Digital Communications (ETT051), exam 20131024

Answers & Hints.

Problem 1.

a) True since $W_{lobe} = \frac{3}{T} = \frac{3 \cdot 5}{4 \cdot 6} R_b = 500$ kHz.

b) False since 16-FSK has better d_{min}^2 than 16-QAM.

c) False since $P_s \approx 3.75\, Q\left(\sqrt{\frac{24}{255} \cdot 382.47}\right) =$

$= 3.75 \cdot Q(6) \approx 3.75 \cdot 10^{-9}$.

d)

$W = \frac{c}{T_s} = \frac{c}{k} R_b$, c is a constant.

If R_b is constant then the statement is True.

e) False since d_{min}^2 is small, the mobile is battery powered and the signal attenuation is large.

Problem 2a) $\frac{D^2}{2N_0} \geq 4.7534^2$

i) $D^2 = A^2 T_b / 2 \Rightarrow R_b \leq \frac{A^2/N_0}{4 \cdot 4.7534^2} \approx 110.6$ kbps

ii) $D^2 = 2A^2 T_b \Rightarrow R_b \leq \frac{A^2/N_0}{4.7534^2} \approx 442.6$ kbps

2b) $D^2 = A^2 \frac{T_b}{2}$, $\mathcal{E}_b = \frac{A^2 T_b/2 + A^2 T_b}{2} = \frac{3}{4} A^2 T_b$

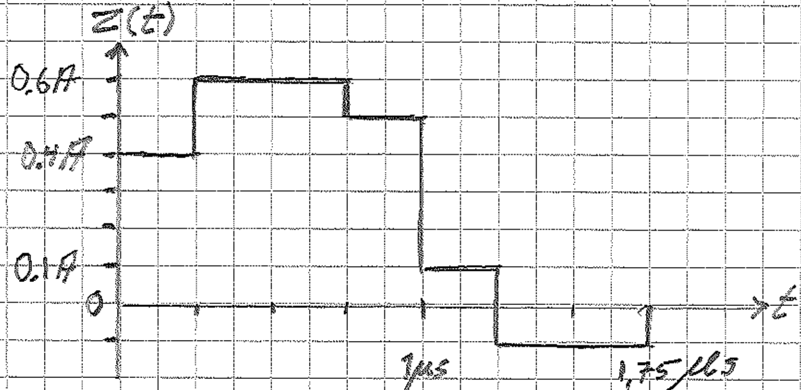
$\Rightarrow d^2 = 1/3$

$P_b = Q\left(\sqrt{\frac{1}{3} 63.096}\right) = Q(4.586) \approx 2.1 \cdot 10^{-6}$

This case is 4.77 dB worse in energy-efficiency compared with binary orthogonal signals.

Problem 3.

a) i) Let z(t) denote the output pulse from the 3-ray channel. The answer is then given by the figure below:



ii) ISI may occurr in the receiver if $T_s < 1.75\,\mu s$. The input pulse has been changed by the channel. Therefore, the receiver needs an estimate of the channel.

b) $$P_{tr} = E_g \frac{M^2-1}{3T_s} = A_M^2 (M^2-1)/6 = \text{constant}$$

$$\rightarrow \quad \frac{A_4}{A_2} = \sqrt{\frac{3}{15}} = 0.447$$

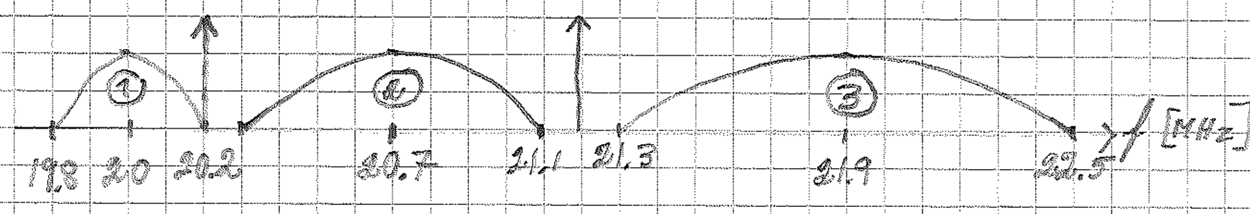
$$\frac{A_8}{A_2} = \sqrt{\frac{3}{63}} = 0.218$$

$$\frac{A_{64}}{A_2} = \sqrt{\frac{3}{64^2-1}} = 0.02707$$

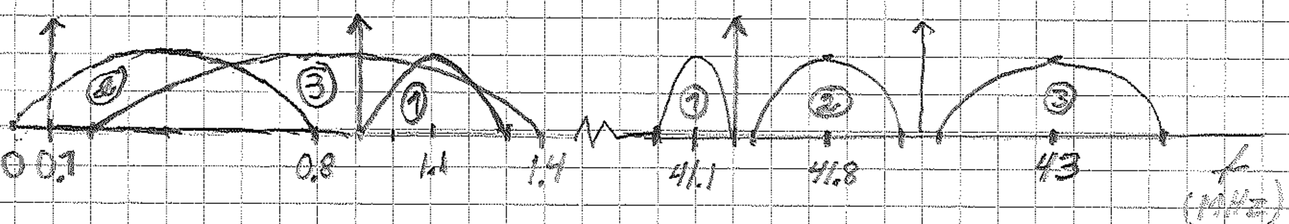
The M amplitudes get closer to each other as M increases, and this means higher sensitivity to noise.

Problem 4.

a) i) The frequency content in r(t) is symmetric around f=0. It is given below for positive frequencies.



ii) The frequency content in y(t) is also symmetric around f=0. For f>0 it is given below, but for low frequencies the final additions are not shown.



iii) 400 kHZ

b) i) At high E_b/N_0 most symbol errors are made between signals separated by D_{min}. Hence, with Gray coding most symbol errors then give only one bit error, and $P_b \approx P_s/k$.

ii) $D_{ij}^2 = (A_i - A_j)^2 E_g$, $E_g = 3A^2T/8$, $D_{min}^2 = E_g$

$$\Rightarrow \frac{D_{min}^2}{2N_0} = 49, \quad C = \frac{1}{16}(1+1+0+0..+0) = \frac{1}{8}$$

$$\text{union bound} = \frac{1}{8} Q(7) \approx 1.6 \cdot 10^{-13}$$

Problem 5.

$W_{lobe} = 2 \cdot 10^{6} \Rightarrow T_s = 10^{-6}$

$P_b \approx P_s \leq 2Q\left(\sqrt{d_{min}^2 \, \mathcal{E}_b/N_0}\right) \leq 2Q(5.612)$

$$2 \log_2(M) \sin^2(\pi/M) \frac{P_z}{R_b N_0} \geq 5.612^2$$

a)

$R_b = 5$ Mbps $\Rightarrow M = 32$

$\Rightarrow \quad \frac{P_z}{N_0} \geq 1.64 \cdot 10^9$

but $\frac{P_z}{N_0}$ must also satisfy

$6.54 \cdot 10^9 > \frac{P_z}{N_0}$ since otherwise a larger value than $M=32$ had been chosen by the system. The value $6.54 \cdot 10^9$ is the smallest value for which $M=64$ can be used.

b, c) Let us test with $M=16$, using the same expression as was used above.

We get that $\frac{P_z}{N_0} \geq 4.14 \cdot 10^8$ but this is too high.

Test with $M=8$ gives $\frac{P_z}{N_0} \geq 1.075 \cdot 10^8$ **OK!**

so $M=8 \Rightarrow R_b = 3$ Mbps.