

P8

a)

$$\begin{aligned} E(p) &= Np(1-p)^{N-1} \\ E'(p) &= N(1-p)^{N-1} - Np(N-1)(1-p)^{N-2} \\ &= N(1-p)^{N-2}((1-p) - p(N-1)) \\ E'(p) = 0 &\Rightarrow ((1-p) - p(N-1)) = 0; p = \frac{1}{N} \end{aligned}$$

b)

$$E(p^*) = N \frac{1}{N} \left(1 - \frac{1}{N}\right)^{N-1} = \left(1 - \frac{1}{N}\right)^{N-1} = \frac{\left(1 - \frac{1}{N}\right)^N}{1 - \frac{1}{N}}$$

$$\lim_{N \rightarrow \infty} \left(1 - \frac{1}{N}\right) = 1 \quad \lim_{N \rightarrow \infty} \left(1 - \frac{1}{N}\right)^N = \frac{1}{e}$$

Thus

$$\lim_{N \rightarrow \infty} E(p^*) = \frac{1}{e}$$

P9

See figure 6.11 and reasoning on pp. 486

$$\begin{aligned} E(p) &= Np(1-p)^{2(N-1)} \\ E'(p) &= N(1-p)^{2(N-2)} - Np2(N-1)(1-p)^{2(N-3)} \\ &= N(1-p)^{2(N-3)}((1-p) - p2(N-1)) \end{aligned}$$

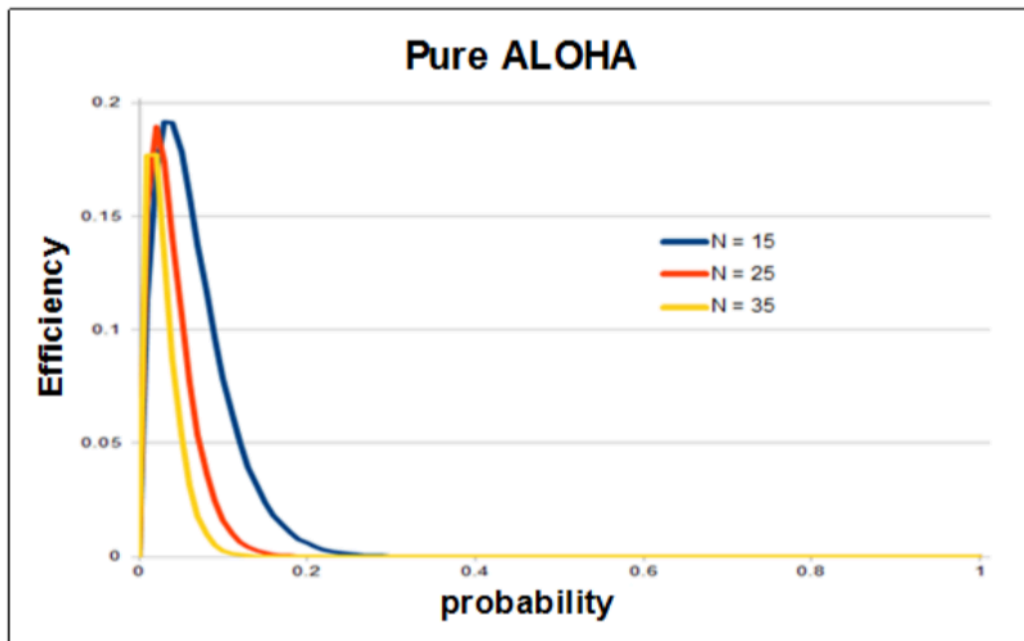
$$E'(p) = 0 \Rightarrow p^* = \frac{1}{2N-1}$$

$$E(p^*) = \frac{N}{2N-1} \left(1 - \frac{1}{2N-1}\right)^{2(N-1)}$$

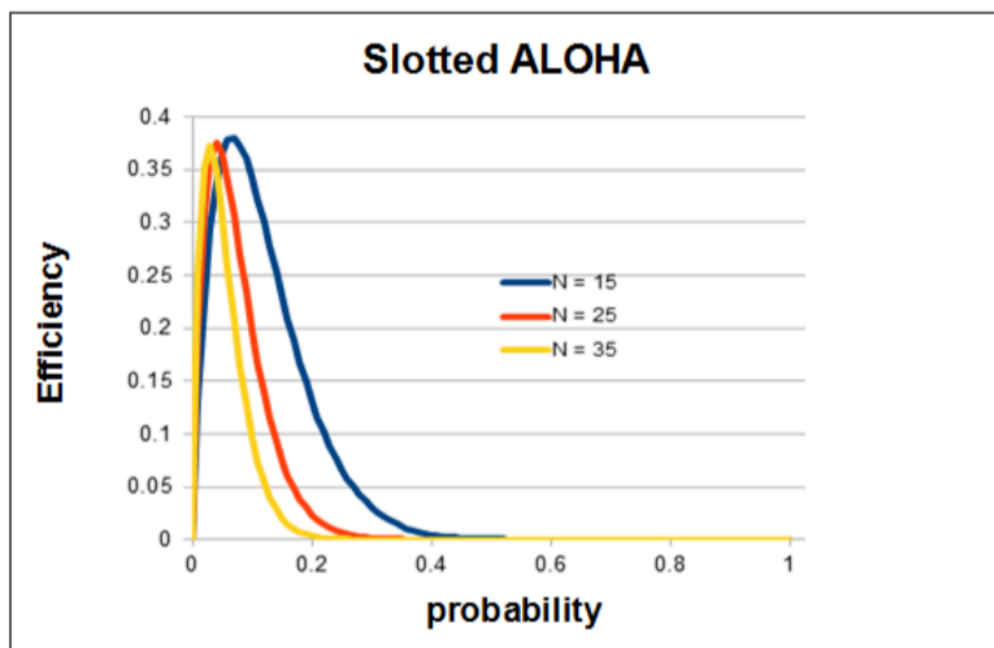
$$\lim_{N \rightarrow \infty} E(p^*) = \frac{1}{2} \cdot \frac{1}{e} = \frac{1}{2e}$$

P12

a)



b)



P13

Time to transmit a polling round is

$$N(Q/R + d_{poll})$$

The number of bits in a polling round is  $NQ$  and the maximum throughput therefore becomes:

$$\frac{NQ}{N(Q/R + d_{poll})} = \frac{R}{1 + \frac{d_{poll}R}{Q}}$$

Note, since all nodes have the same rate it is enough to look at an individual node and therefore remove  $N$ .