

Final exam in  
Digital Communications  
(ETT051)  
on October 20, 2009, 08–13.



Department of Electrical and Information Technology  
Lund University

- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.  
Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.  
If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.
- If You want or if You do not want Your result to appear on the department's web site, please write so on the cover page of the exam.

**Good Luck!**

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**Problem 1:** Determine for each of the five statements below if it is true or false.  
*Observe! As usual, motivations to your answers should be given.*

- “A binary PAM signal constellation can never be orthogonal”
- Assume a conventional M-ary bandpass PAM system. Also assume that  $g(t) = g_{hcs}(t)$  with duration  $3T_s/4$ . “If the bit rate is 800 kbps and  $M = 32$  then the width of the mainlobe is 400 kHz.”
- Assume a conventional M-ary PSK system that uses  $g(t) = g_{rc}(t)$  with duration  $T_s$ , a conventional AWGN channel, and ML receiver. “If  $M = 64$  and  $\mathcal{E}_b/N_0$  is 30.4 dB then  $P_s \approx 2 \cdot 10^{-8}$ .”
- “QPSK is better than BPSK. ”
- “The dominating term in the union bound can never be equal to  $(M - 1)Q(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0})$ .”

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(10 points)

**Problem 2:** Assume a binary communication system that uses equally likely signal alternatives  $s_0(t)$  and  $s_1(t)$ . The received signal alternatives are  $z_0(t) = \alpha s_0(t)$  and  $z_1(t) = \alpha s_1(t)$ , where  $\alpha$  is a fixed and given channel parameter. These signals are disturbed by AWGN noise  $N(t)$  with power spectral density  $R_N(f) = N_0/2$ . It is also given that the ML receiver is used. The signal  $s_0(t) = g_{rec}(t)$ , and  $s_1(t) = 2g_{rec}(t)$ , where the pulse  $g_{rec}(t)$  has amplitude  $A$  and duration  $T$ . It is also given that the bit rate is 600 kbps, and that the average signal power after the channel is  $\bar{P}_z$ .

A requirement is that the bit error probability is at most  $10^{-12}$ .

Another requirement is that the width of the mainlobe of  $g_{rec}(t)$  equals 1.6 MHz.

Determine requirements on the ratios  $A^2/N_0$  and  $\bar{P}_z/N_0$ .

Would these requirements change if the pulse  $g_{rec}(t)$  were changed to  $g_{hcs}(t)$  with the same amplitude  $A$  and duration  $T_1$ ?

How good is the energy efficiency?

(10 points)

**Problem 3:** Assume that the received signal alternatives  $\{z_\ell(t)\}_{\ell=0}^{M-1}$  are conventional M-ary PAM signals. Also assume that a  $g_{rec}(t)$  pulse with amplitude  $A$  and duration  $T = T_s/2$  is used, where  $T_s$  is fixed and given. The communication is disturbed by AWGN  $N(t)$  with power spectral density  $R_N(f) = N_0/2$ , and the ML receiver is used. The ratio  $\bar{P}_z/N_0$  is fixed and given.

The symbol error probability can in this case be upper bounded by  $P_s \leq 2Q(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0})$ . Furthermore, it is known that if  $M = 8$  then  $P_s \leq 2Q(169.03)$ .

Determine the highest bit rate that can be used if the symbol error probability must not exceed  $2 \cdot 10^{-11}$ .

Also explain if, in general, there are any advantages to use QAM instead of bandpass PAM.

(10 points)

**Problem 4:**

- a) Here we consider a three-user digital communication system. The information carrying user signals are denoted  $u_1(t)$ ,  $u_2(t)$  and  $u_3(t)$ , respectively, and they are baseband (low-frequency) signals. The width of the mainlobes of the user information signals  $u_1(t)$ ,  $u_2(t)$  and  $u_3(t)$  are: 800 kHz, 400 kHz and 400 kHz, respectively.

The transmitted multi-user signal is denoted  $s(t)$ , and  
 $s(t) = u_1(t) \cos(2\pi f_1 t) + u_2(t) \cos(2\pi f_2 t) + u_3(t) \cos(2\pi f_3 t)$

The received multi-user signal is denoted  $r(t)$ , and  
 $r(t) = \alpha s(t) + n(t)$

where  $\alpha$  is a given channel parameter, and  $n(t)$  denotes a disturbance.

The receiver first constructs the signal denoted  $y(t)$  as

$$y(t) = r(t) \cos(2\pi f_4 t)$$

and the desired information carrying signal is then obtained by filtering  $y(t)$  in a properly designed low-pass filter.

It is known that  $f_1 = 7.6$  MHz,  $f_2 = 8.4$  MHz, and  $f_3 = 9.2$  MHz. The disturbance is  $n(t) = \cos(2\pi f_A t) + \cos(2\pi f_B t)$  where  $f_A = 8$  MHz and  $f_B = 9$  MHz.

*Note that detailed calculations are not required below. However, the frequency content must be clearly seen in the figures.*

- i) Sketch the frequency content in  $r(t)$ .
- ii) Sketch the frequency content in  $y(t)$  if  $f_4 = 8.6$  MHz.

What are your conclusions concerning the choice of  $f_4 = 8.6$  MHz in the receiver?

- b) Explain in detail:

- i) What ISI is and when it occurs.
- ii) Possible consequences of ISI.
- iii) Methods to reduce the effects of ISI.

(10 points)

**Problem 5:**

- a) In a specific application four signal alternatives are used. Equally likely signal alternatives, AWGN channel and ML reception is assumed.

It is given that all signal alternatives  $z_i(t)$  have the same energy, here denoted  $E$ . The signals  $z_0(t)$ ,  $z_2(t)$  and  $z_3(t)$  are all orthogonal to each other, while  $z_1(t) = -z_0(t)$ .

Find an expression for the union bound in this case.

What are, in general, the advantages and disadvantages with the union bound?

- b) Determine in detail the specific operations performed by the specific ML-receiver that is used in Problem 2, and illustrate with a detailed block-diagram of the receiver. The simpler(=low complexity) implementation the better.

(10 points)