

**Answers & Hints to the exam in the course Digital Communications (ETT051),  
October 22, 2010, 14-19.**

**Problem 1.**

a) The energy efficiency is better for 64-QAM since  $d_{min}^2 = 6/21$ , compared with  $24/255$  for 16-PAM. The bandwidth efficiency for 64-QAM is  $6\rho_{BPSK}$ , and for band pass 16-PAM it is  $4\rho_{BPSK}$  (worse than 64-QAM). However, the bandwidth efficiency for base band 16-PAM is  $8\rho_{BPSK}$  (better than 64-QAM).

When comparing 64-QAM with base band 16-PAM the statement is FALSE.

When comparing 64-QAM with band pass 16-PAM the statement is TRUE.

b) TRUE, since  $W_{lobe} = 4/T = 8R_b/3$ .

c) TRUE, since  $P_s \approx 2Q(\sqrt{d_{min}^2 \mathcal{E}_b/N_0}) = 2Q(5.273) \approx 1.14 \cdot 10^{-7}$ .

d) TRUE, since  $D_{min}^2 = 2\mathcal{E}_b 0.8 = 2(PT_s/4)0.8 = PT_s 0.4$

e) FALSE, since  $d_{min}^2 = 4$  for 16-PPM, and it is smaller for 16-QAM.

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**Problem 2.**

With  $\alpha$  and  $R_b = 384$  kbps it is given that  $P_b = Q(\sqrt{d^2 \mathcal{E}_b/N_0}) = Q(\sqrt{\frac{d^2 \alpha^2 P_{sent}}{R_b N_0}}) = 10^{-8} = Q(5.612)$ .

a)  $P_b = Q(\sqrt{\frac{d^2 \alpha^2 P_{sent}}{100 R_b N_0}}) = Q(0.5612) \approx 0.28$ .

b)  $\frac{d^2 \alpha^2 P_{sent}}{R_b N_0} = 5.612^2$

The ratio  $\alpha^2/R_b$  must be held constant.

i)  $\alpha^2/R_b = (\alpha/10)^2/R_{b,new}$  leading to that  $R_{b,new} = 3.84$  kbps.

ii)  $\alpha^2/R_b = (10\alpha)^2/R_{b,new}$  leading to that  $R_{b,new} = 38.4$  Mbps.

c) It is given that  $P_b = Q(\sqrt{d^2 \mathcal{E}_b/N_0}) = Q(\sqrt{d^2 19.953}) = 10^{-8} = Q(5.612)$ . This gives  $d^2 = 1.578$  which is 1.98 dB better than binary orthogonal signaling.

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### Problem 3.

a) The output signal alternative will be:

0 in the time interval  $t < 0$ .

$1.5A$  in the time interval  $0 < t < 0.1T_s$ .

$0.6A$  in the time interval  $0.1T_s < t < 0.3T_s$ .

$1.2A$  in the time interval  $0.3T_s < t < 0.5T_s$ .

$-0.3A$  in the time interval  $0.5T_s < t < 0.6T_s$ .

$0.6A$  in the time interval  $0.6T_s < t < 0.7T_s$ .

$0.3A$  in the time interval  $0.7T_s < t < 0.8T_s$ .

$-0.3A$  in the time interval  $0.8T_s < t < 1.2T_s$ .

0 in the time interval  $1.2T_s < t$ .

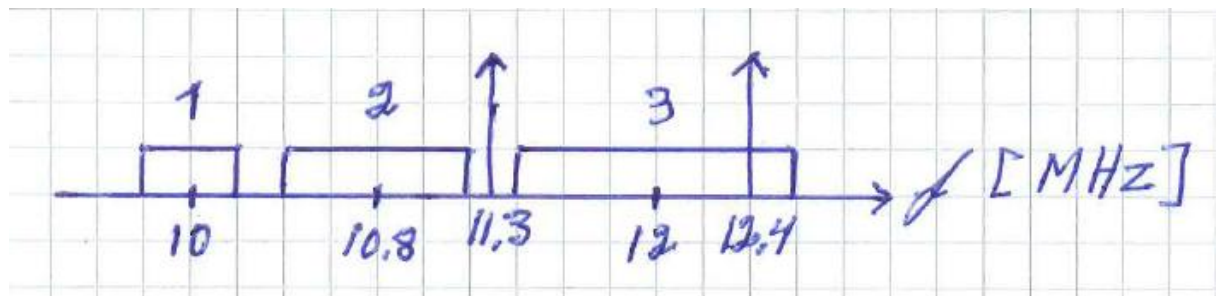
b) See the compendium (e.g. Chapter 6).

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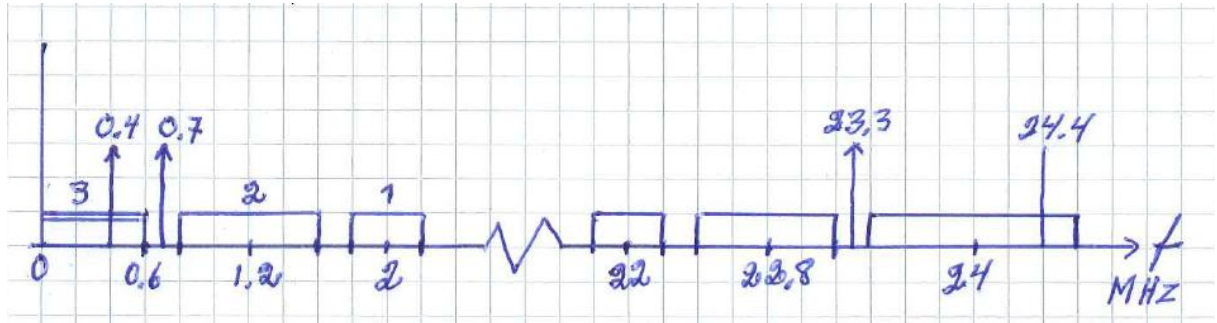
### Problem 4.

a)

i)  $r(t)$ : (symmetry around  $f=0$ )



ii)  $y(t)$ : (symmetry around  $f=0$ )



iii) Correct user signal (user 3) at base band in  $y(t)$ . A disturbance at 400 kHz. Low pass filter bandwidth should be 600 kHz.

b)

i) Close to the true  $P_s$  at medium and high snr. Also then a simple expression.

ii) It determines the energy efficiency (and  $P_s$ ) at medium and high snr.

iii) The bandwidth efficiency is low.

### **Problem 5.**

a)  $d_{min}^2 \mathcal{E}_b/N_0 \geq 6.3613^2 = 40.466$  leads to that

$R_b \leq \frac{3 \log_2(M) 10^9}{(M-1) 40.466}$  which gives maximum allowed bit rates for different values of  $M$ .

The requirement on the width of the main lobe is  $W_{lobe} = 4/T = 16R_b/(3k) = 5 \cdot 10^6$  which can be formulated as  $R_b = 15k10^6/16$ , and from this expression we get the bit rate for different  $k$  (or  $M$ ).

We want as high bit rate as possible, so we calculate  $R_b$  for increasing  $k$ . However, we must not violate the maximum bit rates obtained above!

It is found that  $M = 64$  should be used and the bit rate then is  $R_b = 5.625$  Mbps.

b) From the maximum allowed bit rates in a) we find that we should use  $M = 4$  and the bit rate  $R_b = 49.4$  Mbps.

$W_{lobe}$  will in this case be  $W_{lobe} = (16/6)R_b = 131.7$  Mhz.