Simulation Course: Lab Exercise 2

Solving Linear and Integer Programming Problems

Exercise 1: Product Mix Problem

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\begin{aligned}
&\textit{Maximize} \quad 13x_1 + 11x_2 && \text{(Primal Problem)}\\
&\text{Subject to:}\\
&\qquad 4x_1 + 5x_2 \quad \le 1500 \quad \text{(Storage cosntraint)}\\
&\qquad 5x_1 + 3x_2 \quad \le 1575 \quad \text{(Raw material constraint)}\\
&\qquad x_1 + 2x_2 \quad \le 420 \quad \text{(Production time constraint)}\\
&\qquad x_1 \ge 0,\ x_2 \ge 0
\end{aligned}
$$

$$
\begin{aligned}
&\textit{Minimize} \quad 1500y_1 + 1575y_2 + 420y_3 && \text{(Dual Problem)}\\
&\text{Subject to:}\\
&\qquad 4y_1 + 5y_2 + 1y_3 \quad \ge 13 \quad \text{(D1)}\\
&\qquad 5y_1 + 3y_2 + 2y_3 \quad \ge 11 \quad \text{(D2)}\\
&\qquad y_1 \ge 0,\ y_2 \ge,\ y_3 \ge 0
\end{aligned}
$$

I. Implement two linear programs in MATAB for the primal and dual problems, obtain the optimal solutions for the two problems, and compare them.

II. Implement a sensitivity analysis on the primal problem with respect to the second constraint (i.e. raw material constraint). Run your analysis and answer the following questions:

   a. What is the shadow price of the raw material?

   b. Increase the capacity of raw materials by a step size of 15, starting from the initial point 1575 and ending at 1725. Thus, we will have 11 data points as follows:
   1575 1590 1605 1620 1635 1650 1665 1680 1695 1710 1725
   Run your linear program with each data point as the new capacity of raw materials, then record the value of the objective function for each data point, and plot a line diagram showing the relation between the data points and the objective value.

   c. In your sketched diagram, at which data point(s) does the slope change? What is the slope of each line segment? How does the slope relate to shadow price?

Exercise 2: Staff Scheduling Problem

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\begin{aligned}
&\textit{Minimize} \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \\
&\text{Subject to:} \\
&\qquad x_1 + x_4 + x_5 + x_6 + x_7 \ge 8 \qquad (C1) \\
&\qquad x_1 + x_2 + x_5 + x_6 + x_7 \ge 6 \qquad (C2) \\
&\qquad x_1 + x_2 + x_3 + x_6 + x_7 \ge 5 \qquad (C3) \\
&\qquad x_1 + x_2 + x_3 + x_4 + x_7 \ge 4 \qquad (C4) \\
&\qquad x_1 + x_2 + x_3 + x_4 + x_5 \ge 6 \qquad (C5) \\
&\qquad x_2 + x_3 + x_4 + x_5 + x_6 \ge 7 \qquad (C6) \\
&\qquad x_3 + x_4 + x_5 + x_6 + x_7 \ge 9 \qquad (C7) \\
&\qquad x_i \in Z^+,\ i = 1,\ 2, \ldots 7
\end{aligned}
$$

I. Implement an integer program, solve the program, and obtain the optimal solution.

II. Relax the program to real-valued positive variables. Then modify your code to implement a linear program for this relaxed problem. Run your code, obtain the optimal solution, and compare it with the solution of the integer program in part (I). Which solution is better? Which solution is feasible? Can you propose a general relation between the solutions of the integer and the relaxed problems?