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Lund University

**LABORATORY LESSON 1**  
**Digital Communications (ETT051)**

**Signal alternatives, bit rate, bandwidth,  
and error probability**

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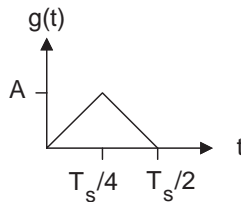
# 1 INTRODUCTION

The purpose of this laboratory lesson is to give an increased understanding of commonly used digital communication methods. Important concepts studied here are bit rate versus bandwidth, and error probability for various communication methods.

## 2 HOME PROBLEMS

(To be solved at home *before* this laboratory lesson.)

1. Read this laboratory instruction carefully, such that you are well aware of what you are expected to do during this laboratory lesson.
2. Calculate the bandwidth, defined as the one-sided width of the mainlobe, if the bit rate is 9600 bps, and if equiprobable M-ary PAM signal alternatives are used. Assume that the possible amplitudes are  $\pm 1, \pm 3, \pm 5, \dots, \pm(M-1)$ , and that the pulse  $g(t)$  shown below is used. Furthermore, assume that:
  - i)  $M = 2$ .
  - ii)  $M = 256$ .



3. Assume bandpass M-ary PAM with carrier frequency  $f_c = 2$  [GHz], and the  $g_{rc}(t)$  pulse with duration  $T = 0.8T_s = 10 \mu s$ . Calculate bit rates, and  $W_{lobe}$  for the cases  $M = 2$ , and  $M = 1024$ .

How much of the signal power  $\bar{P}$  is contained within the mainlobe?  
Is it here better to use  $M = 1024$  than  $M = 2$ ?
4. In a binary communication system with equiprobable signal alternatives the received signal is  $r(t) = z_\ell(t) + N(t)$  if message  $m_\ell$  is sent in  $0 \leq t \leq T_b$ ,  $\ell = 0, 1$ . The noise  $N(t)$  is AWGN with  $R_N(f) = N_0/2$ , and  $z_\ell(t) = \alpha s_\ell(t)$ , where  $\{s_0(t), s_1(t)\}$  are the signal alternatives used by the transmitter. The parameter  $\alpha$  represents the influence of the communication channel. The average signal power in  $z(t)$  is denoted  $P_z$ , and the receiver is ML. If the bit rate is 38.4 kbps, and if BPSK is used then it is known that the bit error probability equals  $10^{-5}$ .
  - i) Calculate the bit error probability if the bit rate is increased to 384 kbps, while keeping  $P_z$  unchanged. Conclusion?
  - ii) What is the relationship between the average transmitted energy per information bit denoted  $\bar{E}_{b, sent}$ , and the average received energy per information bit denoted  $\mathcal{E}_b$ ?  
Is it  $\bar{E}_{b, sent}$  or  $\mathcal{E}_b$  that determines the bit error probability?
  - iii) Is it true that a 20 dB loss in signal energy is obtained if  $\alpha = 1/10$ ?

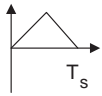
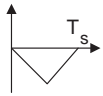
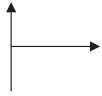
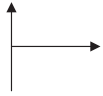
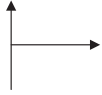
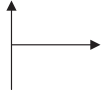
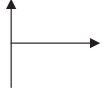
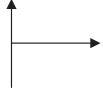
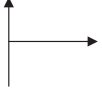
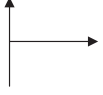
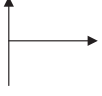
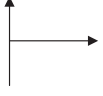
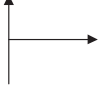
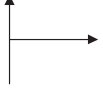
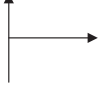
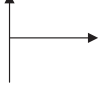
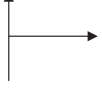
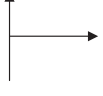
### 3 LABORATORY EXERCISE A: Signal constellations

#### Part A1: Binary signal constellations

Execute the file `binary.m`

Two figures are shown on the screen, each containing four different binary signal sequences. Each sequence has the structure  $s_0(t), s_1(t), s_0(t), s_1(t), \dots$

Classify each sequence by writing an "X" in appropriate places in the table below.

Sequence	$s_0(t)$	$s_1(t)$	PAM	PSK	FSK	QAM	PPM	PWM	Ortho- gonal	Anti- podal	On-Off
Ex			X							X	
1											
2											
3											
4											
5											
6											
7											
8											

## Part A2: 4-ary signal constellations

Execute the file `four.m`

Two figures are shown on the screen, each containing four different signal sequences. Each sequence has the structure  $s_0(t), s_1(t), s_2(t), s_3(t), s_0(t), s_1(t), s_2(t), s_3(t), \dots$

Classify each sequence by writing an “X” in appropriate places in the table below.

Sequence	PAM	PSK	FSK	QAM	PPM	PWM	Orthogonal
1							
2							
3							
4							
5							
6							
7							
8							

**Note! The laboratory assistant must check your answers before continuing with Laboratory exercise B.**



## 4 LABORATORY EXERCISE B:

### The distribution of signal power in the frequency domain for bandpass M-ary PAM signals

Here we study properties in the frequency domain for bandpass M-ary PAM signals. It is assumed that  $M = 2^k$ , equiprobable signal alternatives are used, and that the average amplitude equals zero (see Subsection 2.4.1.1 on page 32 in the compendium). Furthermore, the bit rate is assumed to be  $R_b = 19200$  bps.

In Parts B1 and B2 below, the carrier frequency is  $f_c = 10/T_s$ .

A bandpass PAM signal alternative can be written as  $s_\ell(t) = A_\ell g(t) \cos(\omega_c t)$  in  $0 \leq t \leq T_s$ . The pulse shapes  $g(t)$  studied here are defined in Appendix D in the compendium (see also pages 86–87). Furthermore, in this exercise all pulses have duration  $T = T_s$ .

Execute the file `spektral.m`

Seven figures are shown on the screen.

#### Part B1: Mainlobe and sidelobes

We know from (2.227) on page 93 in the compendium that the power spectral density for bandpass M-ary PAM in this case is,

$$R(f) = \frac{\bar{P}}{2E_g} \left( \underbrace{|G(f + f_c)|^2}_{\text{around } -f_c} + \underbrace{|G(f - f_c)|^2}_{\text{around } +f_c} \right)$$

Figures 1-5 show the power spectral density  $R(f)$  in dB, for frequencies  $f$  around  $+f_c$ . The figures show  $10 \log_{10} (R(f))$ , (normalized with the maximum value) for five different pulse shapes  $g(t)$ .

Note that the frequency content is located around **the carrier frequency  $f_c = 10/T_s$ , and the bit rate equals 19200 bps.**

Assume that  $M = 16$ , and determine for each of the power spectral densities in Figures 1-5: (*Write your answers in the table below.*)

- The double-sided width  $W$  of the mainlobe in kHz.  
(The `zoom` function is very useful.)
- The attenuation of the first sidelobe, relative to the mainlobe, in dB. The attenuation in dB is here denoted  $u$ .
- The pulse shape  $g(t)$ .

$R(f)$	$W$ [kHz]	$u$ [dB]	$g(t)$
Figure 1			
Figure 2			
Figure 3			
Figure 4			
Figure 5			

## Part B2: Classification of pulse shapes

Figure 6 shows the five power spectral densities considered in Part B1. Determine which pulse shape is best (=1), next best (=2), ..., worst (=5) with respect to:

- i) asymptotic decay of the sidelobes      ii) width of the mainlobe.

Write the corresponding pulse shapes in the table below.

Classification of pulse shapes		
	Asymptotic decay	Width of mainlobe
1 (best)		
2		
3		
4		
5 (worst)		

Which pulse shape do you recommend, and why?

.....

## Part B3: Bit rate & Bandwidth

Figure 7 shows normalized power spectral densities in dB for five different values of  $M$ . *The same pulse shape is used in all five cases in Figure 7* (it is one of the pulse shapes studied in Parts B1 and B2).

Observe that the power spectral densities in Figure 7 are shown as functions of  $fT_b$ , and they are located around **the carrier frequency  $f_c = 10/T_b = 192000$  Hz**.

Select two of the five cases in Figure 7. For the two selected cases determine the values of  $M$  that are used (the zoom-function is useful).

Answer: .....

Determine the bandwidth efficiencies for the two selected cases in Figure 7.

Answer: .....

Which  $M$  corresponds to the case with the smallest width of the mainlobe, and how much of the total signal power is confined within the frequency interval  $187900 \text{ Hz} \leq |f| \leq 196100 \text{ Hz}$  in this case?

Answer: .....

**Note! The laboratory assistant must check your answers before continuing with Laboratory exercise C.**





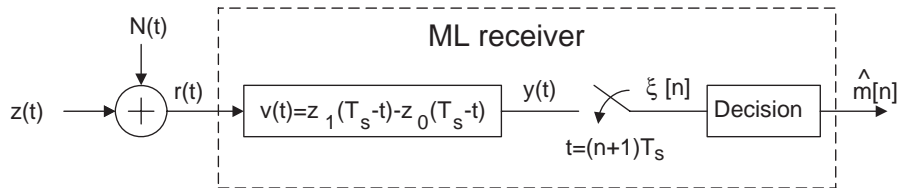
# 5 LABORATORY EXERCISE C:

## The bit error probability

### Part C1: Binary RZ (Return-to-Zero) PAM

Execute the file `brus1.m`

Three figures are shown on the screen. Each figure shows *the information carrying signal*  $z(t)$  ( $z_1(t), z_0(t), z_0(t), z_1(t), \dots$ ), *the additive noise*  $N(t)$ , *the received noisy signal*  $r(t)$ , and *the output signal*  $y(t)$  from the receiver filter  $v(t)$ , see below.



The ML receiver above is identical with Figure 4.10 on page 247 in the compendium. *The average received signal energy per information bit*  $\mathcal{E}_b$  is the same in the three figures, but *the noise parameter*  $N_0$  is different.

Figure 1  $\longleftrightarrow \mathcal{E}_b/N_0 = 1$  (i.e. 0 dB)

Figure 2  $\longleftrightarrow \mathcal{E}_b/N_0 = 5$  (i.e.  $\approx 7$  dB)

Figure 3  $\longleftrightarrow \mathcal{E}_b/N_0 = 10$  (i.e. 10 dB)

Is it possible in Figure 3 to observe the signal  $z(t)$  in the noisy signal  $r(t)$ ? .....

.....

Is it possible to modify the receiver above such that the bit error probability is reduced? .....

.....

Note that the dashed (red)  $y(t)$ -curve is the ideal noise-free version of  $y(t)$ , i.e. the output obtained if there were no additive noise in the received signal. A rectangular input signal gives a triangular output signal. Note also that  $y(t)$  is quite “similar” to the red curve!

*Consequently, the filter  $v(t)$  reduces the effect of the noise significantly!!*

How is the value  $y(5T_s/2)$  used by the receiver above? .....

.....

Is a decision error (i.e. bit error) made at  $t = 7T_b$  for the specific realization of  $y(t)$  in Figure 3?

.....

How many decision errors are made for the specific realization of  $y(t)$  in Figure 3?

.....

Calculate the bit error probability for the receiver in Figure 3 (use a known formula).

$P_b =$  .....

Calculate the bit error probability for the receiver in Figure 1 (use a known formula).

$P_b =$  .....

## Part C2: Binary NRZ PAM

Execute the file **brus2.m**

Three figures are shown on the screen. The difference compared to Part C1 is that now we use NRZ signal alternatives (**N**on-**R**eturn-to-**Z**ero).

Calculate the bit error probability for the receiver in Figure 3 (use a known formula).

$P_b =$  .....

Why are these signal alternatives better than those in Part C1.

.....

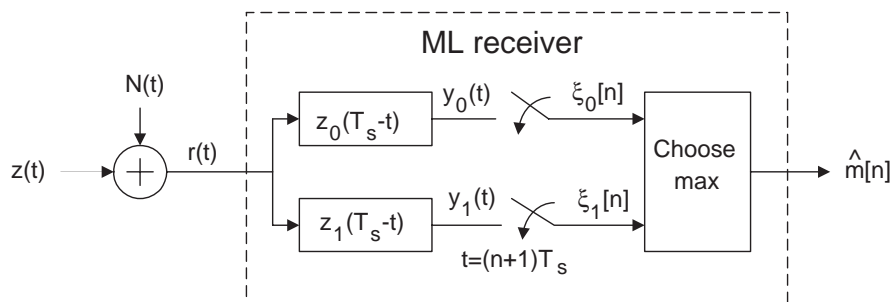
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## Part C3: Binary orthogonal FSK

Execute the file **brus3.m**

Three figures are shown on the screen, as before corresponding to  $\mathcal{E}_b/N_0 = 1$ ,  $\mathcal{E}_b/N_0 = 5$  and  $\mathcal{E}_b/N_0 = 10$  respectively.

The signal alternatives are here binary orthogonal FSK with  $f_1 = 2f_0$ , and the receiver below is used (compare with Figure 4.9 on page 245 in the compendium).



Study the *noise-less versions* of  $y_0(t)$  and  $y_1(t)$  in Figure 3, especially at *the sampling times*, and explain why  $\{y_0(6T_b) > 0, y_1(6T_b) = 0\}$ .

.....

Is a decision error made at  $t = 8T_s$  for the specific realization of  $y(t)$  in Figure 1?

.....

Calculate the bit error probability for the receiver in Figure 1 (use a known formula).

$P_b =$  .....

Calculate the bit error probability for the receiver in Figure 3 (use a known formula).

$P_b =$  .....

Compare the energy efficiency for binary orthogonal FSK with the methods studied in Part C1 and in Part C2. Is it true that binary orthogonal FSK is 2 dB better?

.....

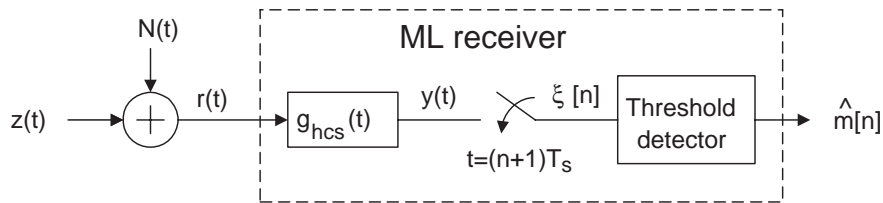
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### Part C4: 4-ary PAM

Execute the file **brus4.m**

Three figures are shown on the screen, as before corresponding to  $\mathcal{E}_b/N_0 = 1$ ,  $\mathcal{E}_b/N_0 = 5$  and  $\mathcal{E}_b/N_0 = 10$  respectively.

The signal alternatives used here are 4-ary PAM with a hcs pulse shape and with  $T = T_s$ . The ML receiver may in this case be implemented as below (compare with Problem 4.24, and also with comments 2–3 in Example 4.4 on pages 242–243 in the compendium).



The symbol error probability is in this case  $P_s = \frac{3}{2} Q(\sqrt{0.8\mathcal{E}_b/N_0})$  (see Table 5.1 on page 361 in the compendium).

Determine the decision thresholds from Figure 3.

$B_0 =$  .....,  $B_1 =$  .....,  $B_2 =$  .....,

Is a decision error made at  $t = 6T_s$  for the specific realization of  $y(t)$  in Figure 1?

.....

Estimate the bit error probability for the situation represented by Figure 3.

.....  $\leq P_b \leq$  .....

What is the advantage, and disadvantage, of using 4-ary PAM instead of binary PAM? .....

**Note! The laboratory assistant must check your answers before ending this Laboratory lesson.**

