# ETSN01/ETSN10 Exam 

March 15th 2018<br>$2 \mathrm{pm}-7 \mathrm{pm}$

## Instructions

- Clearly label each page you hand in with your name or identifier and the page number in the bottom right hand corner.
- Materials allowed: calculator, writing material. No other material or notes are allowed in the examination hall. If your calculator is programmable, the memory must be erased prior to the exam.
- Your answers must be given in clear, legible handwriting. If an answer is not able to be read, it will not be marked.
- All questions should be answered in the booklets provided.
- The exam contains 9 questions and is 9 pages long. It is out of a total of 100 marks.


## Part A: Short answer questions (30 marks)

Question 1 (30 marks)
(a) (2 marks) Give a real-world example that can be modelled with a binomial distribution.
(b) (2 marks) Explain what is meant by a confidence interval
(c) (2 marks) What is the difference between distance vector routing and link state routing?
(d) (2 marks) Explain how bit-round fair queueing approximates processor sharing while still allowing for practical implementation.
(e) (2 marks) For Poissonian traffic, how do the statistical properties of the traffic change at different timescales?
(f) (2 marks) Explain how medium access is controlled using Demand Assigned Multiple Access (DAMA).
(g) (2 marks) Explain the process a node follows to send a packet in p-persistent CSMA.
(h) (2 marks) How are data rate and energy usage related in wireless networks?
(i) (2 marks) Draw a diagram of the topology of a centralised infrastructure network.
(j) (2 marks) In a cellular network, what is meant by macro, micro, and pico cells?
(k) (2 marks) What is meant by a resource block in LTE?
(l) (3 marks) Describe the three target use cases for 5 G .
(m) (3 marks) Give three assumptions used in the Bianchi model of the 802.11 Distributed Coordinatino Function.
(n) (2 marks) What does it mean that TCP's flow control mechanism is credit-based?

## Part B: Long answer questions (70 marks)

Question 2 (4 marks)
The token bucket scheme places a limit on the length of time at which traffic can depart at the maximum data rate. Let the token bucket be defined by a bucket size boctets and a token arrival rate of r octets/second, and let the maximum output data rate by M octets/s.
What is the length of the maximum-rate burst, for $\mathrm{b}=350 \mathrm{kB}, \mathrm{r}=1.5 \mathrm{MB} / \mathrm{s}$, and $\mathrm{M}=20 \mathrm{MB} / \mathrm{s}$ ?
(Note: $1 \mathrm{kB}=1 \times 10^{3} \mathrm{~B}, 1 \mathrm{MB} / \mathrm{s}=1 \times 10^{6} \mathrm{~B} / \mathrm{s}$ )
Question 3 (5 marks)
We have a transmitter T and a receiver R , that communicate over a noisy channel; they can only exchange two symbols $\{0,1\}$, i.e. it is a binary channel; you know from previous measurements that a symbol is accurately detected $72 \%$ of the time (i.e. if you transmit a 1 , it will be correctely detected as a $172 \%$ of the time, the same for a 0 ); you also know that only $20 \%$ of the messages are transmitted as 1. What is the probability that having received a 1 , the symbol is correct?

Question 4 (4 marks)
Consider a link layer protocol which requires lost frames to be retransmitted. The probability of a frame being lost is p . What is the expected number of transmissions to successfully send a frame if acknowledgements are never lost?

Question 5 ( 8 marks)
A router employing weighted fair queuing has three flows. Flow 2 receives 4 times the bandwidth of Flow 1 and Flow 3 receives 2.5 times the bandwidth of Flow 1. Assume the following packets arrive at the router at about the same time and with the same output link. In what order will the packets be sent?

| Packet | Size (bits) | Flow |
| :---: | :---: | :---: |
| 1 | 50 | 1 |
| 2 | 80 | 1 |
| 3 | 30 | 1 |
| 4 | 100 | 2 |
| 5 | 150 | 2 |
| 6 | 125 | 2 |
| 7 | 480 | 3 |
| 8 | 200 | 3 |

Question 6 (11 marks)
Two nodes are communicating using Reservation TDMA. Each node has a mini-slot, in which it transmits the number of packets it has to send. Each node may reserve up to 4 slots in each frame, and unused slots can be used by the other node. The duration of each mini-slot is one tenth the duration of each data slot.

Assume packets arrive at each node according to a Poisson distribution with parameter $\lambda$ in each frame, and that packets that cannot be transmitted in the next frame are dropped.
(a) (5 marks) What is the probability that both nodes reserve their 4 slots in a given frame?
(b) (6 marks) Calculate the utilisation of the system (proportion of time on the channel used to transmit data) for $\lambda=2$.

Question 7 (14 marks)
An LTE base station has a large number of users currently connected in its cell. These users request bearers according to the following traffic model:

- The users collectively make video calls following a Poisson distribution, with an average interarrival time of 1.4 seconds. The call length follows a negative exponential distribution, with an average call length of 4 minutes and 30 seconds.
- The users generate web traffic with an average of 60 concurrent web connections open at any given time. The web traffic has low variance when considered over the set of all users, so can be modelled using only the average value (i.e. a constant, deterministic distribution).

Each video call requires a minimum guaranteed bitrate (GBR) bearer with $480 \mathrm{kbps}\left(480 \times 10^{3} \mathrm{bps}\right)$ of bandwidth allocated to it. Web traffic, on the other hand, uses non-guaranteed bitrate bearers. In total, the base station has $100 \mathrm{Mbps}\left(100 \times 10^{6} \mathrm{bps}\right)$ of capacity available for web and video call traffic.
(a) (3 marks) How many video calls can be served simultaneously?
(b) (4 marks) Write an expression for the probability that a video call will be blocked, that is, there will not be sufficient capacity available to allocate a bearer for the call. (Note: you do not need to solve for this probability numerically.)
(c) (4 marks) If the blocking probability is $1.6 \%$, how much capacity is available for web traffic on average?
(d) (3 marks) If the web traffic were to instead use guaranteed bitrate bearers, with 200 kbps allocated to each connection, would the voice call blocking probability increase or decrease, and why?

Question 8 ( 10 marks)
Tesla recently launched its Falcon Heavy rocket for its first test flight. Now they are planning its successor, the Big Falcon Rocket (BFR), and they have hired you to design the communications network for it. The BFR will have a high-powered radio link back to the mission control site on Earth. When the mission control site is not visible from the BFR, other satellites will be used to relay the signal back to mission control. The end-to-end delays for these links are large: ranging from about 0.25 s (for a direct link) to about 1 s (with multihop relaying via other satellites).
(a) (4 marks) During the test launch, a high definition video stream will be sent from the BFR back to Earth to allow people to follow the launch via the Internet.
How would you design the network (i.e. what protocols, queueing disclipines, or other operating parameters would you recommend) to send the video stream from the BFR to Earth?
(b) (3 marks) What would you change in your design if, instead of a video stream, you needed to communicate control data between the BFR and mission control? The control data requires a fairly steady flow of traffic in both directions, but has a low data rate. The data needs to arrive with as short a delay as possible and with high reliability, both from the BFR to mission control, and from mission control to the BFR.
(c) (3 marks) What would you change in your design if you needed to accommodate both types of traffic (i.e. both a high definition video stream, and the control data) in the same system?

Question 9 (14 marks)
A factory has a wireless network to monitor the operation of all the machines on the factory floor. The network consists of a large number of sensors attached to the machines, which communicate wirelessly to a gateway. From the gateway, there is a wired link to a centralised controller. A diagram of the network is shown below.
There are two wireless channels, which do not interfere with each other. One channel is used for low priority logging data, while the other channel is used for high priority control data. The control data is used for real-time feedback from the machines and so must arrive at the central controller within a maximum delay of 10 ms . The gateway has two radio interfaces and so is able to receive data on both channels simultaneously.
For each type of data (logging and control), the packets generated from all the sensors collectively form a Poisson process. The logging channel operates using a non-persistent CSMA MAC protocol, while the control channel uses 1-persistent CSMA. The logging data traffic has a packet arrival rate of 400


Figure 1: The factory network. Thin arrows represent wireless links; the thick arrow is a wired link.
packets/s, and an average packet length of 2000 bytes. The control traffic has a packet arrival rate of 500 packets/s and an average packet length of 200 bytes. Each wireless channel has a data rate of 8 $\mathrm{Mb} / \mathrm{s}\left(8 \times 10^{6} \mathrm{bits} / \mathrm{s}\right)$.
(a) (6 marks) What will be the normalised offered load and throughput on each of the wireless channels?
(b) (4 marks) The factory's network engineer is trying to decide what kind of queueing discipline to use on the gateway. Both control and logging packets must share the single outgoing link to the centralised controller. There are two possible configurations: either a single queue, into which all packets are placed in order of arrival (first come first served: FCFS), or a priority queueing system, where control packets are placed in a high priority queue, and logging packets are placed in a low priority queue.
If the link to the centralised controller has a data rate of $5 \mathrm{Mb} / \mathrm{s}\left(5 \times 10^{6} \mathrm{~b} / \mathrm{s}\right)$, what will be the occupancy of the single queue using the FCFS discipline? What will be the occupancy of the control data queue using the priority queueing discipline?
You may assume the following:

- that the arrival process in all cases is Poissonian.
- that when the high priority queue is empty, if a new control packet arrives, it can be served immediately.
(In reality, these two assumption will not hold, but you can neglect any differences.)
(c) (4 marks) What is the probability that the control packets meet their maximum delay requirement using the FCFS discipline? What is the probability that they meet it using priority queueing?

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## ETSN10 Formula Sheet

## Probability

$$
\begin{gathered}
P(\bar{A})=1-P(A) \\
P(A \cup B)=P(A)+P(B)-P(A \cap B) \\
P(A \mid B)=\frac{P(A \cap B)}{P(B)}
\end{gathered}
$$

$A, B$ mutually exclusive:

$$
P(A \cap B)=0
$$

$A, B$ independent:

$$
P(A \cap B)=P(A) P(B)
$$

CDF:

$$
F(x)=P(X \leq x)
$$

PDF:

$$
p_{k}=p(k)=P(X=k)
$$

## Discrete random variables

Mean:

$$
\mu=\sum k p_{k}
$$

Variance:

$$
\operatorname{Var}(X)=\sum(k-\mu)^{2} p_{k}=E\left[X^{2}\right]-(E[X])^{2}
$$

Standard deviation:

$$
\sigma=\sqrt{(\operatorname{Var}(X))}
$$

$$
E[a X+b]=a E[X]+b \operatorname{Var}(a X+b)=a^{2} \operatorname{Var}(X)
$$

Bernoulli random variable:

$$
\begin{aligned}
p_{0} & =1-p \\
p_{1} & =p \\
\mu & =p \\
\sigma^{2} & =p(1-p)
\end{aligned}
$$

Geometric:

$$
\begin{aligned}
p_{k} & =(1-p)^{k-1} p \\
\mu & =\frac{1}{p} \\
\operatorname{Var}(X) & =\frac{1-p}{p^{2}}
\end{aligned}
$$

Poisson random variable:

$$
\begin{aligned}
p_{k} & =e^{-\lambda} \frac{\lambda^{k}}{k!} \\
\mu & =\lambda \\
\sigma^{2} & =\lambda
\end{aligned}
$$

## Continuous random variables

Mean:

$$
\mu=\int x p(x) \mathrm{d} x
$$

Variance:

$$
\operatorname{Var}(X)=\int(x-\mu)^{2} p(x) \mathrm{d} x
$$

Negative exponential:

$$
\begin{aligned}
p(x) & =\alpha e^{-\alpha x} \\
P(X \leq x) & =1-e^{-\alpha x} \\
\mu & =\frac{1}{\alpha}
\end{aligned}
$$

$$
\operatorname{Var}(X)=\left(\frac{1}{\alpha}\right)^{2}
$$

Gaussian:

$$
p(x)=\frac{1}{\sigma \sqrt{2 \pi}} e^{-\frac{(x-\mu)^{2}}{2 \sigma^{2}}}
$$

## Multiple random variables

Joint CDF:

$$
F(x, y)=P(X \leq x, Y \leq y)
$$

$X, Y$ independent:

$$
F(x, y)=F(x) F(y)
$$

$$
\begin{aligned}
E[X+Y] & =E[X]+E[Y] \\
E[X Y] & =E[X] E[Y] \\
\operatorname{Var}(X+Y) & =\operatorname{Var}(X)+\operatorname{Var}(Y)
\end{aligned}
$$

Properties of random variables $X, Y$, constants $a, b$ :

$$
\begin{gathered}
E[a X+b]=a E[X]+b \\
\operatorname{Var}(a X+b)=a^{2} \operatorname{Var}(X) \\
E[X+Y]=E[X]+E[Y]
\end{gathered}
$$

Covariance:
$\operatorname{Cov}(X, Y)=E[(X-E[X])(Y-E[Y])]=E[X Y]-E[X] E[Y]$
Correlation coefficient:

$$
r(X, Y)=\frac{\operatorname{Cov}(X, Y)}{\sigma_{x} \sigma_{y}}
$$

## Stochastic Processes

Autocorrelation:

$$
R\left(t_{1}, t_{2}\right)=E\left[x\left(t_{1}\right) x\left(t_{2}\right)\right]
$$

Autocovariance:

$$
C\left(t_{1}, t_{2}\right)=\operatorname{Cov}\left(x\left(t_{1}\right), x\left(t_{2}\right)\right)
$$

## Sampling and random numbers

Sample mean:

$$
\bar{z}=\frac{1}{n} \sum_{i=1}^{n} z_{i}
$$

Sample variance:

$$
\hat{V}=\frac{1}{n} \sum_{i=1}^{n}\left(z_{i}-\bar{z}\right)^{2}
$$

Unbiased sample variance (Bessel's correction):

$$
s^{2}=\frac{1}{n-1} \sum_{i=1}^{n}\left(z_{i}-\bar{z}\right)^{2}
$$

Confidence intervals:

$$
\text { Confidence }=\operatorname{Pr}\left(|\bar{z}-\mu| \leq \alpha \times \frac{\sigma}{\sqrt{n}}\right.
$$

$\alpha=1.96$ gives confidence of $95 \%$. Inverse method:

$$
X=F^{-1}(Y)
$$

## Medium Access Control

FDMA/TDMA rate of work:

$$
\eta=\frac{1}{n \times \nu} \sum_{i=1}^{n} \rho_{i}
$$

Polling efficiency:

$$
E=\frac{T_{t}}{T_{t}+T_{\text {idle }}+T_{\text {poll }}}
$$

ALOHA throughput:

$$
S=G e^{-G}
$$

Slotted ALOHA throughput:

$$
S=G e^{-2 G}
$$

1-persistent CSMA throughput:

$$
S=\frac{G e^{-G}(1+G)}{G+e^{-G}}
$$

Non-persistent CSMA throughput:

$$
S=\frac{G}{1+G}
$$

CSMA utilisation:

$$
\begin{gathered}
A=k p(1-p)^{k-1} \\
E[w]=\frac{1-A}{A} \\
U=\frac{1}{1+2 a+a(1-A) / A}
\end{gathered}
$$

## Queueing Systems

Kendall Notation parameters:

1. Arrival distribution
2. Service distribution
3. Number of servers
4. Total capacity (default: infinite)
5. Population size (default: infinite)
6. Service disciplien (default: FIFO)

Little's Law:

$$
E[R]=\lambda E\left[T_{R}\right]
$$

Occupancy:

$$
\rho=\frac{\lambda}{\mu}
$$

$\mathrm{M} / \mathrm{M} / 1$
Number of items in the system:

$$
E[R]=\frac{\rho}{1-\rho}
$$

Total time in system:

$$
E\left[T_{R}\right]=\frac{1}{\mu(1-\rho)}
$$

Waiting time:

$$
E\left[T_{W}\right]=\frac{\rho}{\mu(1-\rho)}
$$

Delay bound:

$$
\operatorname{Pr}\left(T_{R} \leq t\right)=1-e^{-\mu(1-\rho) t}
$$

$\mathrm{M} / \mathrm{M} / \mathbf{1} / \mathrm{n}$
Blocking probability:

$$
P_{B}=\frac{1-\rho}{1-\rho^{n+1}} \rho^{n}
$$

Carried traffic:

$$
\gamma=\lambda\left(1-P_{B}\right)
$$

Little's Law:

$$
E[R]=\gamma E\left[T_{R}\right]
$$

Number of items in the system:

$$
E[R]=\frac{1-\rho}{1-\rho^{n+1}} \sum_{i=0}^{n} i \rho^{i}
$$

## Erlang (M/M/n/n)

Offered traffic:

$$
A=\lambda h
$$

Service rate:

$$
\mu=\frac{1}{h}
$$

Erlang loss function:

$$
E_{n}(A)=P_{B}=\frac{\frac{A^{n}}{n!}}{\sum_{j=0}^{n} \frac{A^{j}}{j!}}
$$

Carried traffic:

$$
A_{c}=A\left(1-P_{B}\right)
$$

Lost traffic:

$$
A-A_{c}
$$

## Jackson Networks

Probability a packet leaves the network from node $i$ :

$$
1-\sum_{j=1}^{n} r_{i j}
$$

Total arrival rate to node $i$ :

$$
\lambda_{i}=\gamma_{i}+\sum_{j=1}^{n} \lambda_{j} r_{j i}
$$

## Queueing Disciplines

Kleinrock Conservation Law:

$$
\sum_{n=1}^{N} \rho_{n} q_{n}=C
$$

Processor Sharing:

$$
\begin{gathered}
F_{i}^{\alpha}=S_{i}^{\alpha}+P_{i}^{\alpha} \\
S_{i}^{\alpha}=\max \left\{F_{i-1}^{\alpha}, A_{i}^{\alpha}\right\}
\end{gathered}
$$

Generalised Processor Sharing:

$$
F_{i}^{\alpha}=S_{i}^{\alpha}+\frac{P_{i}^{\alpha}}{w_{\alpha}}
$$

## Network Architectures

Packet reception rate:

$$
P R R=\frac{S}{T}
$$

Cellular frequency re-use:

$$
D=R \sqrt{3 K}
$$

## Congestion Control

Token bucket:

$$
R=\rho T+\beta
$$

## TCP

Max bandwidth:

$$
B W_{\max }=\frac{M S S \times C}{R T T \times \sqrt{p}} \quad C=\sqrt{\frac{3}{2}}
$$

Normalised throughput:

$$
S= \begin{cases}1 & W \geq 2 R D \\ \frac{W}{2 R D} & W<2 R D\end{cases}
$$

Expected (average) round trip time:
$\operatorname{ERTT}(K+1)=\frac{K}{K+1} \operatorname{ERTT}(K)+\frac{1}{K+1} \operatorname{RTT}(K+1)$
Smoothed round trip time:
$S R T T(K+1)=\alpha \times S R T T(K)+(1-\alpha) \times R T T(K+1)$
Retransmission timeout with SRTT:
$R T O(K+1)=\min \{U B, \max \{L B, \beta \times S R T T(K+1)\}\}$
Van Jacobson's algorithm:
$\operatorname{DRTT}(K+1)=(1-\alpha) \times D R T T(K)+\alpha \times(S R T T-E R T T)$

$$
R T O=E R T T-4 \times D R T T
$$

Exponential backoff:

$$
R T O_{i+1}=q \times R T O_{i}
$$

## Other useful formulas

Geometric series:

$$
\begin{gathered}
\sum_{k=0}^{n-1} a r^{k}=a \frac{1-r^{n}}{1-r} \\
\sum_{k=0}^{\infty} a r^{k}=\frac{a}{1-r}
\end{gathered}
$$

Expectation of geometric series:

$$
\sum_{k=0}^{\infty} k a r^{k}=\frac{a r}{(1-r)^{2}}
$$

