Digital Communications (ETT051)
Exam October 26, 2012, 14-19.
Answers & Hints.

Problem 1.

a) True since $W_{lobe} = \frac{3}{T} = \frac{3}{0.8T_s} = \frac{3}{4.8}R_b = 6.25 \cdot 10^6$

b) True since $d^2_{min} = 0.8$ for both, and

$W_{4PAM} = \frac{c}{T_s} \Rightarrow \rho_{4PAM} = \frac{2}{c}$, c depends on the pulse.

$W_{16QAM} = \frac{2c}{T_s} \Rightarrow \rho_{16QAM} = \frac{2}{c}$

c) False since $P_s \approx 4Q\left(\sqrt{\frac{3 \cdot 6}{63} \cdot 93.756}\right) = 4Q(5.176) \approx 4 \cdot 10^{-7}$.

d) False since $E_r = E_\gamma = E_g/2$.

e) True since in general MAP $\neq$ ML.

Problem 2.

a)

$$Z_1(t) = 4\, g_{rec}(t), \quad Z_0(t) = g_{rec}(t)$$

$$D_{0,1}^2 = \int_0^{T_b} \left(4 g_{rec}(t) - g_{rec}(t)\right)^2 dt = 9 E_g = 9 A^2 \frac{3 T_b}{4}$$

$$P_b = Q\left(\sqrt{\frac{D_{0,1}^2}{2N_0}}\right) = Q\left(\sqrt{\frac{27}{8}\frac{A^2 T_b}{N_0}}\right) = Q\left(\sqrt{\frac{27}{8} \cdot 8.64}\right) =$$

$$= Q(5.4) = 3.3 \cdot 10^{-8}$$

b)

The same answer as in a)
since $D_{01}^2 = 9E_g$ is the same as in a).

c)

We need to find d^2 for each case.

Case a) $\quad \mathcal{E}_b = \frac{16E_g + E_g}{2} = \frac{17}{2} E_g$

$$d_{a)}^2 = \frac{D_{0,1}^2}{2\mathcal{E}_b} = \frac{9E_g}{17E_g} = 9/17$$

case b) $\quad \mathcal{E}_b = \frac{4E_g + E_g}{2} = \frac{5}{2} E_g$

$$d_{b)}^2 = \frac{9E_g}{5E_g} = 9/5$$

So, case b) is $10\log_{10}\left(\frac{9/5}{9/17}\right) = 5.31\,dB$
better than case a).

Problem 3.

a)

$E_0 = E_g$, $E_1 = \frac{E_g}{4}$, $E_2 = \frac{E_g}{9}$

The signal alternatives $z_0(t)$, $z_1(t)$ and $z_2(t)$ are orthogonal to each other since they exist in different time slots.

$D^2_{0,1} = E_0 + E_1 = \frac{5}{4} E_g = 1.25 E_g = D^2_{2}$

$D^2_{0,2} = E_0 + E_2 = \frac{10}{9} E_g = 1.11 E_g = D^2_1$

$D^2_{1,2} = E_1 + E_2 = \frac{13}{36} E_g = 0.361 E_g = D^2_{min}$

$C_{min} = \frac{1}{3}(0+1+1) = 2/3$

$C_1 = \frac{1}{3}(1+0+1) = 2/3$

$C_2 = \frac{1}{3}(1+1+0) = 2/3$

union bound $= \frac{2}{3} Q\left(\sqrt{\frac{0.361 E_g}{2N_0}}\right) + \frac{2}{3} Q\left(\sqrt{\frac{1.11 E_g}{2N_0}}\right) + \frac{2}{3} Q\left(\sqrt{\frac{1.25 E_g}{2N_0}}\right)$

b)

The pulse shape is changed.

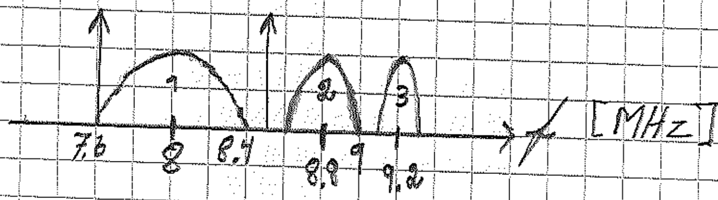
The pulse duration changes from T to $T+\tau_{max}$.

If $T_s < T+\tau_{max}$ overlapping signals appear after the channel which may cause ISI in the receiver.

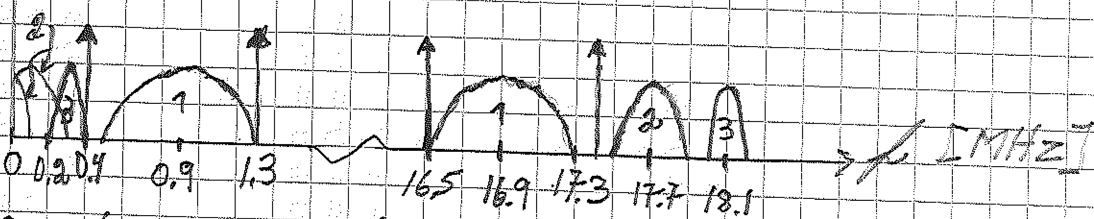
Problem 4

a)

i) Frequency content in $r(t)$ (symmetric around $f=0$):



ii) Frequency content in $y(t)$ (symmetric around $f=0$) if $f_4 = 8.9$ MHz.



Overlapping spectra are added in reality.

iii) $f_4 = 9.2$ MHz.

b)

i) $r_G(t) = Gz(t) + GN(t)$

Let us denote the signal alternatives after the amplifier as $z_i'(t) = Gz_i(t)$, and the noise as $N'(t) = GN(t)$.

Then we have $r_G(t) = z'(t) + N'(t)$, and we can use the receiver in Figure 4.8 in the compendium, but we should replace $z_i(t)$ with $Gz_i(t)$ and E_i with G^2E_i, $i = 0, 1, \ldots, 15$.

ii) $P_s \approx 3\, Q\left(\sqrt{0.8 \frac{\mathcal{E}_b'}{N_0'}}\right) = 3\, Q\left(\sqrt{0.8 \frac{\mathcal{E}_b}{N_0}}\right)$

since $\mathcal{E}_b' = G^2 \mathcal{E}_b$ and $N_0' = G^2 N_0$.

Hence, an amplifier does not change the signal-to-noise ratio.

Problem 5.

i) It is found that with the pulse $g_{rc}(t)$ ρ is:

$$\rho = \frac{R_b}{W_{99.9}} = \frac{R_b T_s/2}{3.46} = \frac{k}{2 \cdot 3.46} \approx 0.4335$$

So, $k=3$ $(M=8)$.

$$d_{min}^2 \frac{E_b}{N_0} \geq 5.612^2$$

$$\frac{6 \log_2(M)}{M^2-1} \cdot \frac{P_Z}{R_b N_0} = \frac{6}{M^2-1} \cdot \frac{T_s P_Z}{N_0} \geq 31.495$$

$$\frac{T_s P_Z}{N_0} \geq 31.495 \cdot \frac{M^2-1}{6} = \begin{cases} 330.7, & M=8 \\ 1338.5, & M=16 \end{cases}$$

So, $$330.7 \leq \frac{T_s P_Z}{N_0} < 1338.5$$

ii)

$\frac{P_Z}{N_0 R_b} > 5.1$ is the same as $\frac{E_b}{N_0} > 5.1$

We know from i) that

$$\frac{E_b}{N_0} \geq \frac{31.495 (M^2-1)}{6 \log_2(M)} \underset{\substack{\uparrow \\ M=8}}{=} 110.2$$

So, the person is wrong since $\frac{E_b}{N_0} > 5.1$ is not enough.