to the  $M$  signal alternatives will typically influence the bandwidth and/or the symbol error probability.”
- “To obtain the  $P_s$  versus  $\mathcal{E}_b/N_0$  curve a practical and efficient method is to use computer simulations, especially for  $P_s \approx 10^{-18}$  and smaller.”

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(10 points)

**Problem 2:** Assume a binary communication system that uses equally likely signal alternatives, a conventional AWGN channel, and an ML receiver.

i) It is given that  $z_0(t) = g_{rec}(t)$  with amplitude  $A$  ( $A$  is a given fixed value) and duration  $T$ . It is also given that  $z_1(t) = -0.5z_0(t)$ . Furthermore,  $A^2/N_0$  is 83 dB and  $T = 0.6T_b$ .

The bit error probability is  $10^{-7}$ .

Calculate the bit rate.

ii) Calculate the difference in energy efficiency, in dB, for the signal alternatives in i) and binary FSK (orthogonal). Which has best energy-efficiency?

iii) Create another pair of signal alternatives compared with i). One signal alternative should be rectangular with amplitude  $A$ , and the other signal alternative should also be rectangular shaped. The new pair of signal alternatives should be chosen such that the bit rate is higher than in i), and the higher bit rate you obtain the better.

The value of  $A$  should be the same as in i) and  $A^2/N_0$  is 83 dB.

The bit error probability should be  $10^{-7}$ , i.e. the same value as in i).

(10 points)

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**Problem 3:**

a) The communication channel is here a 2-ray channel. For the first signal path the multiplication parameter is  $\alpha$  and the delay is zero. For the second signal path the multiplication parameter is  $-\alpha/2$  and the delay is  $10^{-7}/2$  s.

Conventional baseband 4-ary PAM signals are sent from the transmitter. The pulse  $g(t)$  used is a rectangular pulse with amplitude  $A$  and duration  $10^{-7}$  s.

i) Calculate and sketch the output signal from the 2-ray channel if the signal alternative  $3g(t)$  is transmitted in the time interval  $0 \leq t \leq 10^{-7}$ , and if nothing else is transmitted.

ii) Consider now the sequence of transmitted 4-PAM signal alternatives (starting at  $t=0$ ):  $3g(t)$ ,  $-g(t)$ ,  $g(t)$ ,  $-g(t)$ ,  $3g(t)$ , and  $T_s = 10^{-7}$  s.

Determine and clearly sketch the output signal from the 2-ray channel in the time interval  $T_s \leq t \leq 3T_s$ .

What are your conclusions?

b) i) Explain important special properties of M-ary FSK (orthogonal). Also give examples of special application requirements where it could be motivated to use a large M, e.g., 64-FSK.

ii) Determine an expression for the complete union bound for a 4-ary modified PPM signal constellation. Equally likely signal alternatives are assumed. The expression should contain  $\mathcal{E}_b/N_0$ .

In contrast to conventional 4-PPM, where all signal alternatives have equal energy, we here assume that two of the signal alternatives have energy  $E$ , and the other two have energy  $E/4$ .

Also explain the significance of  $d_{min}^2$ .

(10 points)

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#### Problem 4:

- a) Here we consider a three-user digital communication system. The information carrying user signals are denoted  $u_1(t)$ ,  $u_2(t)$  and  $u_3(t)$ , respectively, and they are baseband (low-frequency) signals. The width of the mainlobes of the user information signals  $u_1(t)$ ,  $u_2(t)$  and  $u_3(t)$  are: 600 kHz, 400 kHz and 600 kHz, respectively.

The transmitted multi-user signal is denoted  $s(t)$ , and  
$$s(t) = u_1(t) \cos(2\pi f_1 t) + u_2(t) \cos(2\pi f_2 t) + u_3(t) \cos(2\pi f_3 t)$$

The received multi-user signal is denoted  $r(t)$ , and  
$$r(t) = \alpha s(t) + n(t)$$

where  $\alpha$  is a given channel parameter, and  $n(t)$  denotes a disturbance.

The receiver first constructs the signal denoted  $y(t)$  as

$$y(t) = r(t) \cos(2\pi f_4 t)$$

and the desired information carrying signal is then obtained by filtering  $y(t)$  in a properly designed low-pass filter.

It is known that  $f_1 = 29.3$  MHz,  $f_2 = 30$  MHz, and  $f_3 = 30.8$  MHz. The disturbance is  $n(t) = \cos(2\pi f_A t) + \cos(2\pi f_B t)$  where  $f_A = 29.7$  MHz and  $f_B = 30.3$  MHz.

*Note that detailed calculations are not required below. However, the frequency content must be clearly seen in the figures.*

- i) Sketch the frequency content in  $r(t)$ .
  - ii) Sketch the frequency content in  $y(t)$  if  $f_4 = 30.2$  MHz.
  - iii) The choice of  $f_4 = 30.2$  MHz is not a correct choice but it can occur due to a malfunction in the receiver. Determine the bandwidth of the lowpass filter if instead  $f_4 = 30.8$  MHz, which is an example of a correctly chosen frequency.
- b) i) Assume, for the communication link studied in Chapter 6 in the compendium, that a too high level of ISI has been observed. Suggest possible changes in the choice of communication parameters in the M-ary PAM system such that the ISI is reduced.

ii) Test measurements were performed on a communication link. Conventional PSK were used, and the following measured values were obtained: bandwidth 600 kHz, bit error probability  $10^{-5}$ , bit rate 1.2 Mbps, and  $SNR_r$  equal to 11.614 dB.

A person claims that some of the measured values are wrong, and that this is a result of errors in the test procedure/equipment.

Determine if the person is correct, or not.

(10 points)

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**Problem 5:** Consider a conventional M-ary band-pass PAM communication link that contains attenuation and AWGN (with power spectral density  $N_0/2$ ). The ML receiver is assumed to be used.

The 99.9 % definition of bandwidth is here used and it equals 8 MHz. The carrier frequency is 4 GHz.

The symbol error probability can here be approximated with  $2Q(\sqrt{D_{min}^2/(2N_0)})$ .

The bit error probability is at most  $2 \cdot 10^{-9}$ .

The duration of the pulse used is equal to the symbol time.

When the communication distance of the communication link is such that  $P_z/N_0$  increases to 93.49 dB then the system adaptively changes to the next suitable M.

Analyse the system given above and determine the bit rate used when  $P_z/N_0$  is slightly larger than 93.49 dB.

Also determine the bit rate used when  $P_z/N_0$  is slightly smaller than 93.49 dB.

Are there any differences in the symbol error probability just before and after a new M has been chosen?

(10 points)

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