Final exam in Digital Communications (ETT051) on October 22, 2010, 14–19.

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- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.
  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.
  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.

Good Luck!

Problem 1: Determine for each of the five statements below if it is true or false.
Observe! As usual, motivations to your answers should be given.

a) " 64-ary QAM is better than 16-PAM."

b) Assume a conventional M-ary QAM system. Also assume that $g(t) = g_{rc}(t)$ with duration $T_s/4$. "If the bit rate is 300 kbps and $M = 64$ then the width of the mainlobe is 800 kHz."

c) Assume a conventional M-ary PAM system that uses $g(t) = g_{rec}(t)$ with duration T_s, a conventional AWGN channel, and ML receiver. "If $M = 64$ and $\mathcal{E}_b/N_0$ is 35 dB then $P_s \approx 10^{-7}$."

d) "For 16-QAM we have that $D^2_{min} = 8T_sP/(5k)$, where P is the average signal power in the signal constellation."

e) "The union bound for 16-PPM is the same as the union bound for 16-QAM."

(10 points)

Problem 2: Assume a binary communication system that uses equally likely signal alternatives $s_0(t)$ and $s_1(t)$. The received signal alternatives are $z_0(t) = \alpha s_0(t)$ and $z_1(t) = \alpha s_1(t)$, where α is a fixed and given channel parameter. These signals are disturbed by AWGN noise $N(t)$ with power spectal density $R_N(f) = N_0/2$. It is also given that the ML receiver is used. $\mathcal{E}_b$ denotes the average received signal energy per information bit.

The average transmitted signal power $\bar{P}_{sent}$ has a given constant value.

For the given communication link above it is known that if the bit rate is 384 kbps then $\mathcal{E}_b/N_0$ is 13 dB and the bit error probability is equal to 10^{-8}.

a) Calculate the bit error probability if the bit rate is 384 kbps, and if the communication distance is changed such that α is decreased to $\alpha/10$.

b) It is required that the bit error probability must be equal to 10^{-8}. Which bit rates must then be used if the communication distance is changed such that α is changed to;

i) $\alpha/10$.

ii) 10α.

c) Compare the energy efficiency for the signal alternatives used above with binary orthogonal signal alternatives.

(10 points)

Problem 3:

Assume that M-ary PAM signals are sent from the transmitter. Here, a sent signal alternative has the form $A_l g_{rec}(t)$, where $g_{rec}(t)$ has duration $T = T_s/2$ and amplitude A.

The sent signal alternative above is the input signal to a 4-ray channel (multi-path). The parameters of the 4-ray channel are:

$\alpha_1 = 0.5$, $\tau_1 = 0$
$\alpha_2 = -0.3$, $\tau_2 = T_s/10$
$\alpha_3 = 0.2$, $\tau_3 = 3T_s/10$
$\alpha_4 = -0.1$, $\tau_4 = 7T_s/10$

a) Assume that the input signal alternative is $3g_{rec}(t)$. Determine and make a detailed sketch of the output signal alternative from the 4-ray channel.

b) Explain which consequences that in general may appear due to a multi-path (or filtering) channel (focus on the receiver).

(10 points)

Problem 4:

a) Here we consider a three-user digital communication system. The information carrying user signals are denoted $u_1(t)$, $u_2(t)$ and $u_3(t)$, respectively, and they are baseband (low-frequency) signals. The width of the mainlobes of the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are: 400 kHz, 800 kHz and 1200 kHz, respectively.

The transmitted multi-user signal is denoted $s(t)$, and
$s(t) = u_1(t)\cos(2\pi f_1 t) + u_2(t)\cos(2\pi f_2 t) + u_3(t)\cos(2\pi f_3 t)$

The received multi-user signal is denoted $r(t)$, and
$r(t) = \alpha s(t) + n(t)$
where α is a given channel parameter, and $n(t)$ denotes a disturbance.

The receiver first constructs the signal denoted $y(t)$ as
$y(t) = r(t)\cos(2\pi f_4 t)$
and the desired information carrying signal is then obtained by filtering $y(t)$ in a properly designed low-pass filter.

It is known that $f_1 = 10$ MHz, $f_2 = 10.8$ MHz, and $f_3 = 12$ MHz. The disturbance is $n(t) = \cos(2\pi f_A t) + \cos(2\pi f_B t)$ where $f_A = 11.3$ MHz and $f_B = 12.4$ MHz.

Note that detailed calculations are not required below. However, the frequency content must be clearly seen in the figures.

i) Sketch the frequency content in $r(t)$.

ii) Sketch the frequency content in $y(t)$ if $f_4 = 12$ MHz.

iii) What are your comments to your answer in ii)?

b) Explain in detail:

i) The advantages of the union bound.

ii) Why d^2_{min} is important.

iii) A disadvantage of 256-ary FSK.

(10 points)

Problem 5:
Assume that the equally likely received signal alternatives $\{z_\ell(t)\}_{\ell=0}^{M-1}$ are conventional M-ary QAM signals ($M = 4, 16, 64, 256, ...$). Also assume that a $g_{tri}(t)$ pulse with amplitude A and duration $T = 3T_s/4$ is used. A bandwidth requirement is that the width of the main lobe should be $W_{lobe} = 5$ MHz. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used. The ratio $\bar{P}_z/N_0 = 10^9$.

The symbol error probability can in this case be upper bounded by $P_s \leq 4Q(\sqrt{d^2_{\min}\mathcal{E}_b/N_0})$.

It is a requirement that the symbol error probability must not exceed $4 \cdot 10^{-10}$.

a) Determine the highest bit rate that can be used.

b) Assume here that we do not have any requirements on W_{lobe}. Hence, assume that the requirement on W_{lobe} given above is removed. For this case determine the highest bit rate that can be used. How large is W_{lobe} in this case?

(10 points)