LUND UNIVERSITY
Electrical and Information Technology

Communication and Networks

Problems Framinmg

2016

Problems

1. Assuming even parity, determine the value of the parity bit for each of the following bit sequences:

   (a) 1001001
   (b) 1100111
   (c) 1001011
   (d) 1110111

2. Calculate the CRC for the following messages if the generator polynomial is $g(x) = x^3 + x^2 + 1$.

   (a) 00111010
   (b) 1010011110
   (c) 111000111

(d) 1100110011

3. Assume that a four bit CRC with generator polynomial $C(x) = x^4 + x^3 + 1$ has been used.Check if the following messages are correctly received.

   (a) 11010111
   (b) 10101101101
   (c) 10001110111

4. Remove the bits stuffed from the following sequences when the flag 01111110 is used.

   (a) $010101111101011101111100\dots$
   (b) $0101011111101011101111110|ldots$

5. Bit stuff the following sequences when the flag 01111110 is used.

   (a) 0001111110111110011111001
   (b) 000111111111111111111111111111110011111001

6. Consider a data frame of length N. The bits in the frame can be assumed to be i.i.d. with $P(d_i = 0) = P(d_i = 1) = \frac{1}{2}$. Before transmission the standard start and stop flags are appended, i.e. Flag $=$ 01111110, and bit-stuffing performed. That is, we transmit the frame according to Figure 1.

Flag	Bit-stuffed frame	Flag

Figure 1: A transmitted frame with start and stop flags and bit-stuffing.

Derive the expected overhead added by framing and bit-stuffing. What happens as N becomes large?

Comment: Bit-stuffing will occur in positions after the patterns 01^5, 01^{10}, 01^{15}, ..., where the notation 1^n means n consecutive 1s. The probabilities for the patterns 01^{10}, 01^{15}, ... are very small and can be neglected in the derivations. Furthermore, if the frame starts with 1^5 this will also result in a stuffed bit.

7. Assume that a Go-back-N ARQ uses a window of size 15. How many bits are needed to define the sequence number?

8. In Go-back-N ARQ, the size of the sender window must be less than 2^m, where m is the number of bits used for the representation of sequence numbers. Show in an example, by drawing a message sequence, why the size of the sender window must be strictly less than 2^m.

9. A Selective Repeat ARQ is using 7 bits to represent the sequence numbers. What is the maximum size of the sliding window?

10. Assume that host A sends frames to host B and uses sequence numbers coded with 3 bits. A Go-back-N ARQ is used with a sliding window of size 4. Show the content of the window in the following cases:

    (a) Before A has sent any frames.

    (b) After A has sent frames 0,1 och 2; B has sent ACK for frames 0 and 1; and these ACKs have been received by A.

    (c) After A has sent 3, 4, 5 and 6; B has sent ACK for 4; and this ACK has received by A.

11. Host A uses a stop-and-wait ARQ when sending frames to host B. Assume that the distance between A and B is 4000 km. Answer the following questions.

    (a) After how long time can host A receive an ACK for a frame? Use the sped of light as propagation velocity and assume that the time for processing at node B is zero.

(b) How long is the transmission time for a frame of 1000 Bytes if the transmission rate is 100 000 kbps?

(c) Use the answers in a) and b) in order to determine the percentage o time that host A is idle.

12. Consider a 1 km long point-to-point link with maximum bit rate of 10 Gbps. The propagation time is approx 1/3 of the speed of light in vacuum. Equal sized frames are used in both directions.

    (a) What will the windows size (in frames) be if the link shall be utilised to its max?

    (b) How many bits are needed for the sequence number?

    (c) What is the maximum windows size in a Go-Back-N system given this number of bits in the sequence number?

Solutions

1. Add a bit so the weight of the vector is even. That means the modulo 2 sum equals 0.

   (a) 1
   (b) 1
   (c) 0
   (d) 0

2. With a CRC polynomial of degree k you should add k bits to the vector, representing the reminder when the vector is divided by g. The derivation of the extra bits is done by represented by the vector as a polynomial, $p(x)$. Then the polynomial representation of the extra bits, $r(x)$, is the reminder when $p(x)x^k$ is divided by $g(x)$, i.e. $R_{g(x)}(p(x)x^k)$. Then the codeword that to be transmitted is $c(x) = p(x)x^k + r(x)$,. The codeword is divided by the CRC polynomial, i.e. the reminder when $c(x)$ is divided by $g(x)$ is zero. In this problem $k = 3$ and three bits are added.

   (a) $R_{x^3+x^2+1}\big((x^5 + x^4 + x^3 + x) \cdot x^3\big) = x \rightarrow$ 00111010010 where the last three bits are the CRC bits.
   (b) $R_{x^3+x^2+1}\big((x^9 + x^7 + x^4 + x^3 + x^2 + x) \cdot x^3\big) = x^2 + 1 \rightarrow$ 1010011110101
   (c) $R_{x^3+x^2+1}\big((x^8 + x^7 + x^6 + x^2 + x + 1) \cdot x^3\big) = x^2 + x + 1 \rightarrow$ 111000111111
   (d) $R_{x^3+x^2+1}\big((x^9 + x^8 + x^5 + x^4 + x + 1) \cdot x^3\big) = x^2 + 1 \rightarrow$ 1100110011101

3. The CRC can be checked by a division since the generator polynomial $g(x)$ divides a polynomial iff it is a codeword, i.e. $g(x)\Big|\big(p(x)x^k + r(x)\big)$. So if the reminder is zero the vector is accepted.

   (a) $R_{x^4+x^3+1}\big(x^7 + x^6 + x^4 + x^2 + x + 1\big) = x^2 + x \Rightarrow$ Not accepted
   (b) $R_{x^4+x^3+1}\big(x^{10} + x^8 + x^6 + x^5 + x^3 + x^2 + 1\big) = x^3 + x^2 + 1 \Rightarrow$ Not accepted

(c) $R_{x^4+x^3+1}\big(x^{10}+x^6+x^5+x^4+x^2+x+1\big)=0 \Rightarrow$ Accepted

4. (a) $0101011111\underline{0}10111011111\underline{0}0\ldots \Rightarrow 010101111110111011111 0\ldots$

   (b) $0101\,\underbrace{01111110}_{\text{flag}}\,10111011111\underline{0}\ldots \Rightarrow 010101111110101110111111\ldots$

5. (a) $00011111\underset{\underset{0}{\uparrow}}{}1011111\underset{\underset{0}{\uparrow}}{}0011111\underset{\underset{0}{\uparrow}}{}001 \Rightarrow 0001111101011111000111110001$

   (b) $00011111\underset{\underset{0}{\uparrow}}{}11111\underset{\underset{0}{\uparrow}}{}11111\underset{\underset{0}{\uparrow}}{}11111\underset{\underset{0}{\uparrow}}{}11111\underset{\underset{0}{\uparrow}}{}1110011111\underset{\underset{0}{\uparrow}}{}001$

   $\Rightarrow 0001111101111101111101111101111101110011111 0001$

6. The first position there can be a bit stuffed is after the fifth bit. This happens if the first five bits are all 1, which happens with probability $\frac{1}{2^5}$. There can also be stuffed bits after positions 6 to N, each with probability $\frac{1}{2^6}$ for the preceding pattern 011111. That is for a frame with length N the expect number of stuffed bits is

$$E\big[BS\big] = 1\frac{1}{2^5} + \sum_{n=6}^{N} 1\frac{1}{2^6} = \frac{1}{2^5} + \frac{N-6}{2^6} = \frac{N-4}{2^6}$$

When N grows it is interesting to consider the average number of stuffed bits per bit in the frame, i.e.

$$\frac{1}{N}E\big[BS\big] = \frac{1}{N}\Big(\frac{N-4}{2^6}\Big) = \frac{1}{2^6} + \frac{1}{N2^4} \to \frac{1}{2^6}, \quad N\to\infty$$

That is, for large frames the average number of bit stuffed is approximately $\frac{N}{2^6}$.

7. Number bits $= \lceil \log_2 15 \rceil = 4$

8. In a communication system, the sequence number is represented by $m = 2$ bits and is thus able to hold $2^m = 4$ frames. If the sliding window size also was four, and the sender has transmitted all four frames in the sequence but the acknowledgement for frame 0 has not been received. Nevertheless, all frames were received on the receiver side. If the sender was now to retransmit the first frame, number 0, then the receiver would treat that as the first frame in the subsequent sequence. The sender would then treat the resulting acknowledgment as as if though it was intended for the first frame from the first sequence. The sender would proceed with sending the first frame from the second sequence, which will be discarded by the receiver as there it thinks it has already received that frame. On the other hand, if the sliding windows was instead of size 3, i.e. $2^m - 1$, then the receiver would be bounded to the first sequence and the problem resolved. See Figure 2 for an example when the window size 2^m does not work.

Answer:

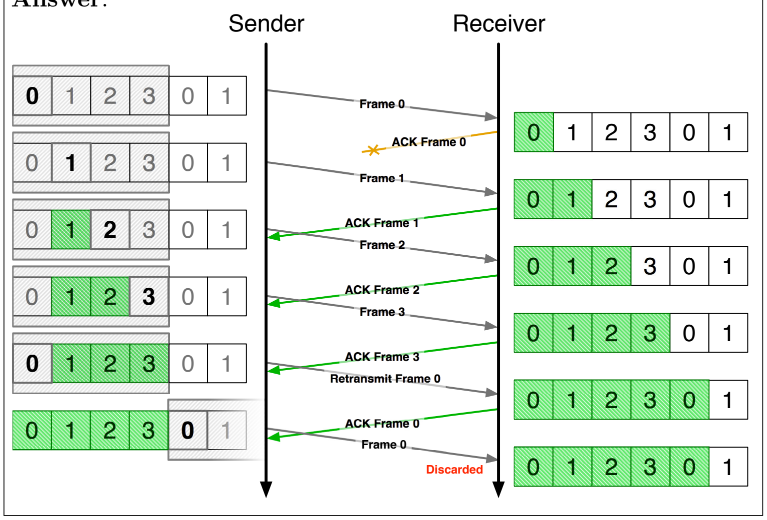
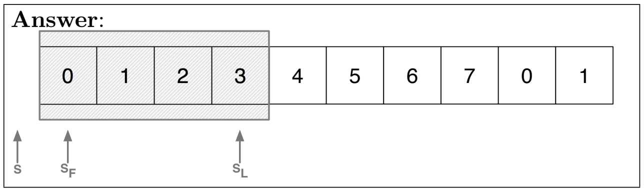


Figure 2: Example showing the window size in Go-back-N must be strictly less than 2^m.

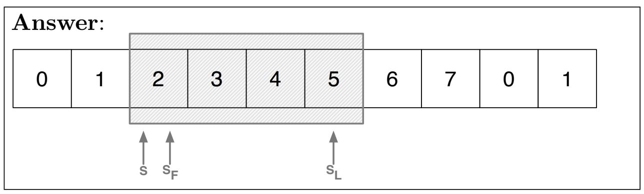
9. In Selective repeat-ARQ the windows size can at the most be half of the largest possible sequence number. It is stated that the sequence number is represented by 7 bits, thus a total of $\frac{2^7}{2} = \frac{128}{2} = 64$ frames can be monitored.

10. Since the sequence number is represented by 3 bits, it can represent 8 frames. The S marker marks the last transmitted frame, S_F , where the window starts, i.e. one frame after the last consecutively acknowledge frame. Moreover, S_L marks the end of the window.

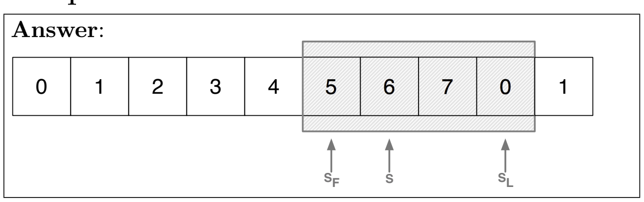
(a) Before anything has been sent, the windows will envelope the first 4 frames.



(b) ACKs have been received for frames 0 and 1, hence the S_F marker will be at frame 2, the earliest unacknowledged frame.



(c) The S marker will point to packet 6, as it is the highest sent frame. Nevertheless, only frame 4 has been acknowledged, as such SF will not mark the beginning of the open window at frame 5.



11. (a) The speed of light in vacuum is $c = 299792458$ m/s. Sending a packet in one direction thus takes $\frac{4000000}{c} = 13.34$ ms. Assuming that the process time is 0 seconds, the total time will henceforth be packet round trip of $2 \cdot 13.34 = 26.68$ ms.

(b) As one byte contains 8 bits, 1000 bytes are 8000 bits. Transmission time is thus $\frac{8 \cdot 10^3 \text{ b}}{100 \cdot 10^6 \text{ bps}} = 80\ \mu$s. Note that bytes are mainly used when referring to data storage as one byte is historically the size of an ASCII symbol, and thus the smallest addressable space in a computers memory.

(c) Out of the total transmission (0.08 ms) and propagation (26.68 ms) time the sender is only occupied while transmitting. Hence, for the event the that sender sends multiple consecutive frames it is vacant $\frac{26.68}{26.68+0.08} = 99.7\%$ of the time.

12. (a) $RTT = 2 \times (1 \times 10^3/100000 \times 10^3 + 8000/10 \times 10^9) = 21.6 \times 10^{-6}$
    Number of frames that can be sent during RTT $21.6 \times 10^{-6}/0.8 \times 10^{-6} = 27$

    (b) 5 since $2^5 = 32 > 27$

    (c) $2^5 > \text{winsize}_{\text{max}} = 31$