23/10-12/GL

Digital Communications
Exam October 21, 2011
Answers & Hints

Problem 1.

a) True, since $W_{lobe} = \frac{4}{T} = \frac{10}{T_s} = 2$ MHz.

b) False, since $(d^2_{min})_{8FSK} = 0.879$ is larger than $(d^2_{min})_{16QAM} = 0.8$

c) True, since $P_s \approx 2Q\left(\sqrt{0.3045 \cdot 118}\right) = 2Q(6)$

d) False, since two signals can never be both antipodal and orthogonal.

e) False, since for M-FSK $P_s \approx (M-1)\,Q\left(\sqrt{k\,15.849}\right)$ and P_s is decreased as k is increased.

Problem 2.

$$P_b = Q\left(\sqrt{d^2\, E_b/N_0}\right)$$

a)

i) $Q(4.3) = Q\left(\sqrt{d^2 \cdot 61.63}\right) \Rightarrow d^2 = 0.3$

So, $P_b = Q\left(\sqrt{0.3 \cdot 192.53}\right) = Q(7.6) = 1.48 \cdot 10^{-14}$

ii) 5.23 dB worse than orthogonal signals.

b)

$$E_0 = \frac{A^2 T_b}{2}, \quad E_1 = \frac{A^2 T_b}{18}, \quad D^2 = \frac{4A^2 T_b}{18}$$

$$d^2 = \frac{D^2}{2E_b} = 0.4 > 0.3$$

So, these signals will give a smaller P_b.

Problem 3.

$$W_{99} = \frac{2.36}{T} = 4\cdot 10^{6} \Rightarrow T_s = 2.36\cdot 10^{-6} = \frac{k}{R_b}$$

If $R_b \approx 3.39$ Mbps $\Rightarrow k = 8$ (in i))

$$d_{min}^2 \frac{\mathcal{E}_b}{N_0} \geq 40.466$$

$$\Rightarrow \frac{3\log_2(M)}{M-1} \cdot \frac{P_\Sigma}{R_b N_0} \geq 40.466$$

i) $k = 8 \Rightarrow \frac{P_\Sigma}{N_0} \geq 1.46\cdot 10^{9}$

ii) $\frac{3 T_s P_\Sigma}{40.466 N_0} + 1 \geq M$

$\Rightarrow$ $34.99 + 1 \geq M$

$\Rightarrow$ $M = 16, \; k = 4$

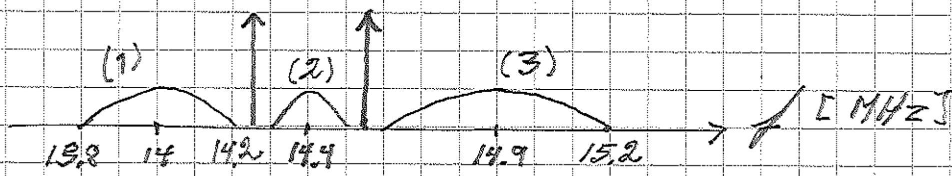
$\Rightarrow$ $R_b = \frac{k}{T_s} = 1.695$ Mbps

iii) Please see the compendium.

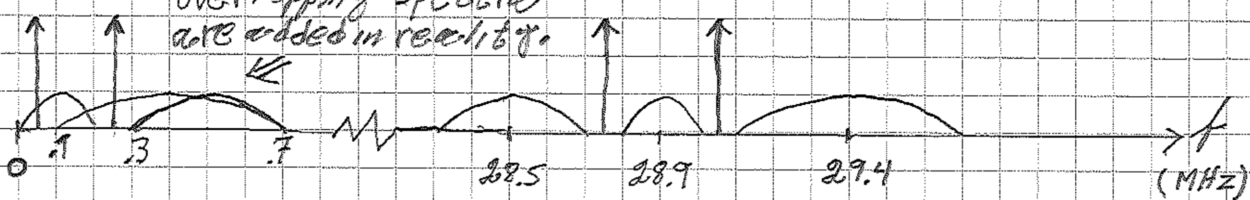
Problem 4.

a)

i) Frequency content in r(t):



ii) Frequency content in y(t) if $f_4 = 14.5$ MHz



iii) $f_4 = 14.4$ MHz

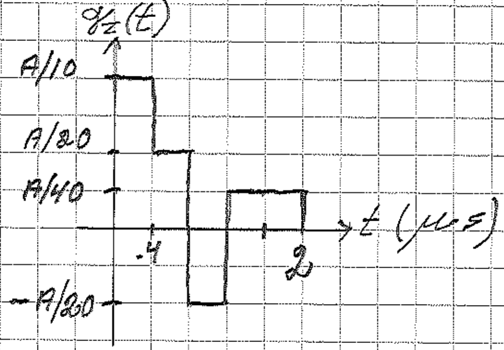
b) Please see chapter 6 in the compendium.

Problem 5.

a) To avoid overlapping signals $T_s = kT_b \geq 2 \cdot 10^{-6}$, so $R_b \leq 1.5$ Mbps.

b) If the input is $3g_{rec}(t)$ then the output is $3g_z(t)$, where $g_z(t)$ is the output pulse shape after the 3-ray channel.

$g_z(t)$ is the output when the input is $g_{rec}(t)$, and $g_z(t)$ is found to be:



c) $D_{min}^2 = 4E_{g_z}$ (since it is M-PAM)

$$E_{g_z} = \int_0^{T_z} (g_z(t))^2 dt \ldots = \frac{13}{2} A^2 \cdot 10^{-9}$$

$D_1^2 = 4D_{min}^2$, $D_2^2 = 9D_{min}^2$, $D_3^2 = 16D_{min}^2$, $D_4^2 = 25D_{min}^2$

$D_5^2 = 36D_{min}^2$, $D_6^2 = 49D_{min}^2$