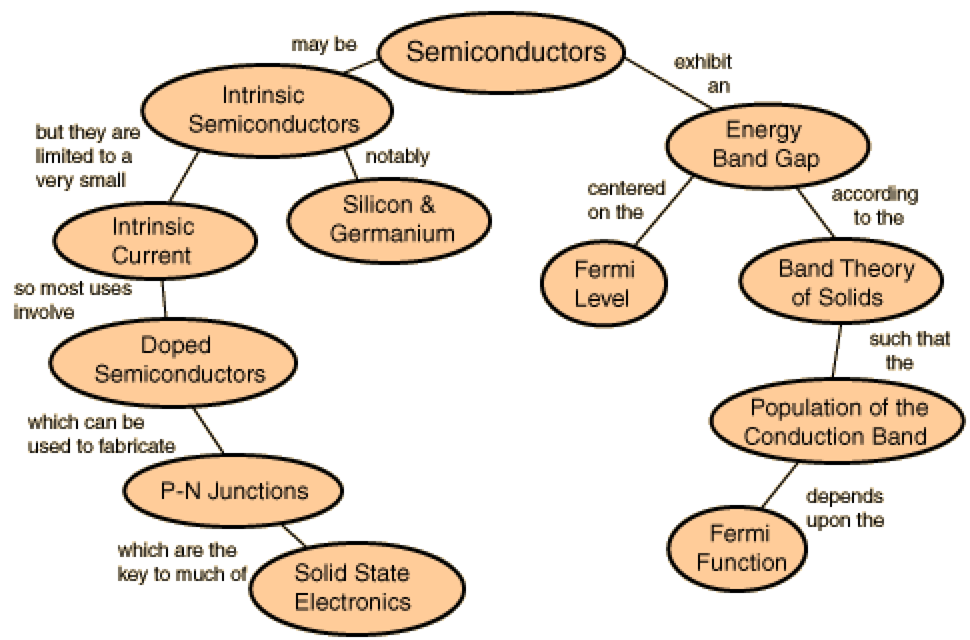
Semiconductors – a brief introduction

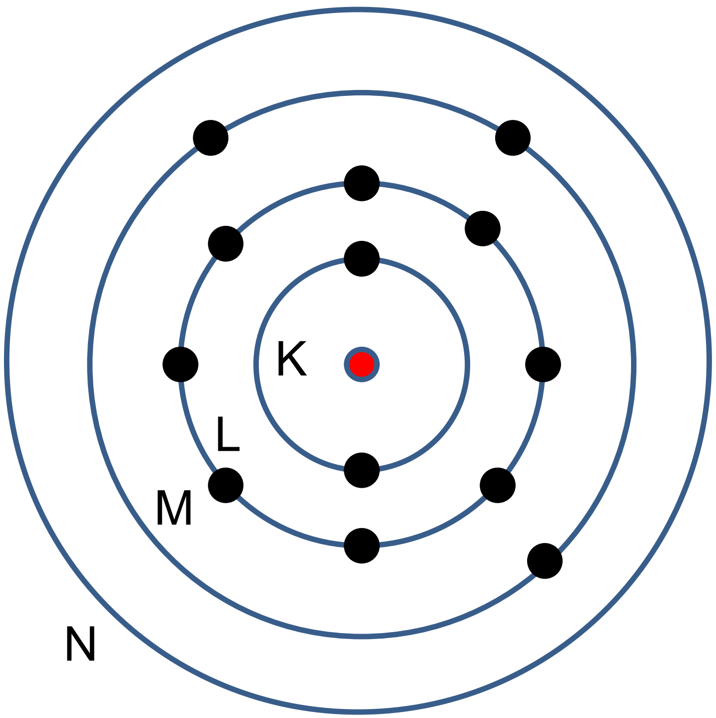
- Band structure – from atom to crystal
- Fermi level – carrier concentration
- Doping
- Transport (drift-diffusion)

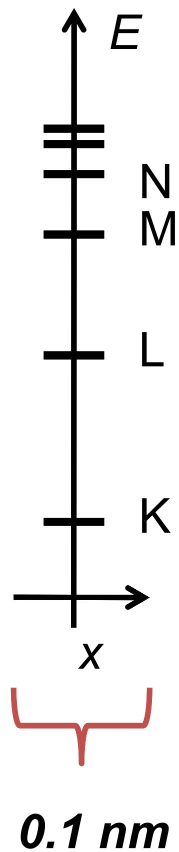
Reading: (Sedra/Smith 7th edition) 1.7-1.9

Hyperphysics (link on course homepage) basic introduction to semiconductors (almost no equations)

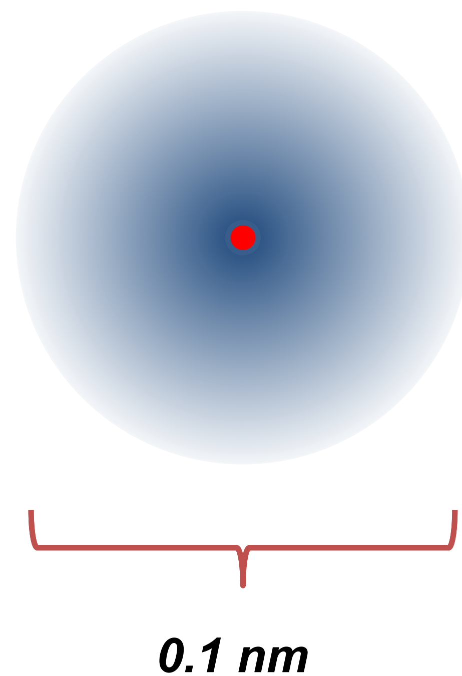


Atomic energy levels

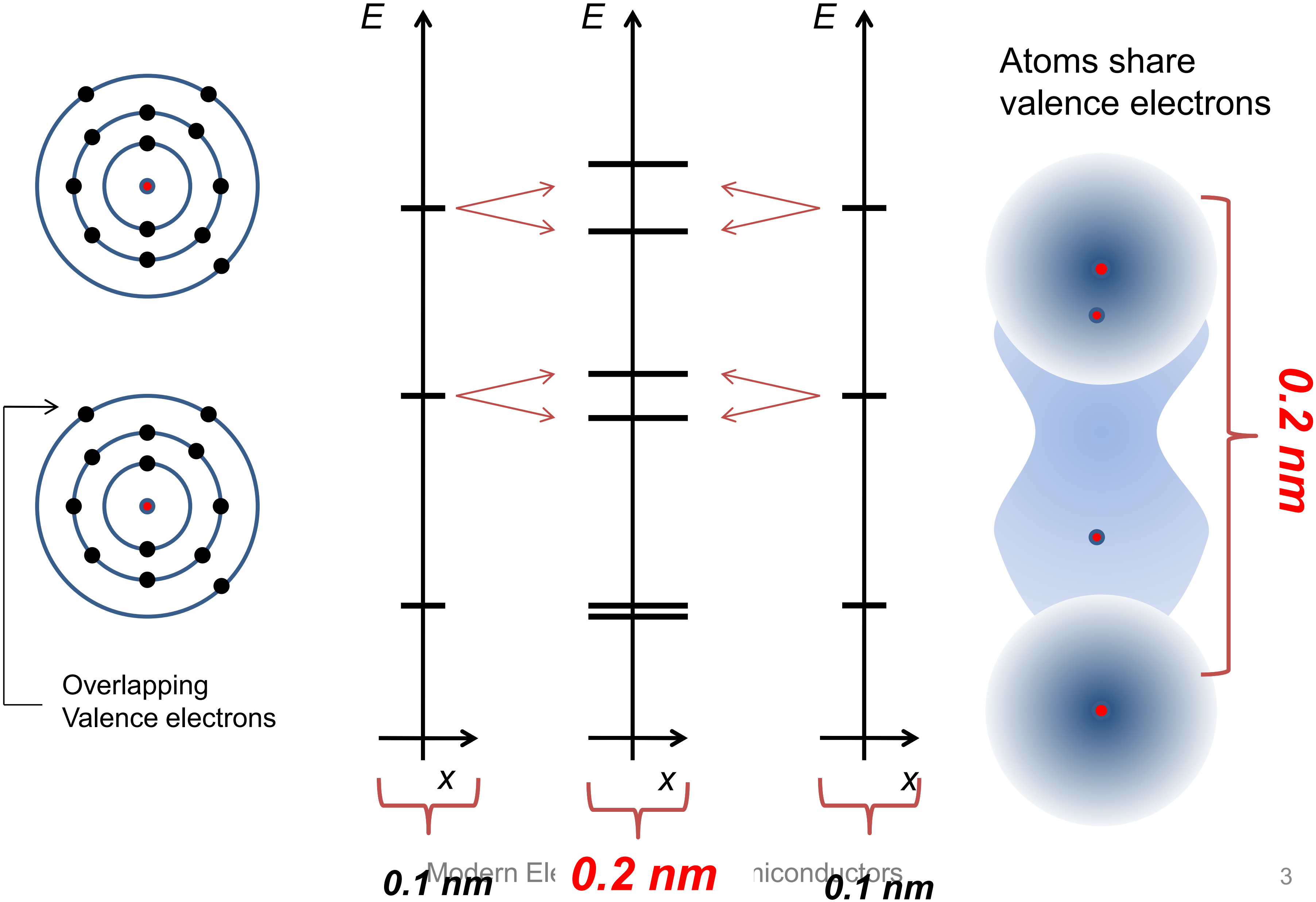




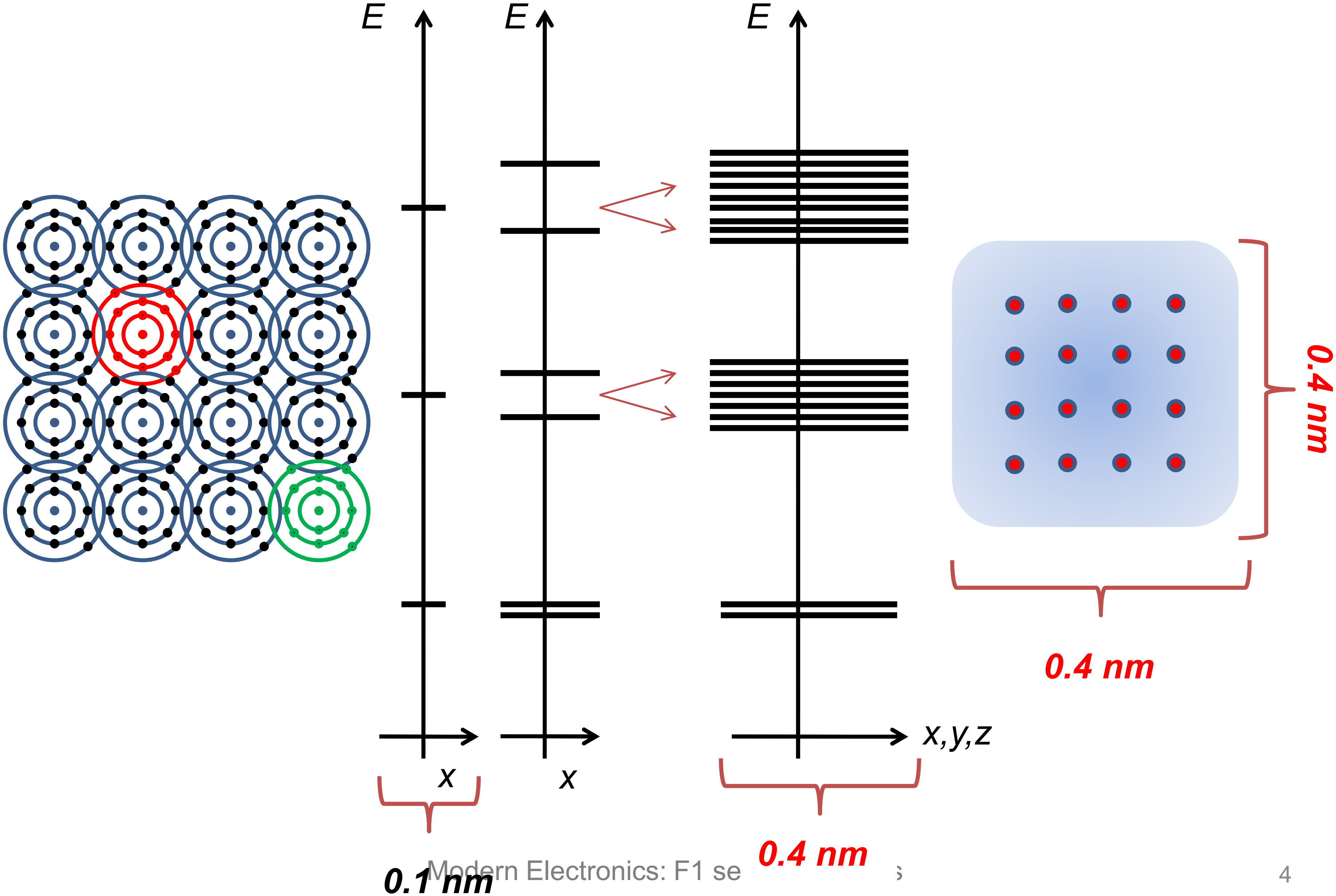
Quantum mechanics:
Wavefunction gives describes probablility to find electron



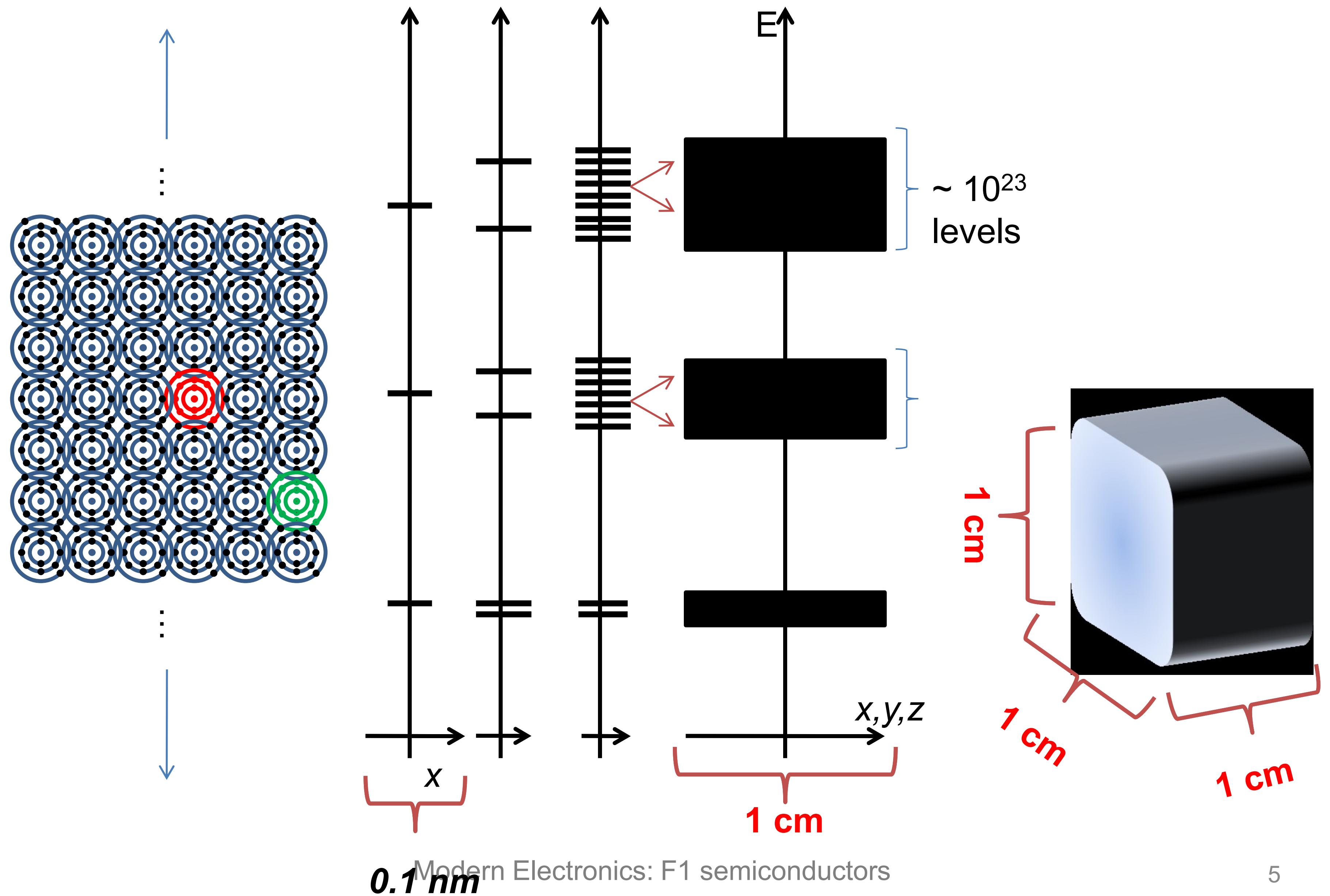
2-atomic molecule – Pauli principle



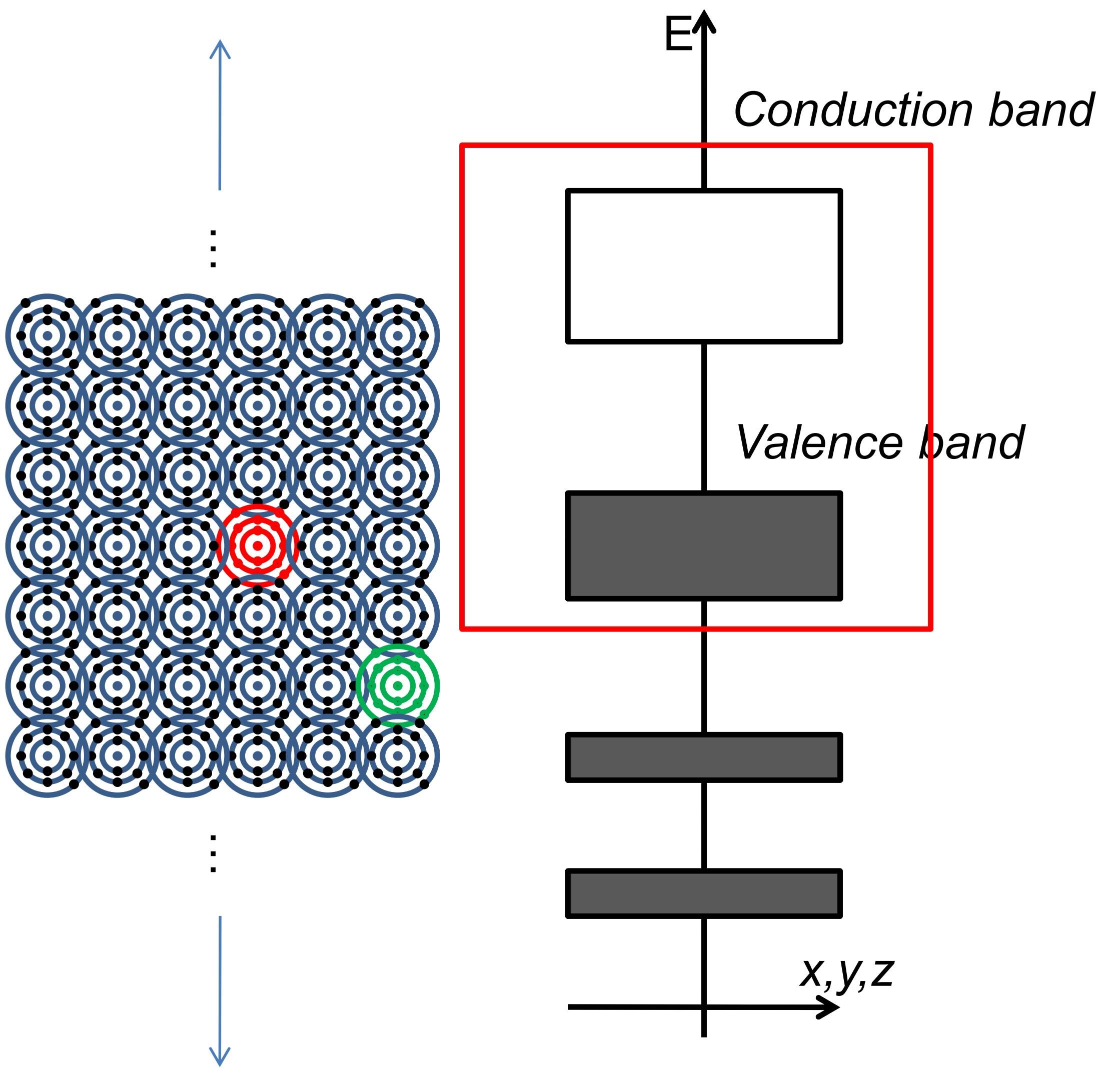
16-atomic molecule



10^{23}-atomic molecule – energy bands



Valence and conduction bands



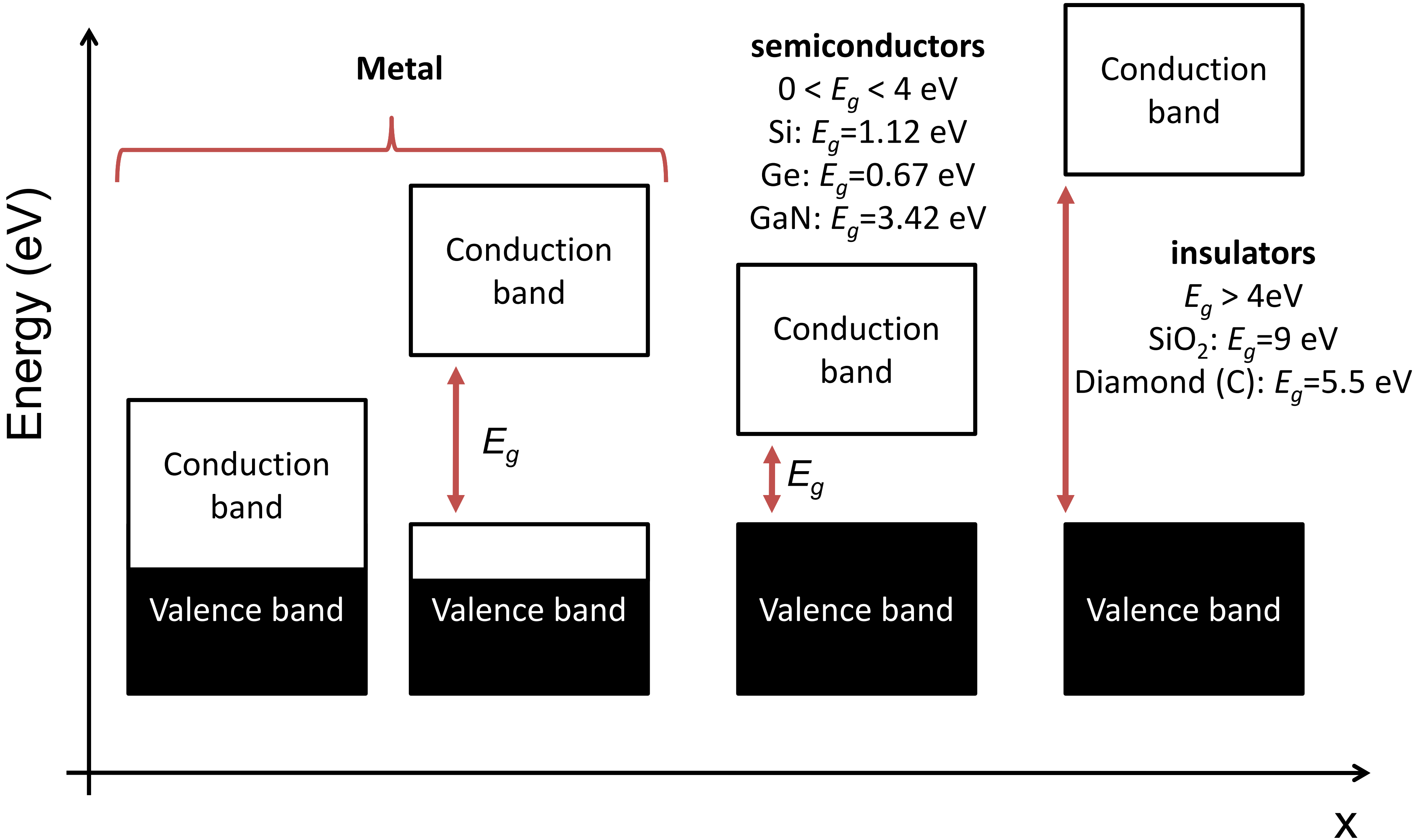
Valence Band: The highest band that has electrons

Conduction Band: The next band with higher energy

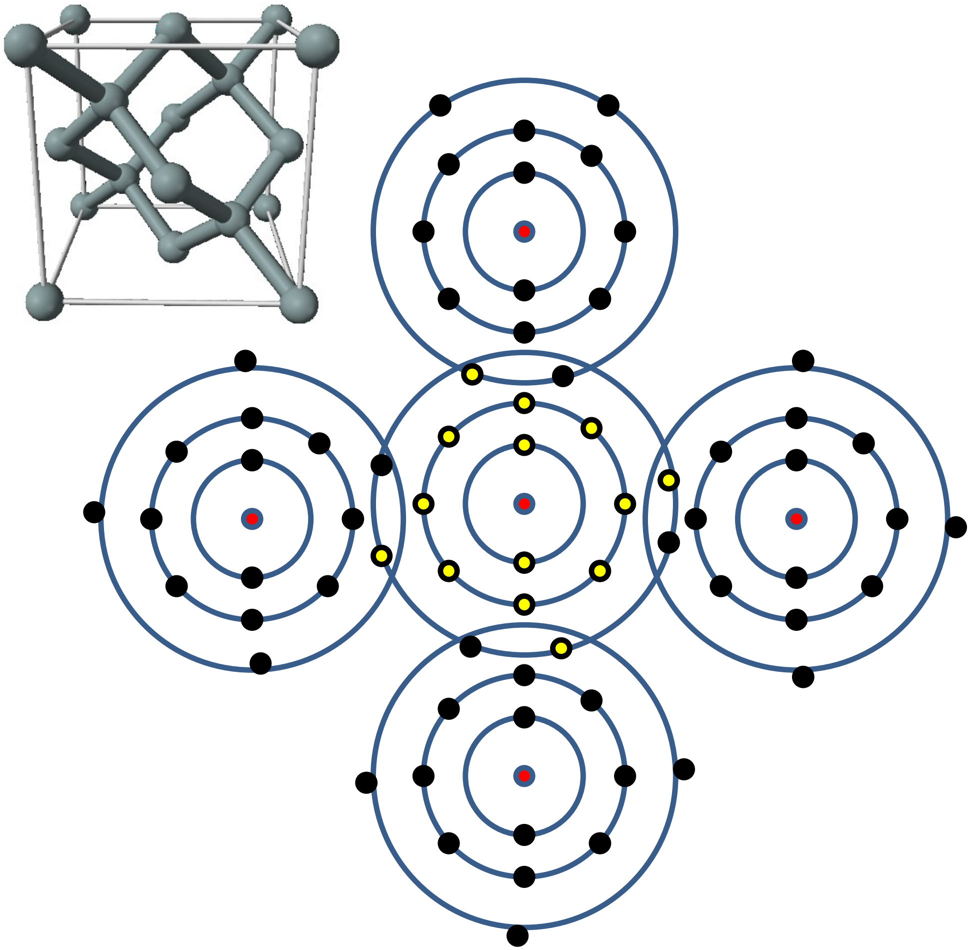
Metal: The valence band is partially filled with electrons

Semiconductor / Insulator: The valence band is filled

metals – semiconductors - insulators



What materials are semiconductors?



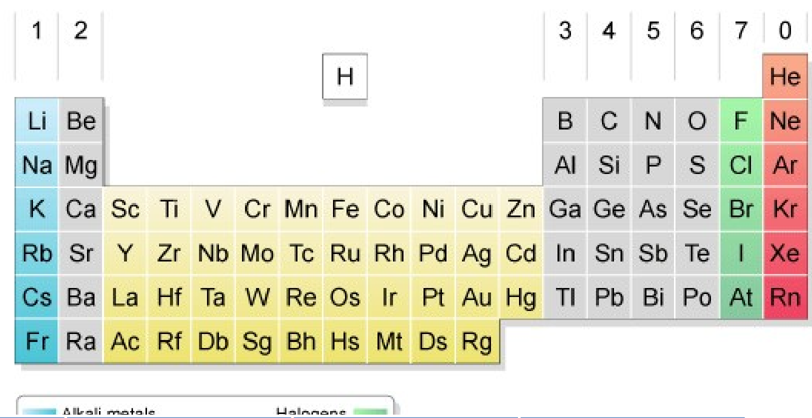
Two atom basis (4 neighbours)

4 own valence electrons

total 8 shared electrons in valence shell -> filled!

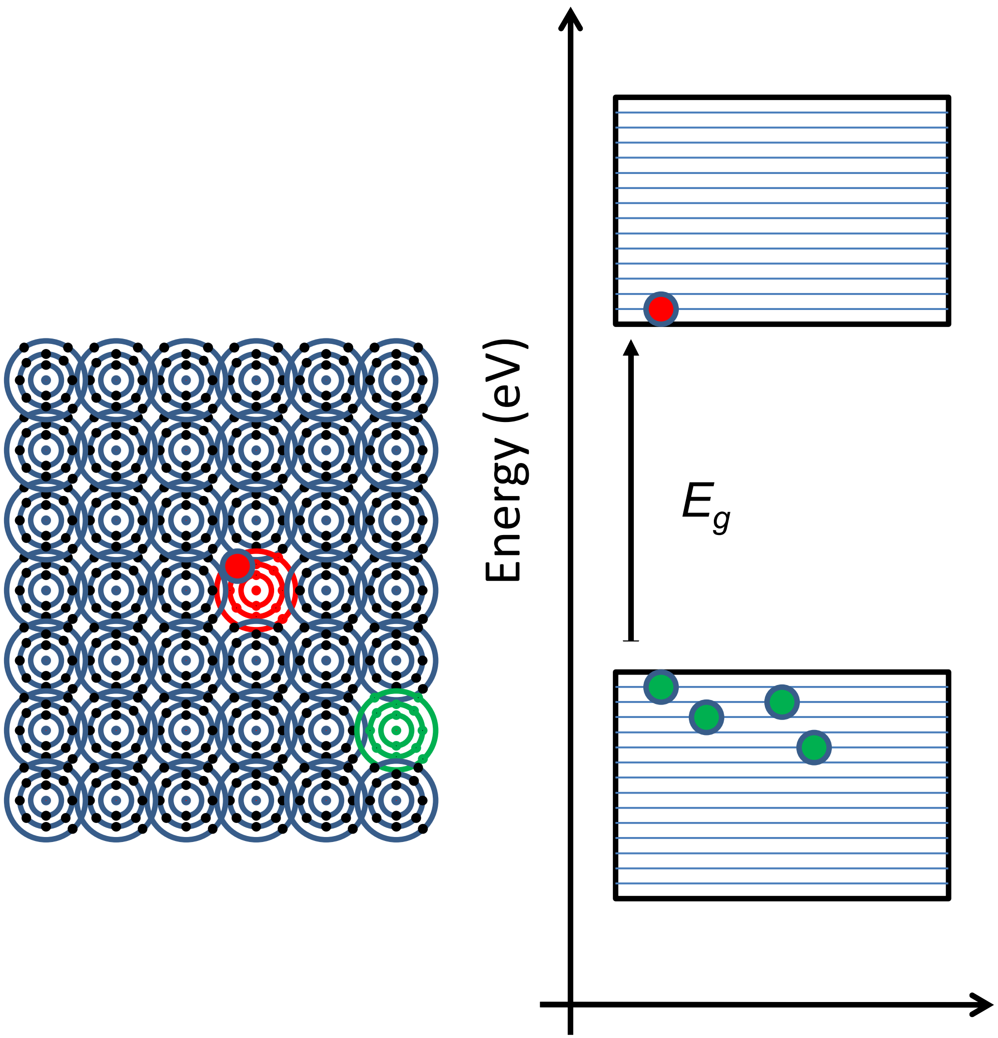
$4+1\times4=8$
$3+5=8$

III	IV	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb



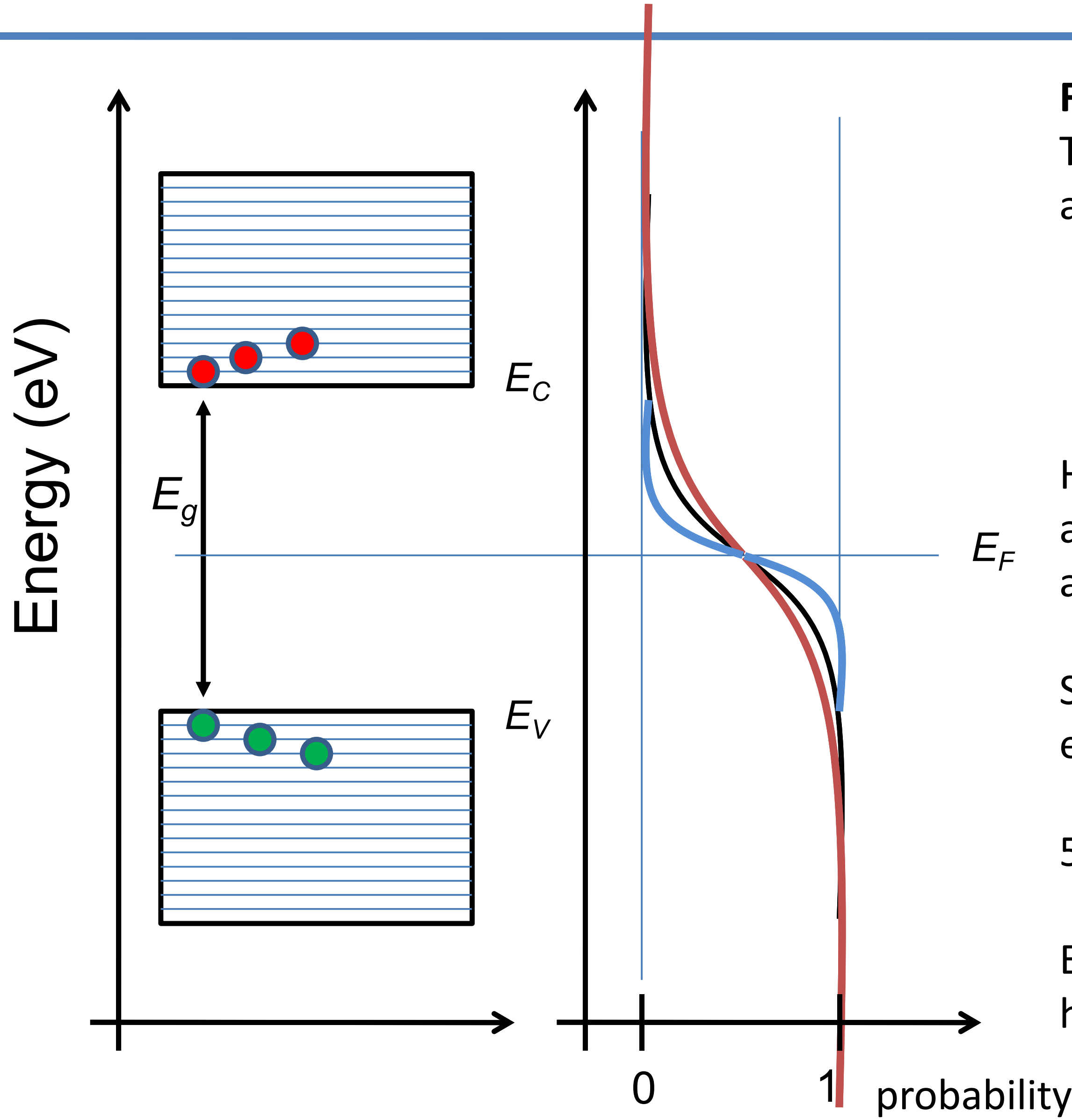
	C	GaP	Si	Ge	InAs	Sn
E_g	5.5 eV	2.24 eV	1.12 eV	0.67 eV	0.34 eV	0
Type	insulator	**Semi**	**cond**	**uc**	**tors**	metal

Thermal excitation



- Each electrons gets (average) kinetic energy $E_{kin}=3/2 \cdot kT$
- An electron can be excited to the conduction band
- Higher T or smaller E_g -> more electrons
- electron density in conduction band = ***n*** (cm^{-3})
- electron density in valence band = **10^{24} - *n*** (cm^{-3})
- ***p*** (cm^{-3}): hole density in valence band
- **n=p** without doping

Thermal excitation – Fermi level



Fermi-Dirac distribution:
The probability of an electron at an energy level E.

$$f(E) = \frac{1}{\exp((E - E_F)/kT) + 1}$$

Higher T – higher probability that a level in the conduction band has an electron.

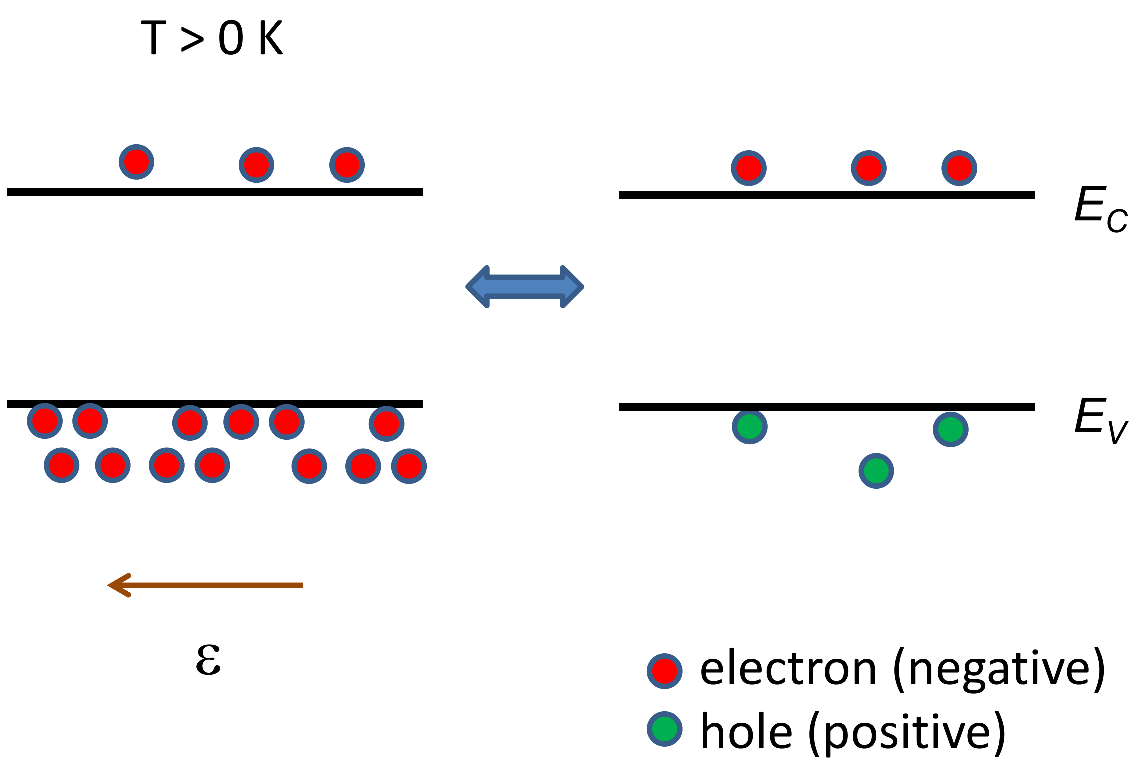
Symmetrical about E_F (Fermi energy).

50% chance to have electron at E_F

Excited electron leave a positive hole in valence band

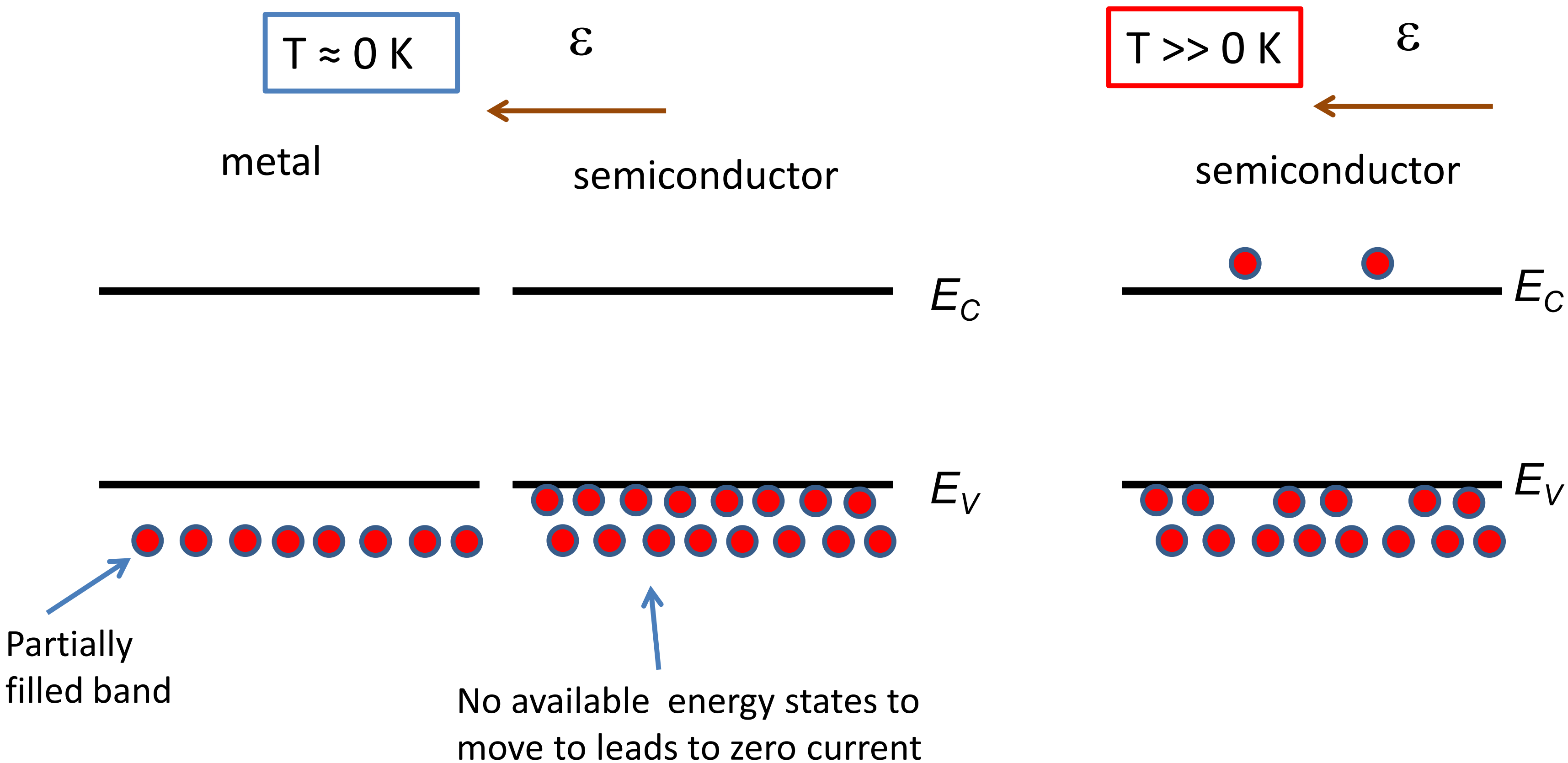
Holes vs electrons

Instead of describing all electrons remaining in valence band, positive holes (missing electrons) are introduced and treated as particles.



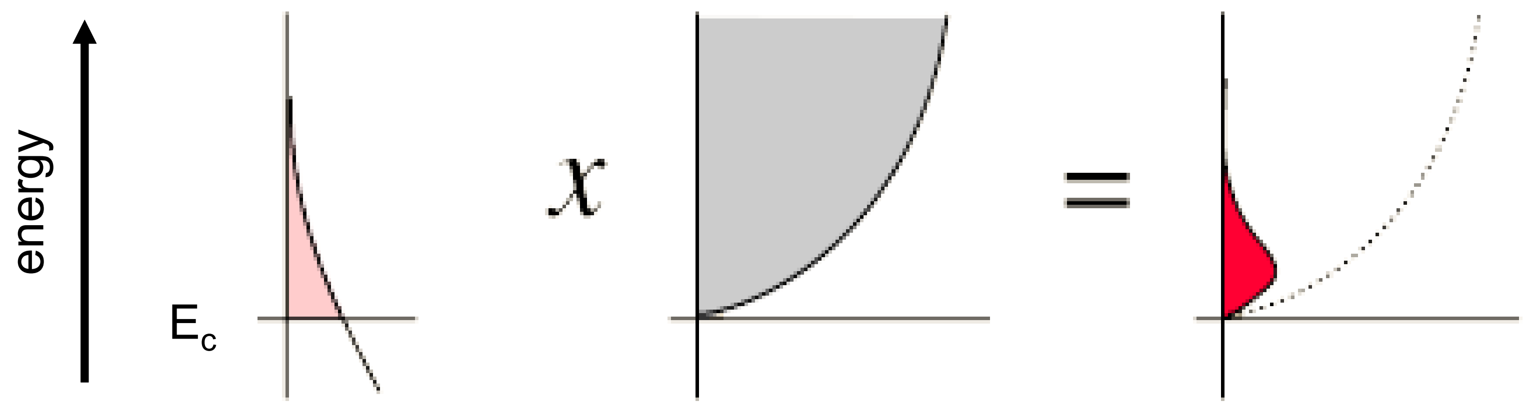
Electron transport

- Apply voltage -> electric field moves charged electrons
- Filled band: ½ of electrons move in one direction, ½ of electrons in the other.
- Need to change velocity of some electrons to get current but all states are filled i.e. need electrons in conduction band.



Carrier concentration

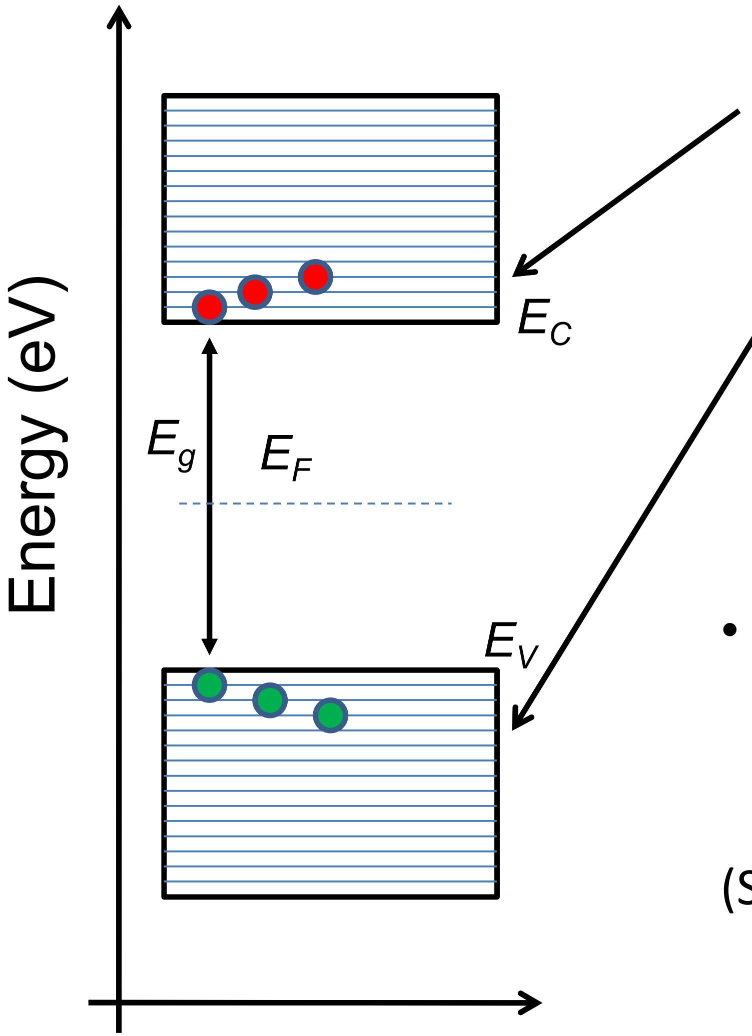
probability of occupying a state x number of available states = population of conduction band



Fermi-Dirac distribution x density of states = carrier concentration

$$\int_{Ec}^{\infty} dE\big(D(E)\,f(E_f, E)\big) = n$$

Carrier concentration - simplified



$$n = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

- Ignore real density of states, introduce N_C, N_V – effective density of states at band edges (describes how dense levels are)

(Si: $N_c=3.2*10^{19}$ cm^{-3} / $N_v=1.8*10^{19}$ cm^{-3})

Intrinsic carrier concentration

Each electron excited from the valence band to the conduction band become a free carrier available for conduction

Intrinsic semiconductor:
- $n=p=n_i$ (intrinsic carrier concentration)
- E_F is in the middle of the band gap

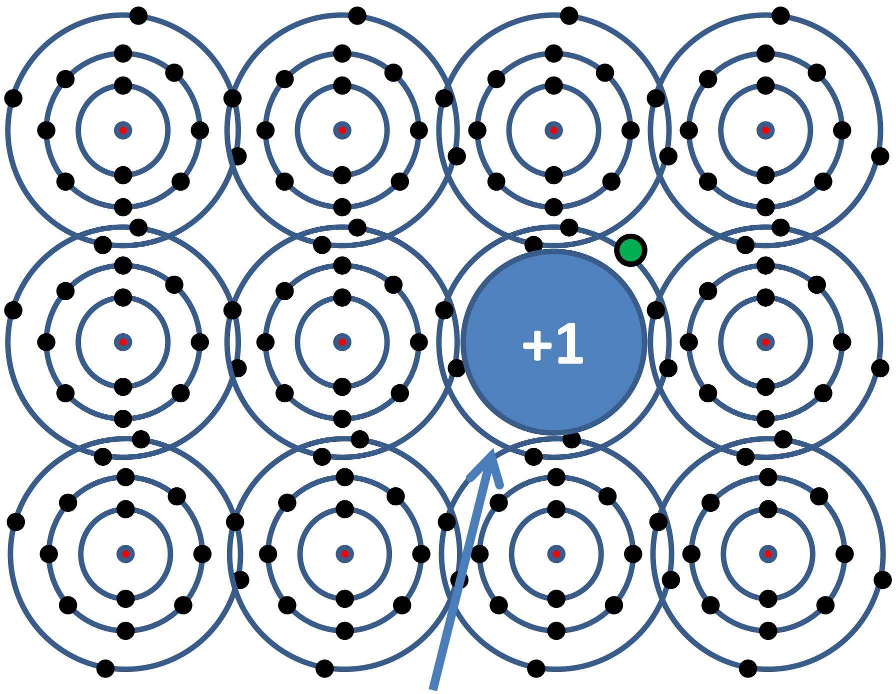
T=300K

Si
E_g=1.11eV
$n_i=1\times10^{10}$ cm^{-3}

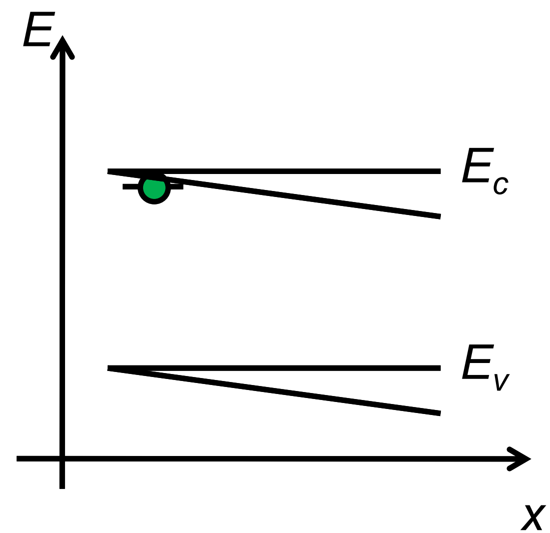
Ge
E_g=0.67 eV
$n_i= 2\times10^{13}$ cm^{-3}

$$n_i = \sqrt{n \cdot p} = \sqrt{N_c N_v}\exp\left(\frac{-E_g}{2kT}\right)$$

Doping with donor atoms: n-type



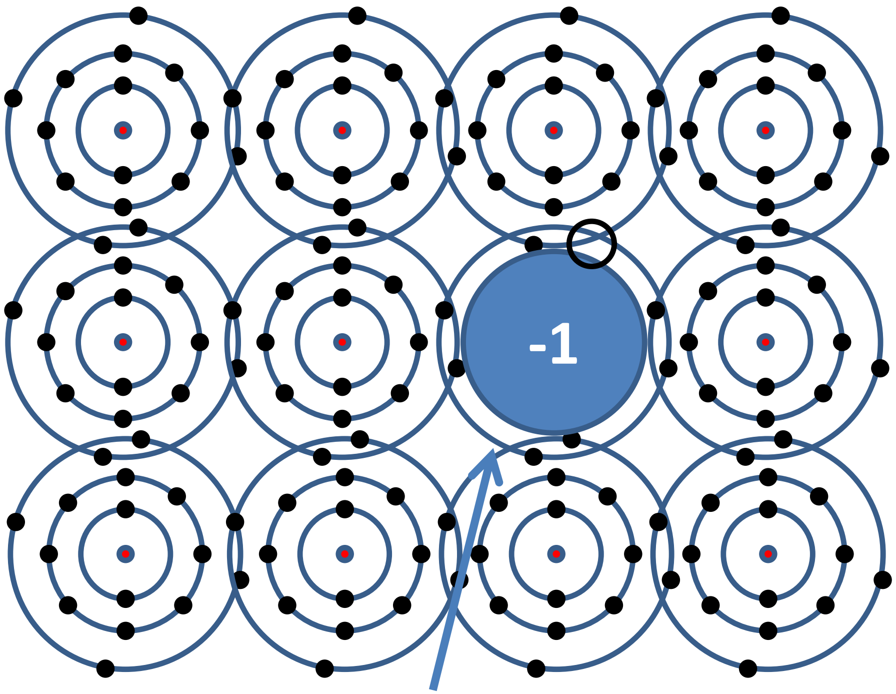
P – atom
5 valence electrons



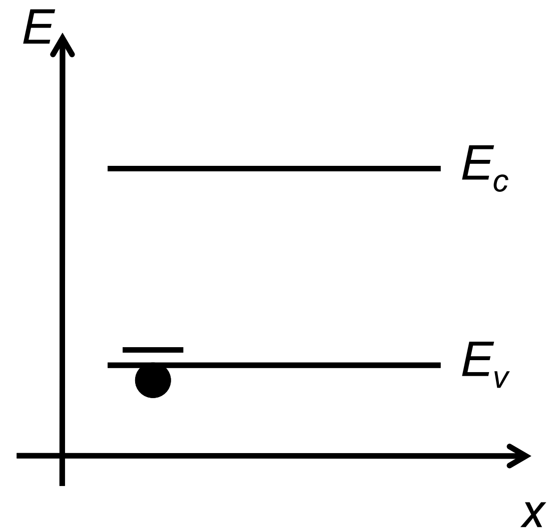
III	IV	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb

Donates electron to the conduction band (mobile)
Ionized atom – positive (immobile)

Doping with acceptor atoms: p-type



Al – atom
3 valence electrons



III	IV	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb

Captures electron -> extra hole to the valence band (mobile)
Ionized atom – negative (immobile)

Doping – extrinsic semiconductor

$$n = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

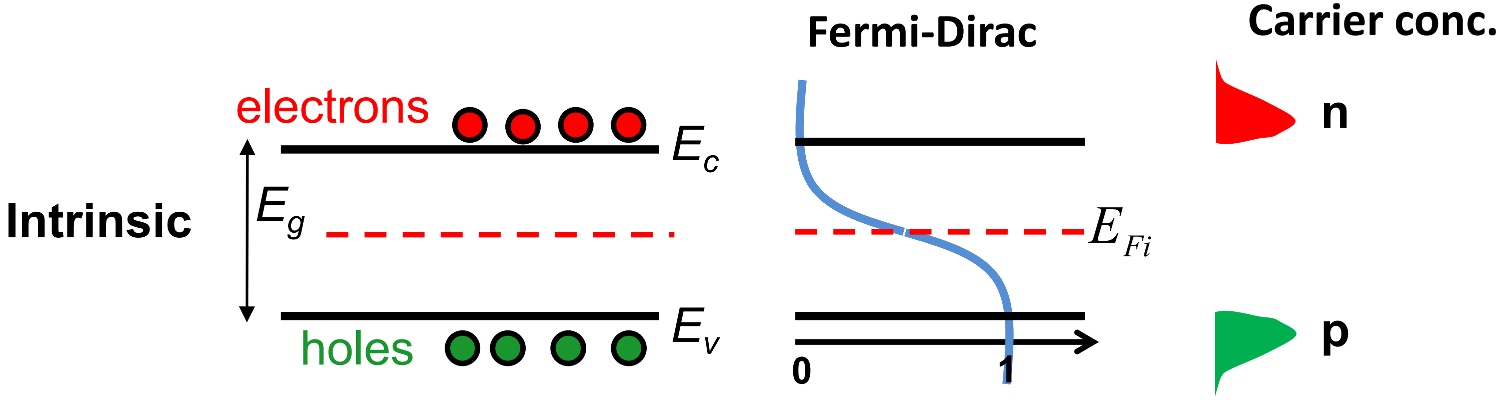
$$np = N_c N_v \exp\left(\frac{-E_g}{kT}\right) = n_i^2$$

Mass action law $$n \cdot p = n_i^2$$ Independent of doping

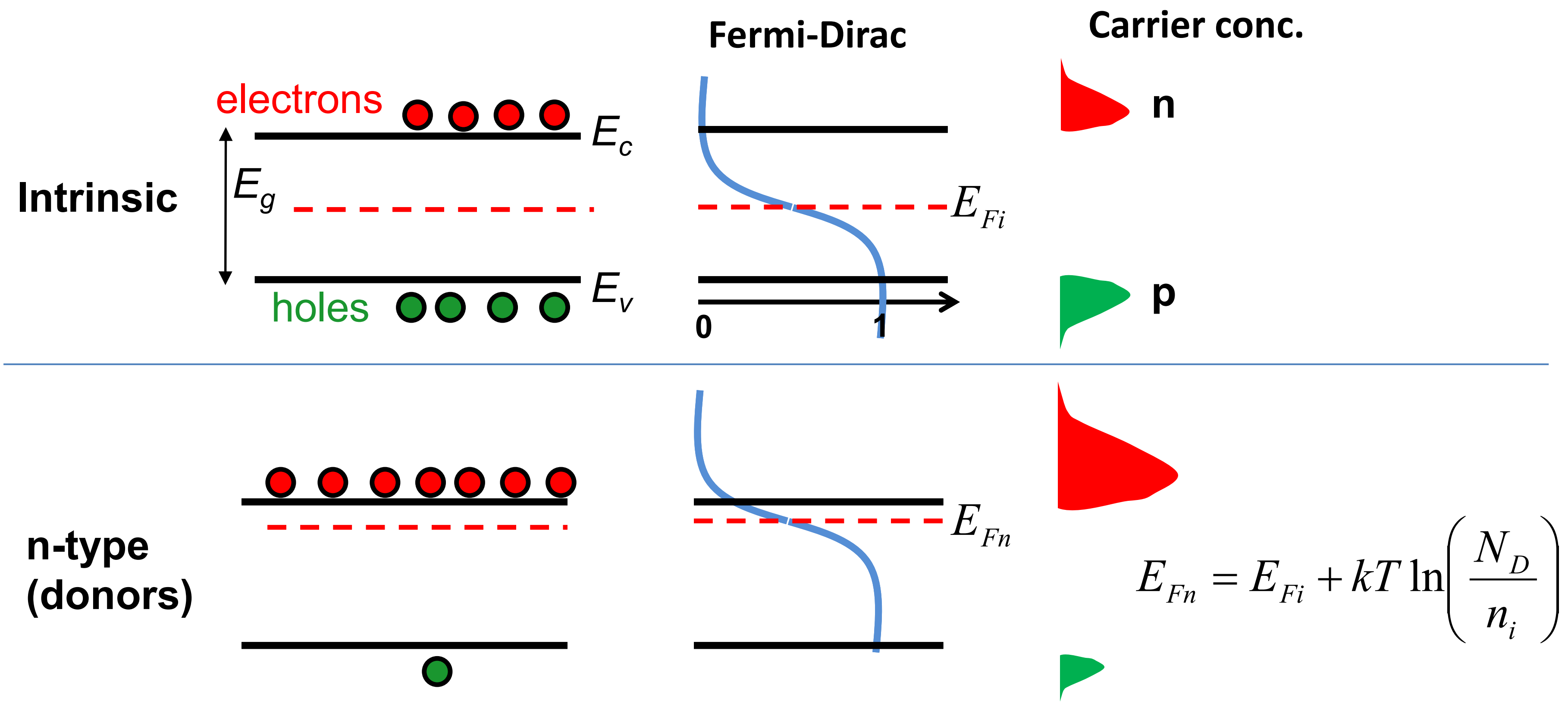
- N_D: donor atom density (cm^{-3}) / N_A: acceptor atom density (cm^{-3})
- For $N_D >> n_i$ ($N_A=0$) the doping dominates majority carrier concentration

$$n=N_D \text{ and } p=n_i^2/N_D$$

doping – carrier density

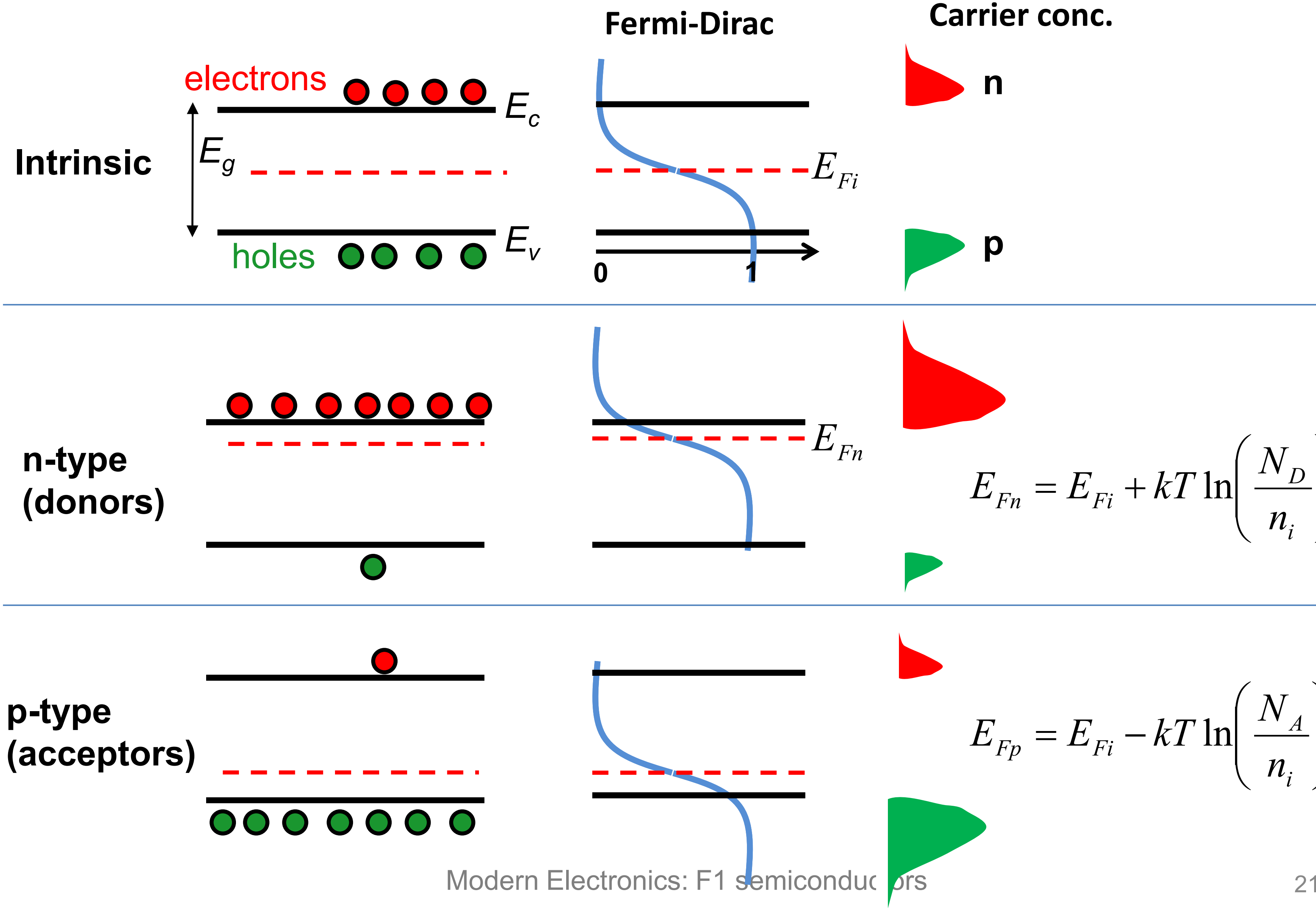


doping – carrier density



$$E_{Fn} = E_{Fi} + kT \ln\left(\frac{N_D}{n_i}\right)$$

doping – carrier density



$$E_{Fn} = E_{Fi} + kT \ln\left(\frac{N_D}{n_i}\right)$$

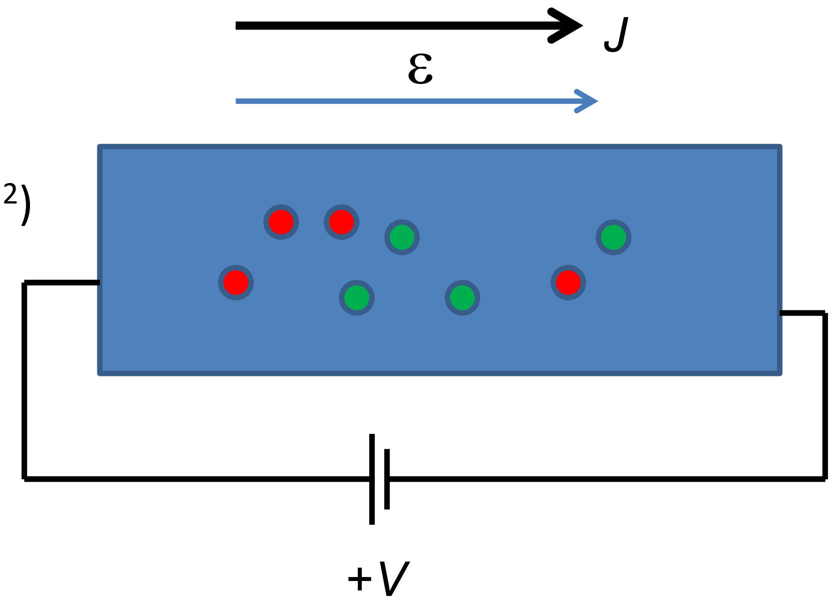
$$E_{Fp} = E_{Fi} - kT \ln\left(\frac{N_A}{n_i}\right)$$

Transport - drift

$$J_p = qp\overbrace{\mu_p \varepsilon}^{v_d}$$ Hole current density (A/cm^2)

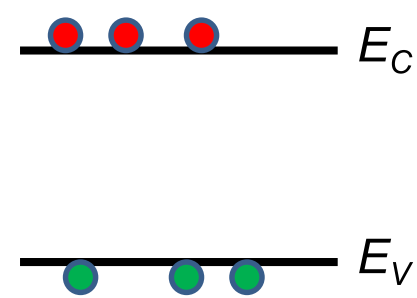
$$J_n = qn\mu_n \varepsilon$$ Electron current density (A/cm^2)

$$J = J_n + J_p$$



$$\sigma = q\left(\mu_n n + \mu_p p\right)$$

$$\rho = \frac{1}{q\left(\mu_n n + \mu_p p\right)}$$



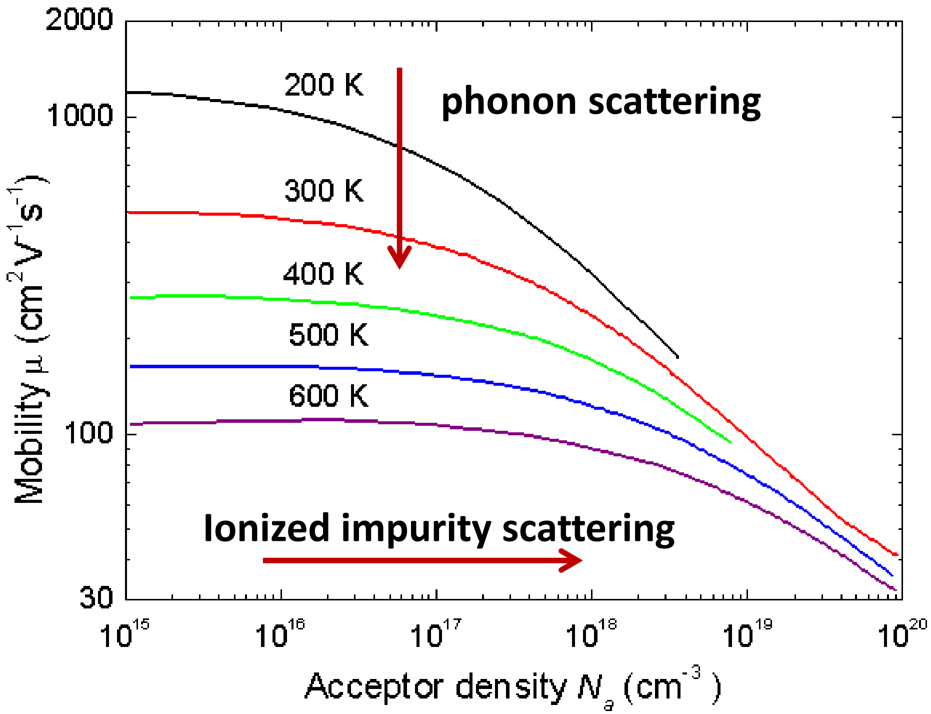
μ_n / μ_p– electron / hole mobility (cm^2/Vs)
n / p – electron / hole concentration (cm^{-3})
σ =1/ρ=conductivity (S/m)
ρ = resistivity (Ωm)

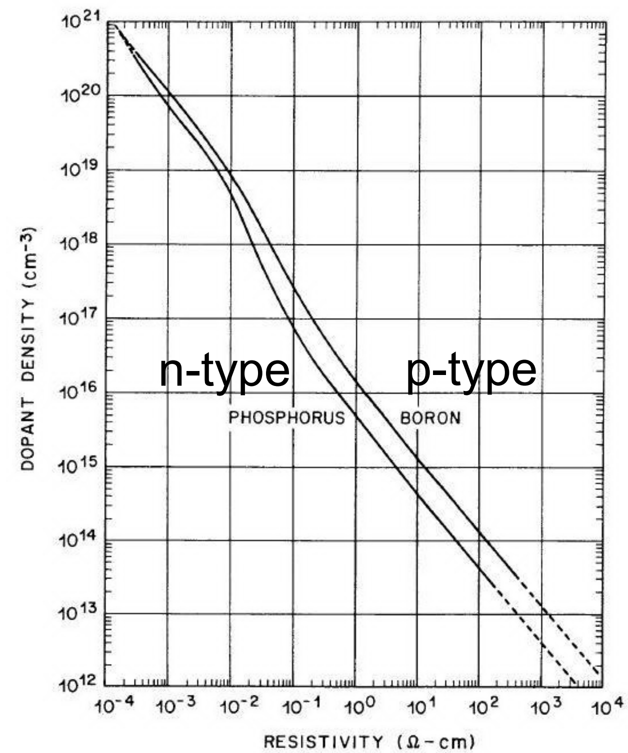
Transport – effect of doping

$$J_n = qnv_d = qn\mu_n\varepsilon$$

n: controlled by doping

μ_n: determined by intrinsic semiconductor properties + scattering





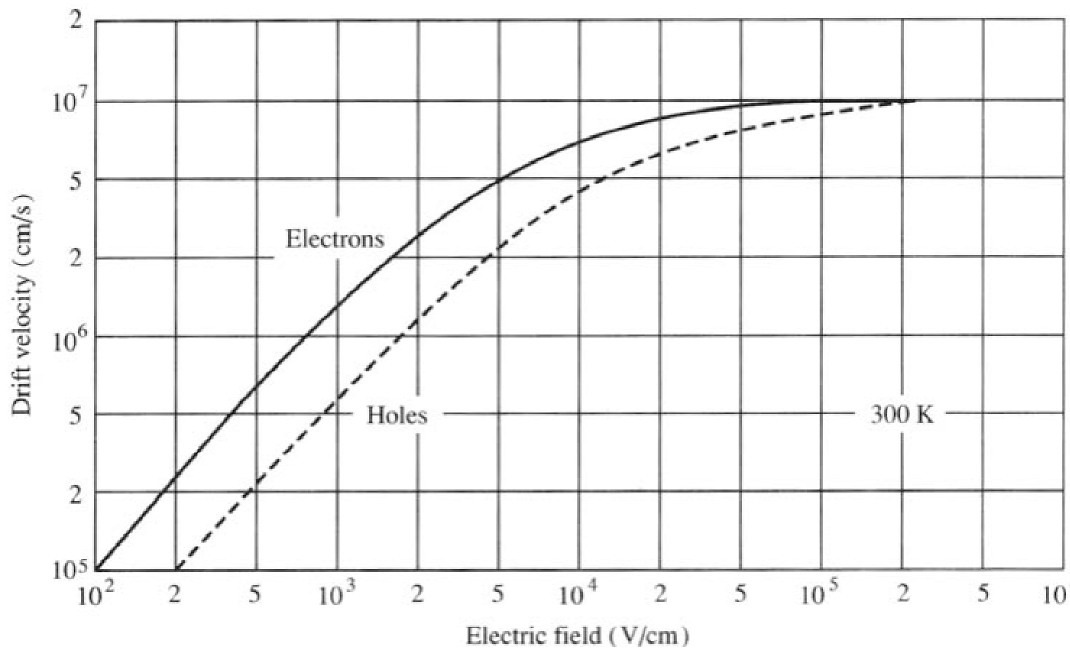
Conversion between resistivity and dopant density in silicon.

Velocity saturation

- At high electric fields ($\varepsilon_c \approx 1.5 \times 10^6$ V/m) electrons can emit optical phonons (lattice vibrations) -> velocity saturates

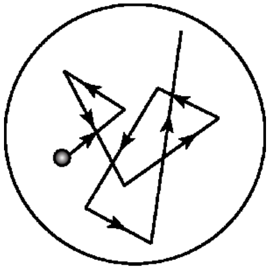
$$J = qnv_d = qn\mu_n(\varepsilon)\varepsilon$$

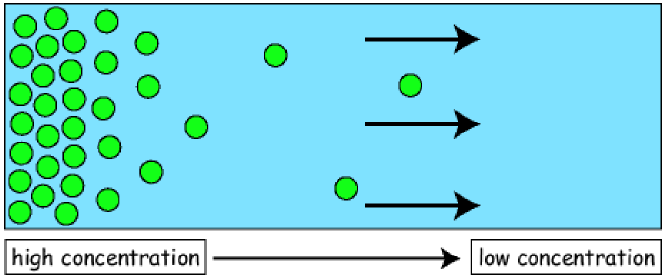
$$v_d = \frac{\mu_n \varepsilon}{1 + \varepsilon / \varepsilon_c} \qquad \begin{cases} \varepsilon << \varepsilon_c \rightarrow v_d \approx \mu_n \varepsilon \\ \varepsilon >> \varepsilon_c \rightarrow v_d \approx \mu_n \varepsilon_c = v_{sat} \end{cases}$$

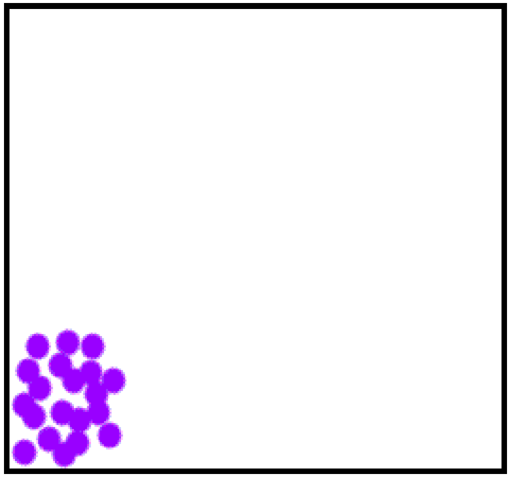


Transport - diffusion

- Random thermal motion gives movement of particles from high to low concentration
- No external forces
- No particle interaction
- Rate depends on concentration gradient





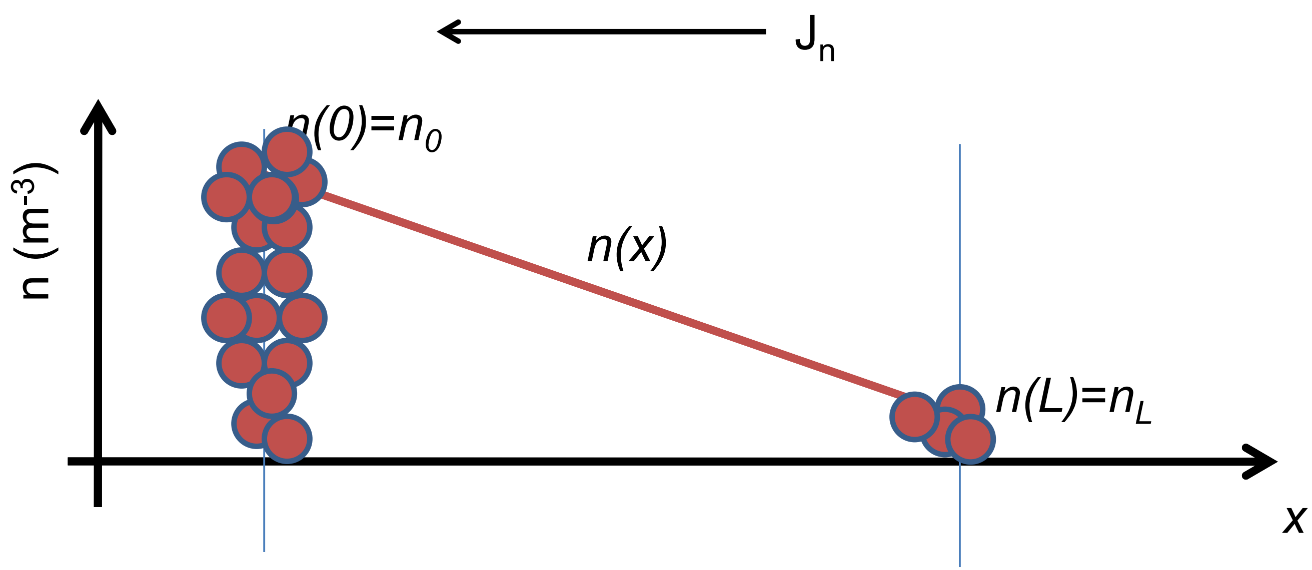


Transport - diffusion

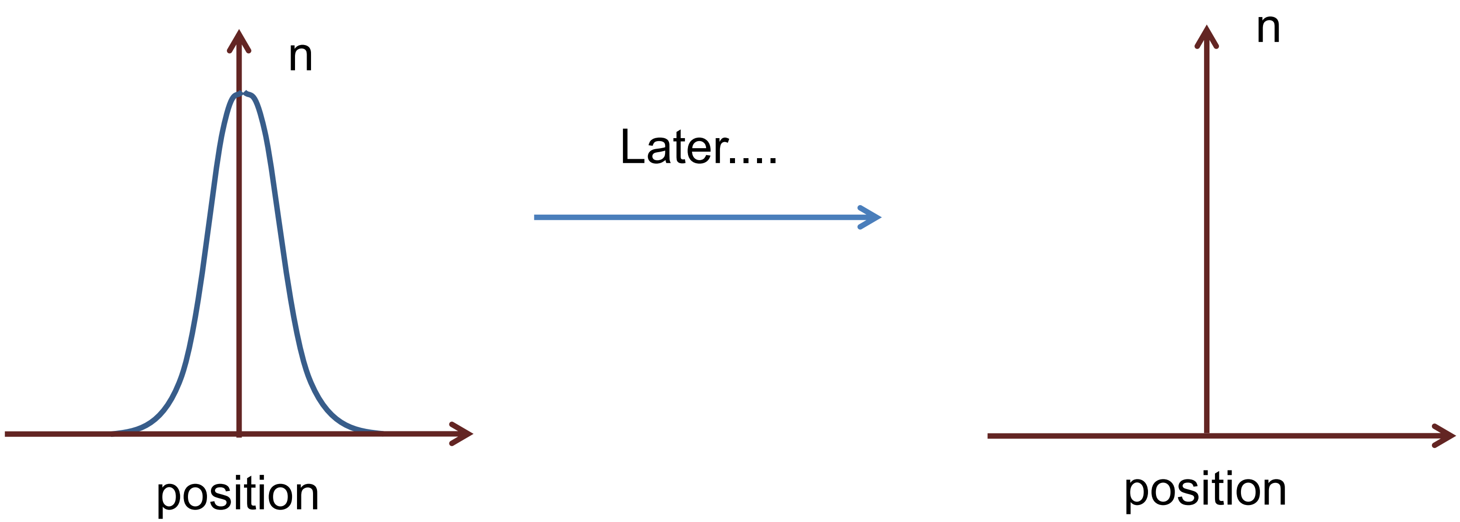
Diffusion current given by gradient of electron (or hole) concentration

$$J_n = qD_n \frac{dn(x)}{dx} = q\left(\frac{kT}{q}\mu_n\right)\frac{dn(x)}{dx} = qV_T\mu_n \frac{dn(x)}{dx}$$

No addition/removal of carriers between 0 < x < L -> constant current -> linear concentration decrease



2 min exercise – drift/diffusion



Sketch carrier distribution later with

1. No electric field
2. E-field in pos. direction (->)

Summary

- Discrete atomic energy levels become bands when atoms are joined in a crystal.

- Semiconductors = filled valence band, no electrons in conduction band (at T=0). Band gap with no allowed states.

- Carrier concentration (n) = Fermi-dirac distribution * density of states

$$n = N_c \exp(\frac{E_F - E_c}{kT})$$

- Doping:
  - Donors (group V): adds mobile negative electrons and positive static ions. Moves Fermi level towards conduction band.
  - Acceptors (group III): adds mobile positive holes and negative static ions. Moves Fermi level towards valence band.

$$n \cdot p = n_i^2$$ $n=N_D$ and $p=n_i^2/N_D$ for $N_D>>n_i$

- Carrier transport:
  - Drift: electric field moves charged particles. Depends on field strength.
  - Diffusion: random motion moves particles from high to low concentration. Depends on concentration gradient.

$$J_n = J_{n,drift} + J_{n,diff} = qn\mu_n \boldsymbol{\varepsilon} + qD_n \frac{dn}{dx}$$

pn- junctions

Charge distribution -> electric field -> potential

Current transport

Breakdown mechanisms

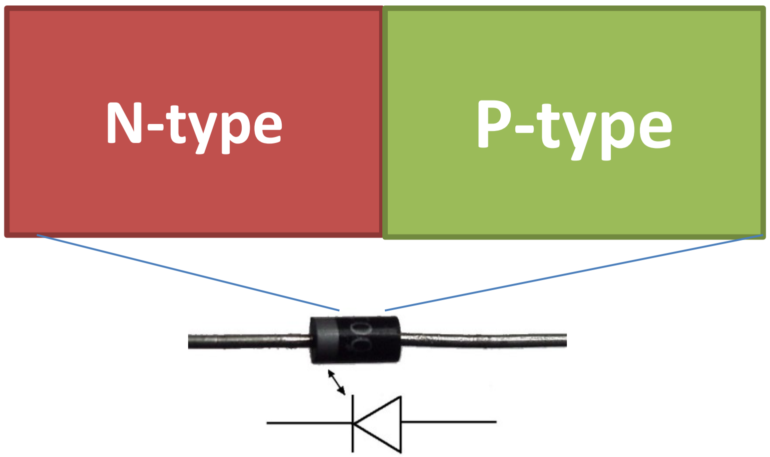
Small-signal model

Depletion / diffusion capacitances

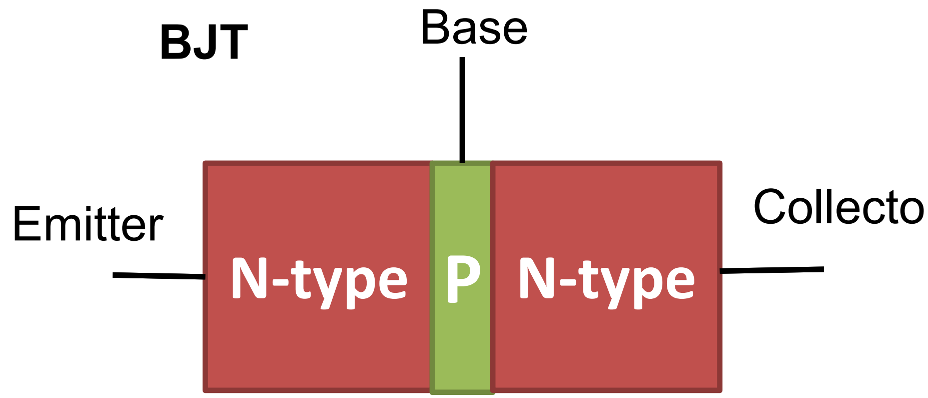
Reading: (Sedra/Smith 7th edition)
1.10-1.12, 3.1 - 3.3

Why pn-junctions?

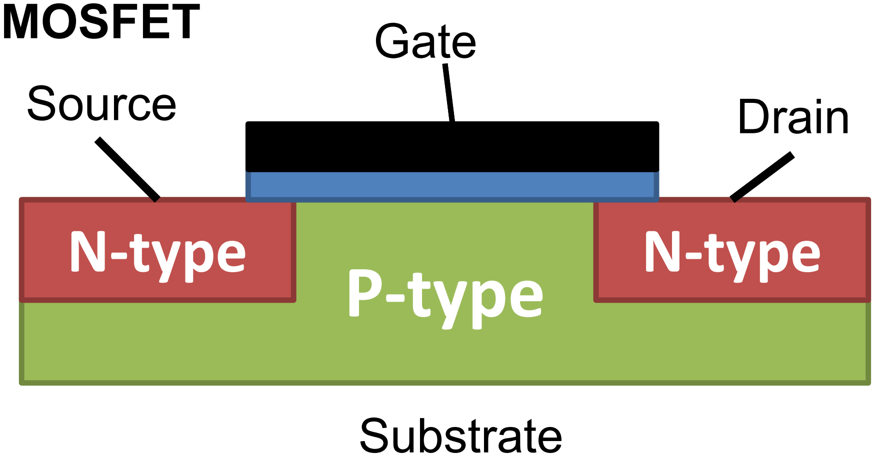
diode
LED
Solar cell



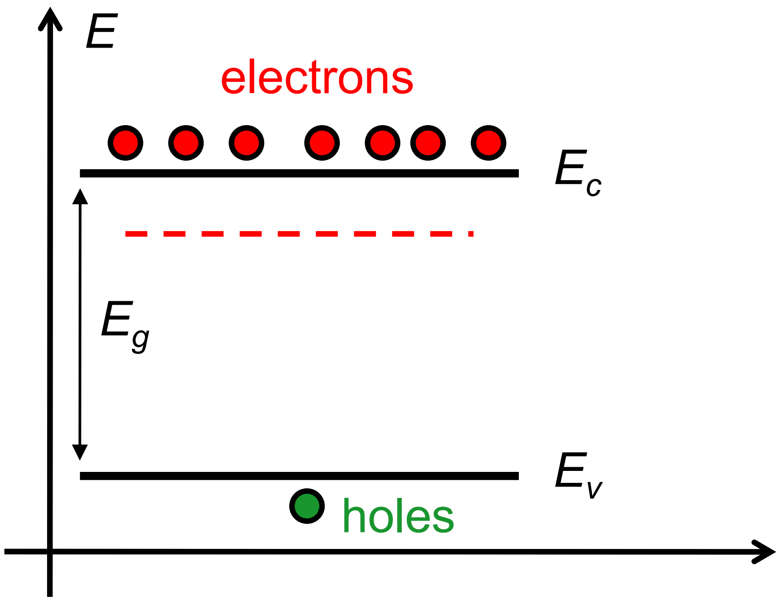
BJT



MOSFET



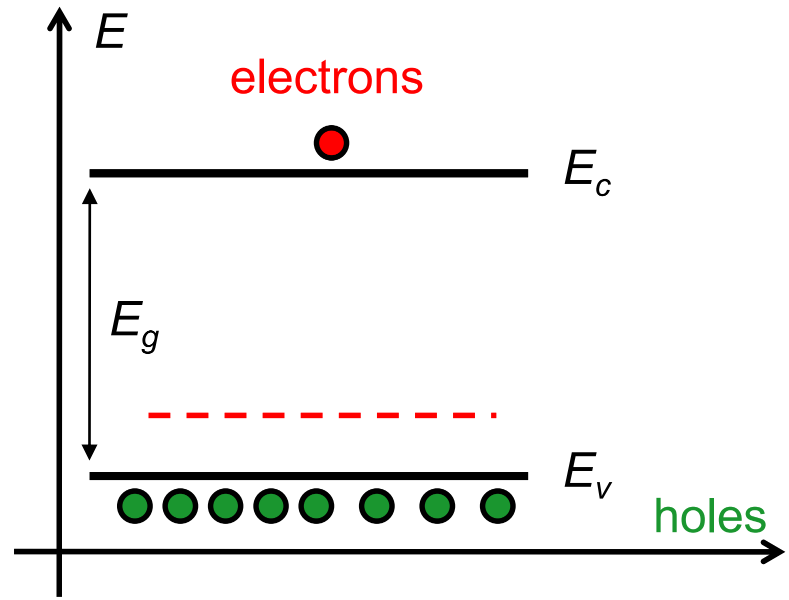
N - type



N_D – donor concentration
n_{n0} – electron (majority) concentration
p_{n0} – hole (minority) concentration

Electrons*:* mobile, negative
Ionized donors: not mobile, positive

P - type



N_A – acceptor concentration
p_{p0} – hole (majority) concentration
n_{p0} – electron (minority) concentration

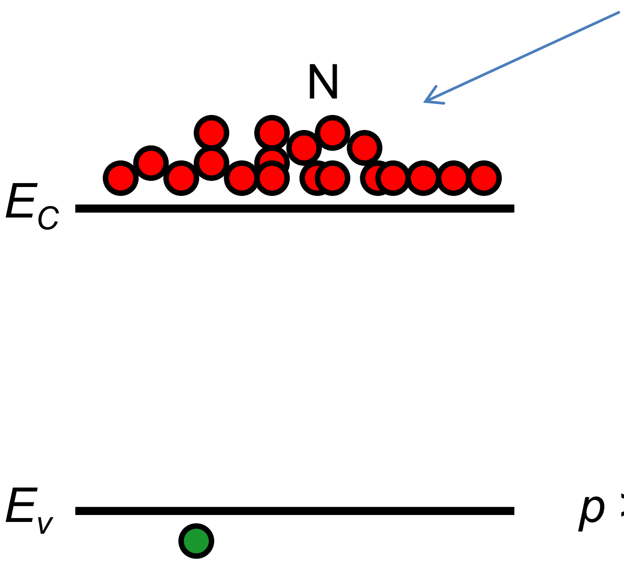
holes*:* mobile, positive
Ionized acceptors: not mobile, negative

Recombination

$$n_0 \cdot p_0 = n_i^2$$ In equilibrium

Important for base current in BJTs

If there is an excess number free carrriers ***np* > *n_i^2*** electrons can recombine with holes to reach equilibrium



$n_{n0}=N_D^+$

$n_{n0} < N_D^+$

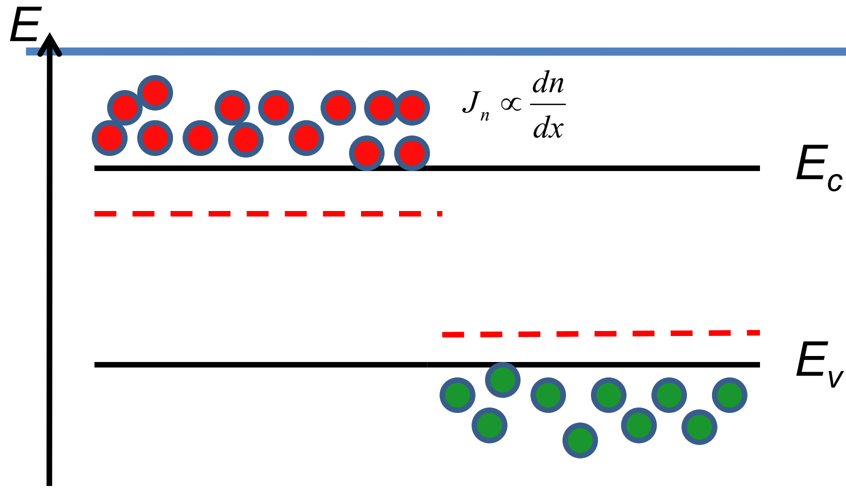
Three electrons recombine leaving three positive ionized donor atoms

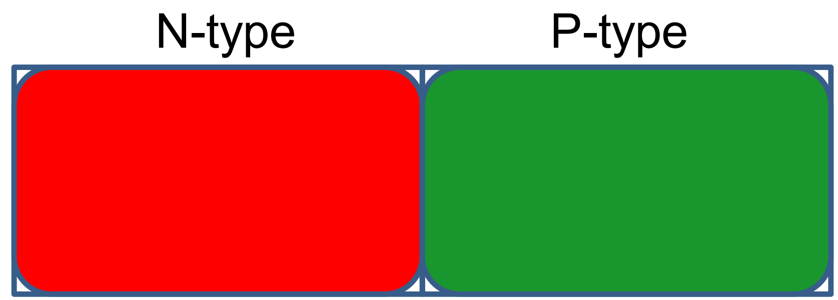
$p > p_0$

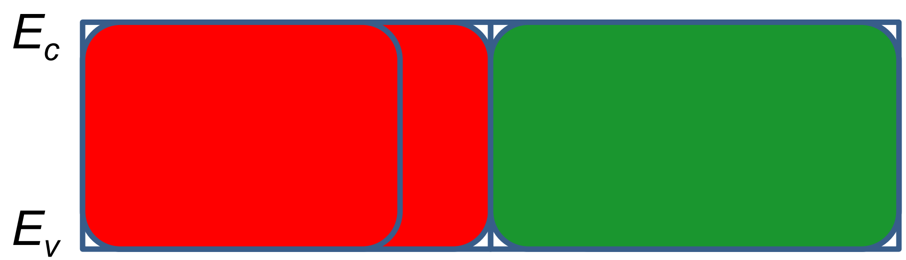
PN-junction – band structure

+ Positive donor
- Negative Acceptor

Free electrons
Free holes





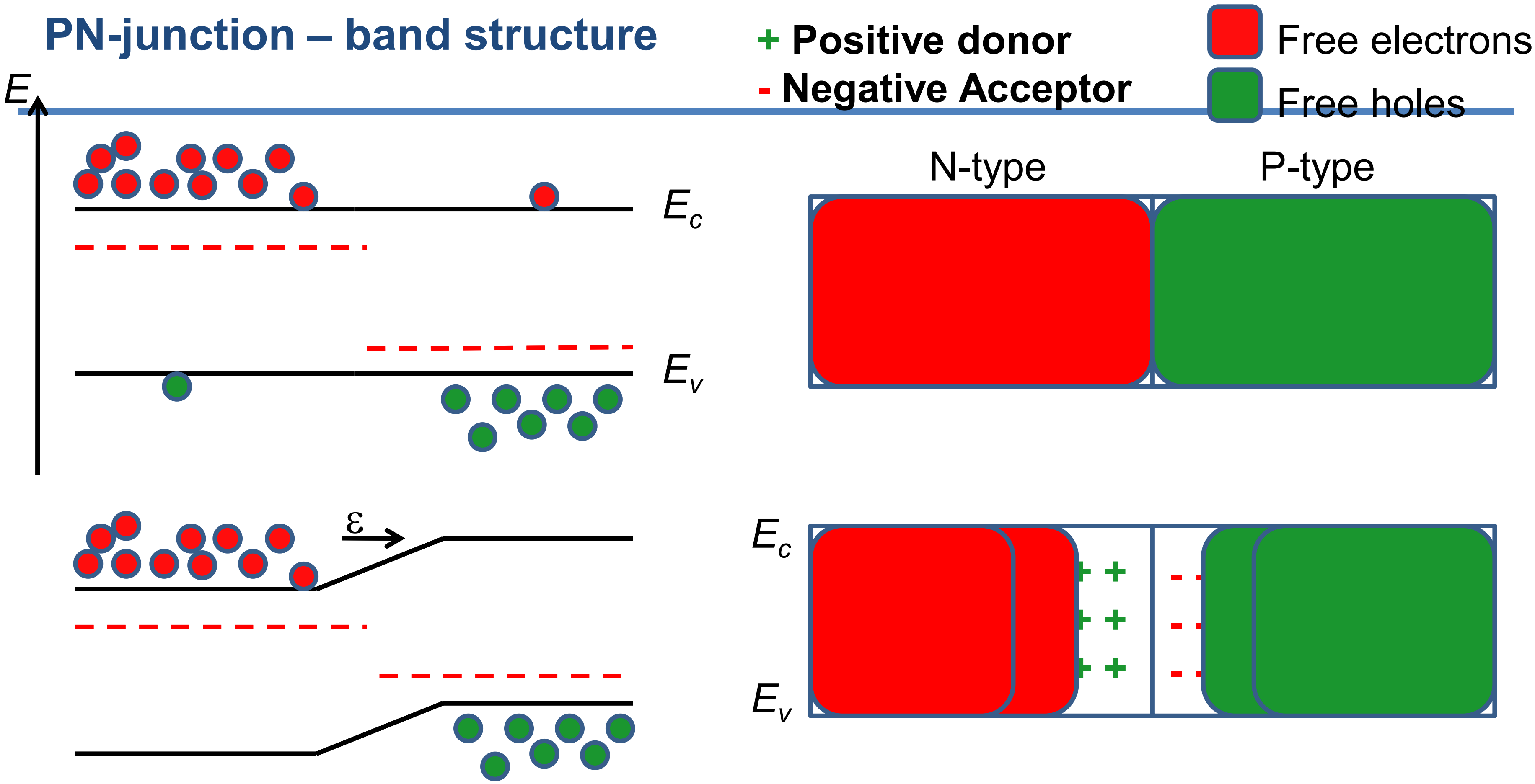


Large conc. difference -> large diffusion current
No ε-field – no drift current

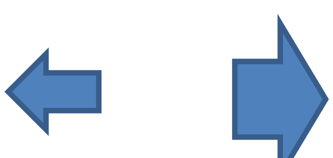
$$I_n = qA\mu_n\left[n \cdot \varepsilon + V_T \frac{dn(x)}{dx}\right]$$

PN-junction – band structure

+ **Positive donor**
- **Negative Acceptor**



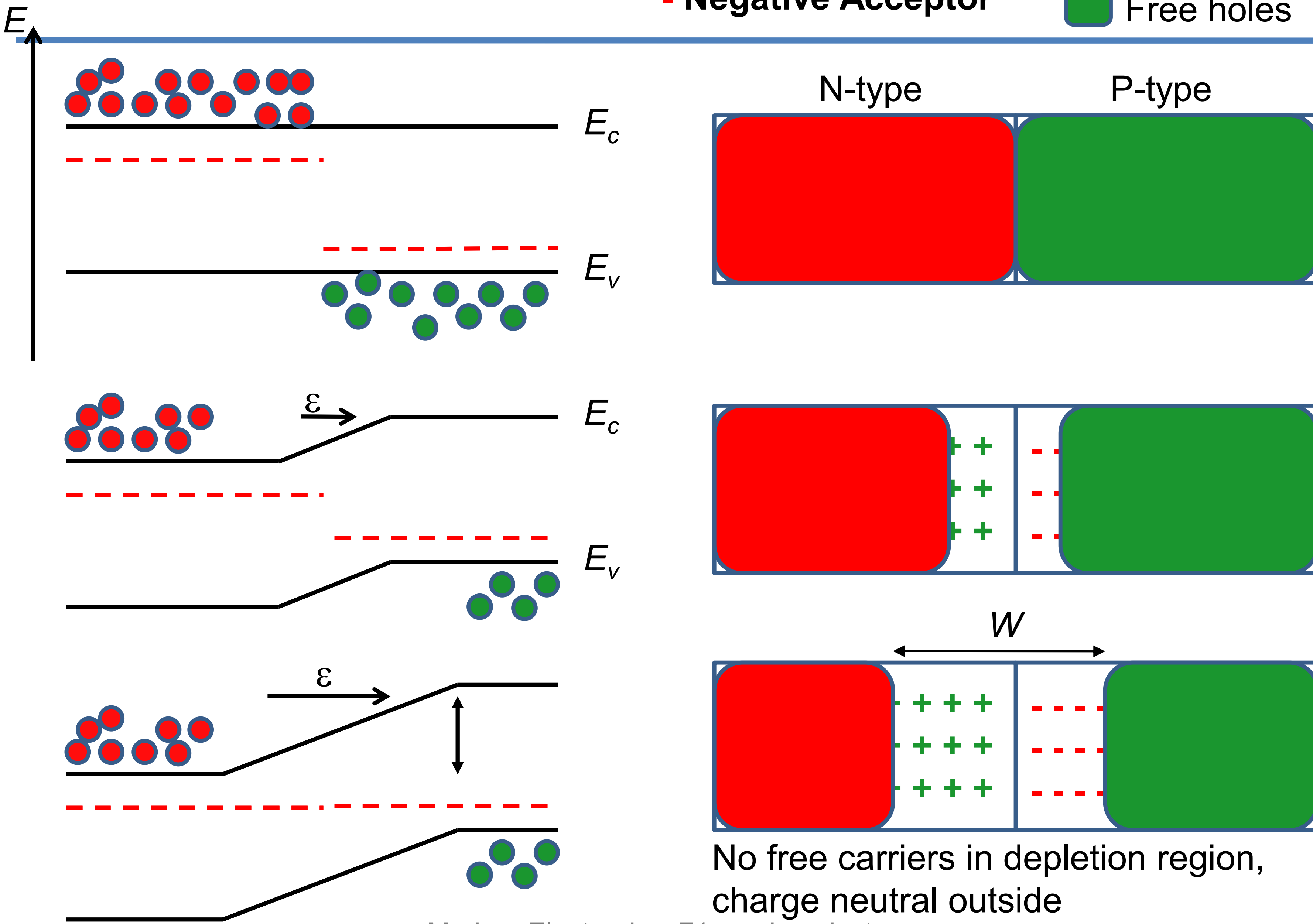
$$I_n = qA\mu_n\left[n\cdot\varepsilon + V_T\frac{dn(x)}{dx}\right]$$



PN-junction – band structure

+ **Positive donor**
- **Negative Acceptor**

Free electrons
Free holes



No free carriers in depletion region, charge neutral outside

charge density -> electric field -> potential

Poisson equation

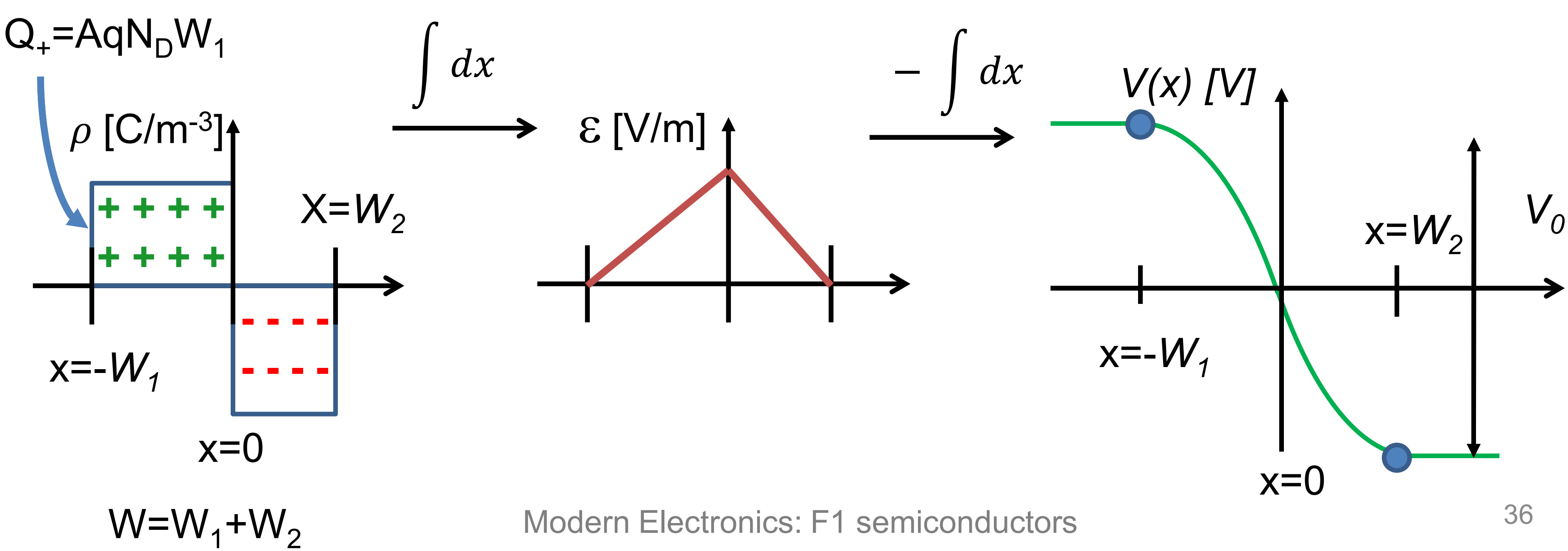
$$\frac{\rho(x)}{\epsilon_s} = \frac{d\varepsilon(x)}{dx} = -\frac{d^2V(x)}{dx^2}$$

Depletion width

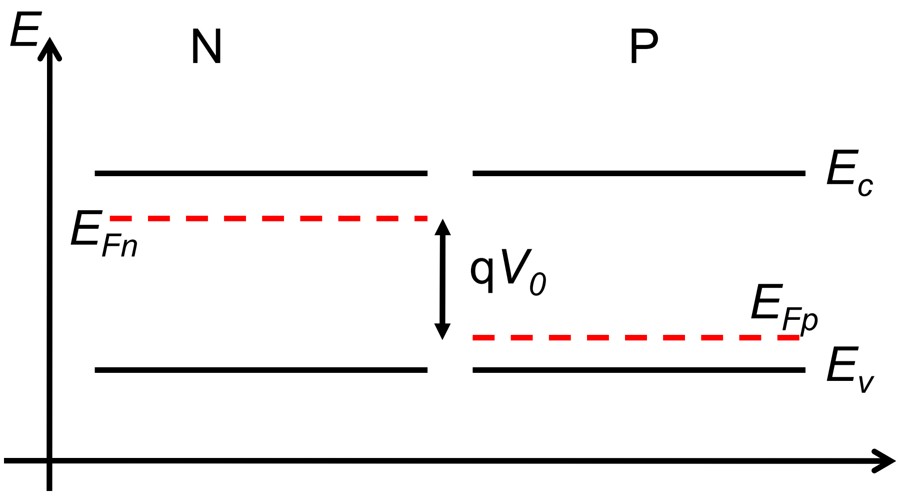
$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$$

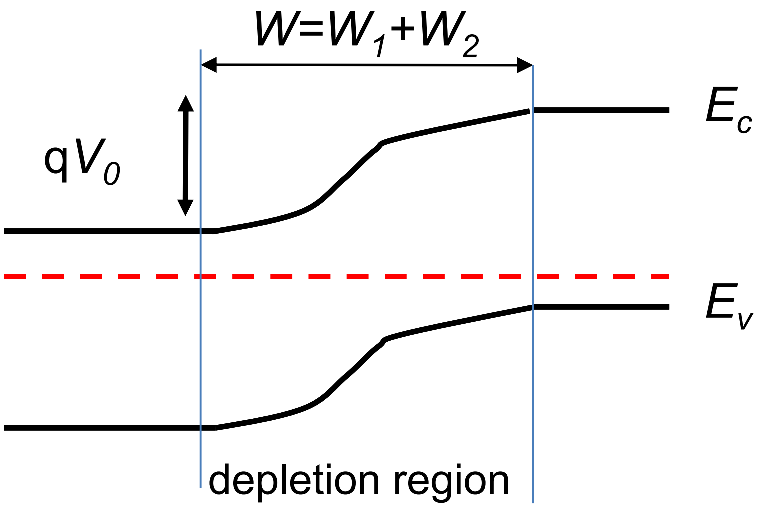
- ρ charge density [C/m^{-3}]
- ϵ_s permitivity [F/m]
- ε electric field [V/m]
- V potential [V]

- V_0 built-in potential [V]



Built-in potential





$$qV_0 = E_{Fn} - E_{Fp}$$

(built-in) Potential barrier for electrons and holes!

$$E_{Fn} = E_{Fi} + kT\ln\left(\frac{N_D}{n_i}\right)$$

$$E_{Fp} = E_{Fi} - kT\ln\left(\frac{N_A}{n_i}\right)$$

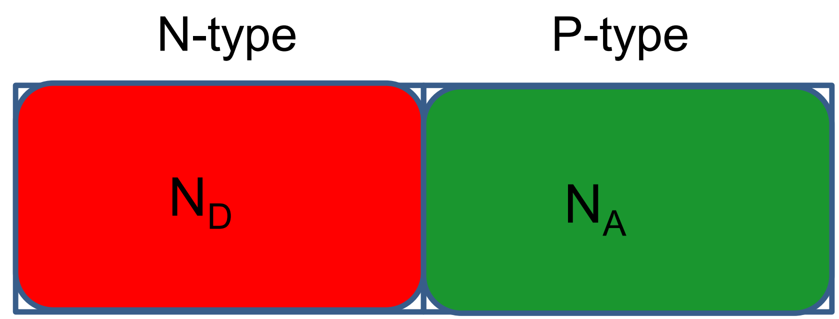
$$qV_0 = kT\left[\ln\left(\frac{N_D}{n_i}\right) + \ln\left(\frac{N_A}{n_i}\right)\right] = kT\left[\ln\left(\frac{N_D N_A}{n_i^2}\right)\right]$$

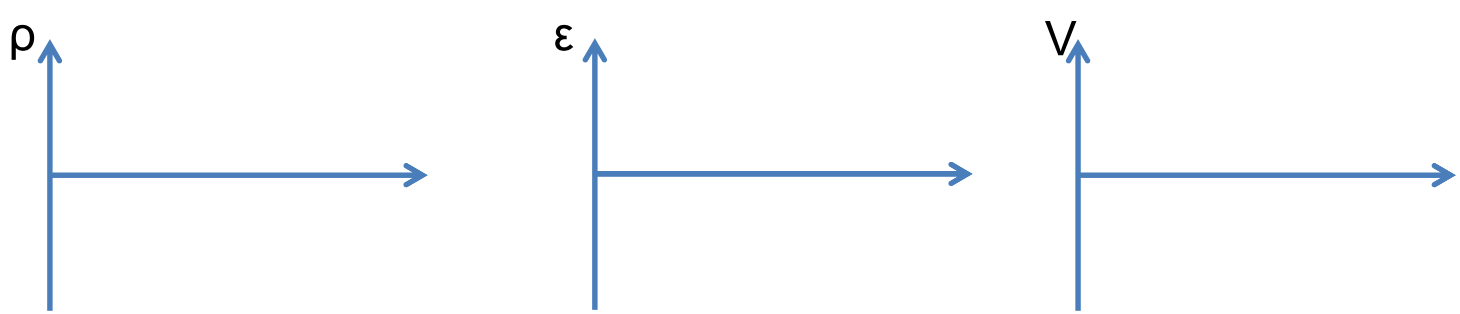
---

homework: calculate depletion width for Si pn-junction with $N_D=N_A=10^{17}$ cm^{-3} at bias voltages V=0 V and V= -1 V.

2 min exercise – assymetric pn-junction

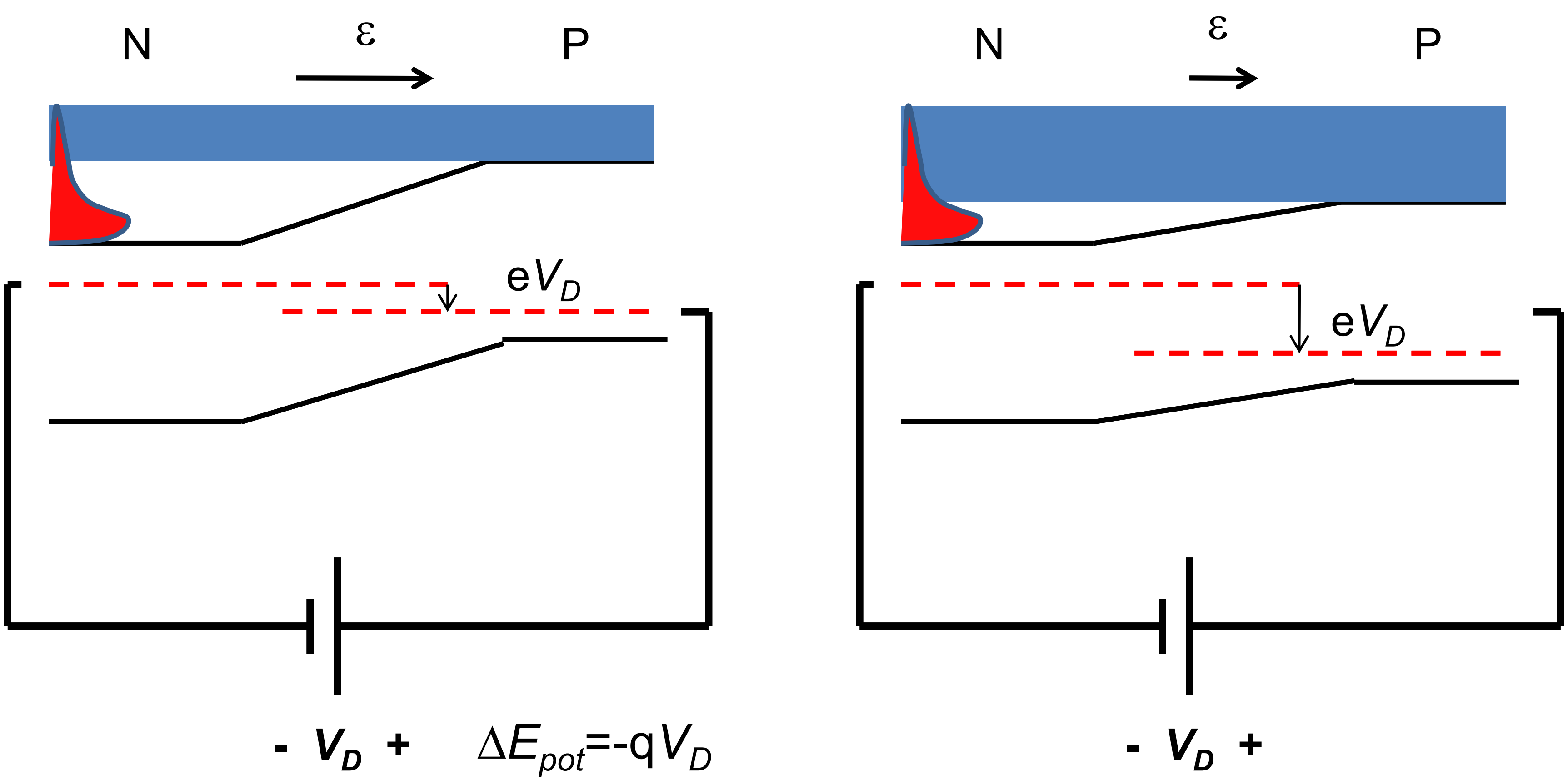
- Consider a pn-junction with $N_D = 5*N_A$ (donors > acceptors)
- Sketch the
  1) charge distribution
  2) electric field
  3) potential



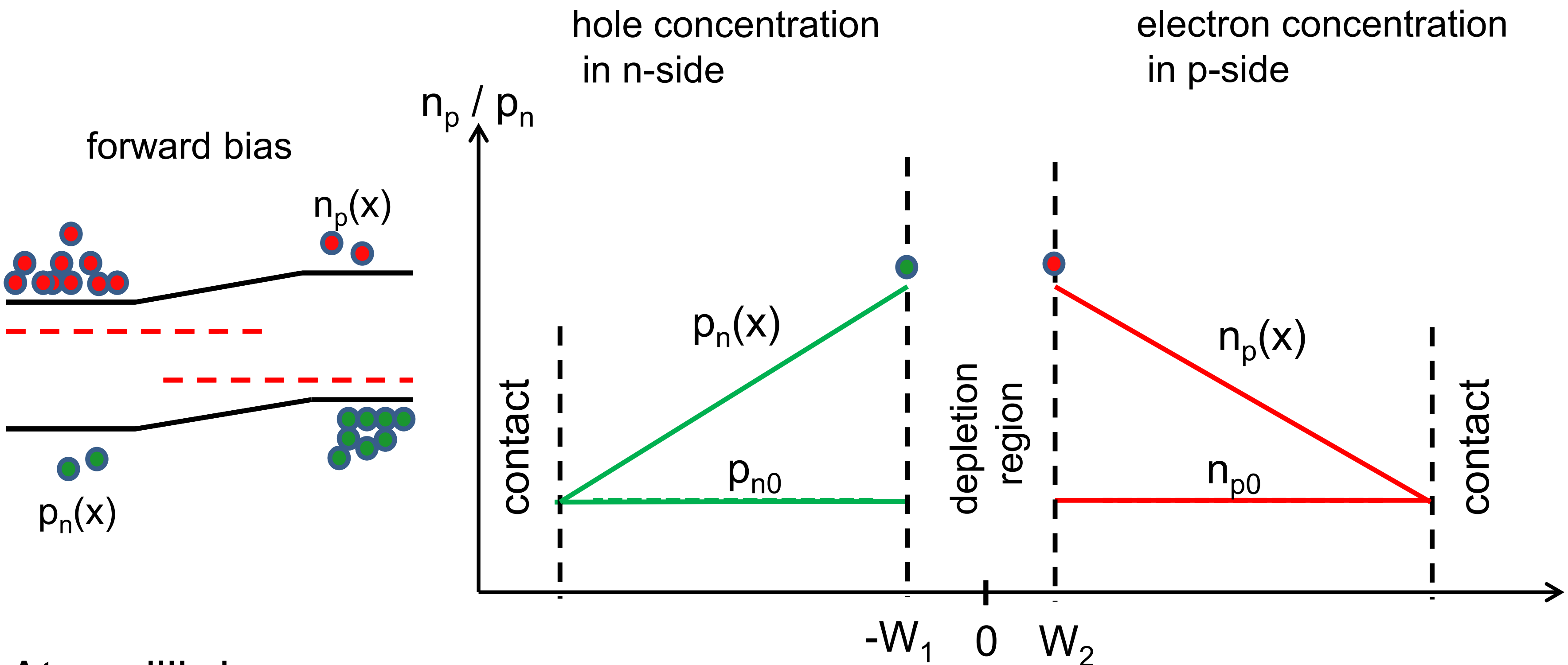


Diode – forward bias

- Only top of the electron distribution can pass over the barrier
- Increase bias -> exponentially increasing amount of electrons can pass
- Reduce electric field but counteracting built-in potential
- Depletion width is reduced



Minority carrier density at depletion edges at forward bias



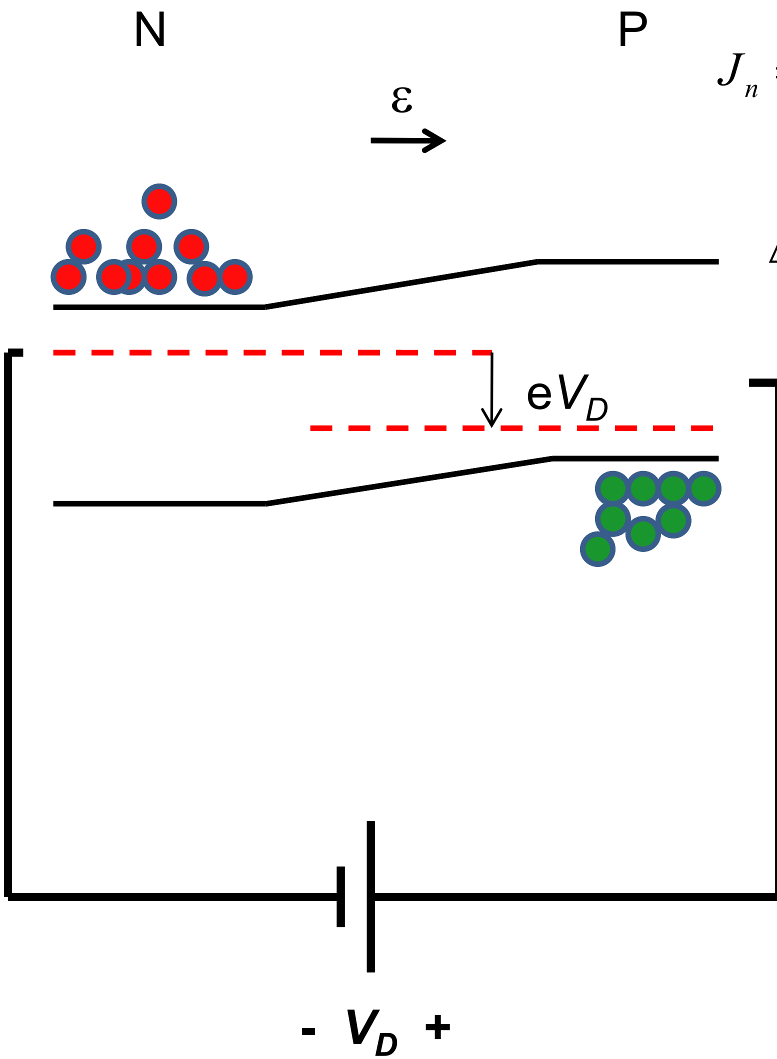
At equilibrium:

$n_{p0}=n_i^2/N_A$

$p_{n0}=n_i^2/N_D$

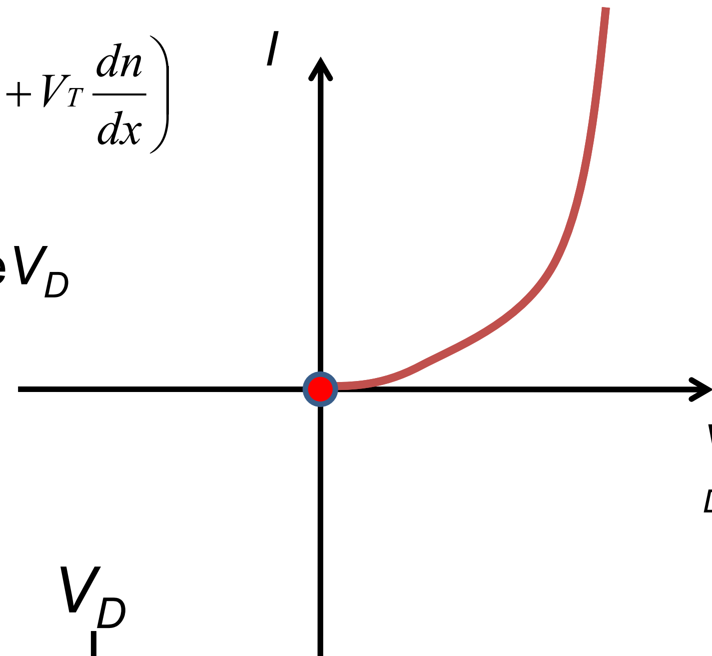
$$p_n(-W_1) = p_{n0}\exp\left(\frac{qV}{kT}\right) \qquad n_p(W_2) = n_{p0}\exp\left(\frac{qV}{kT}\right)$$

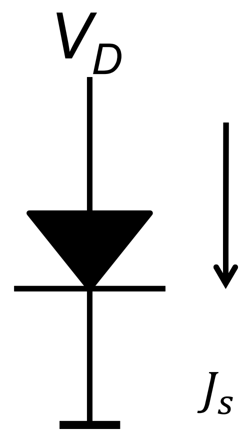
Diode – forward bias



$$J_n = q\mu_n\left(n\varepsilon + V_T\frac{dn}{dx}\right)$$

ΔE_{pot}=-eV_D

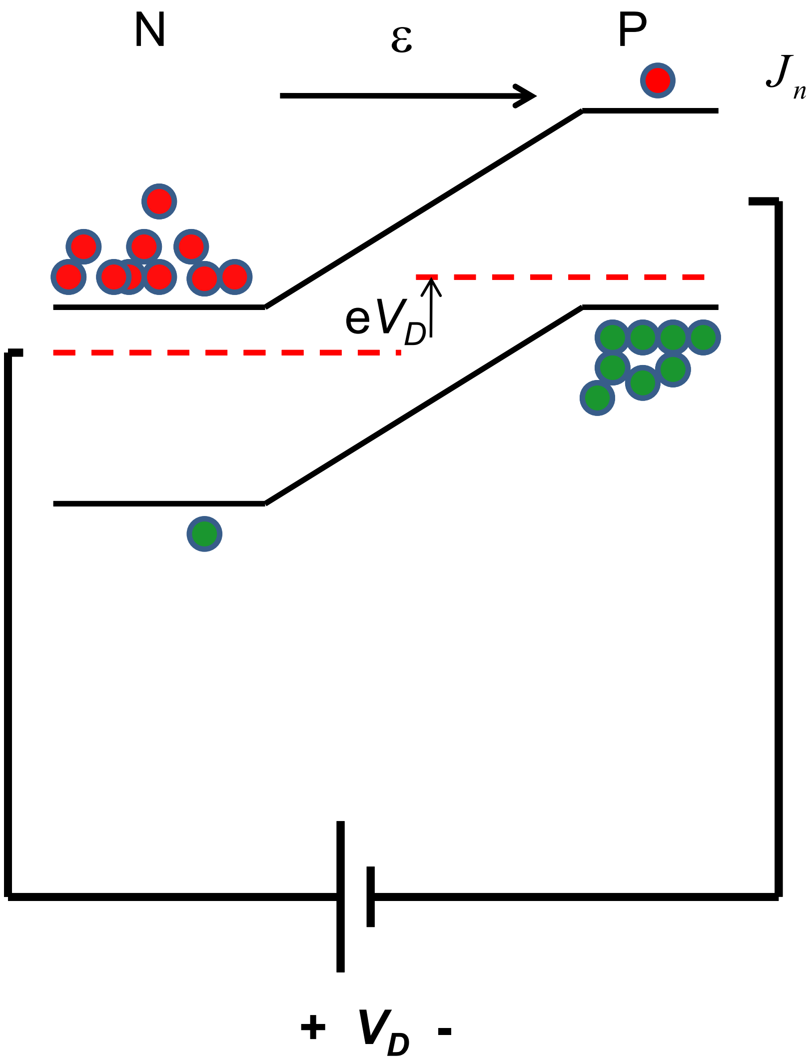




$J = J_S(\exp(V/V_T)\text{-}1)$

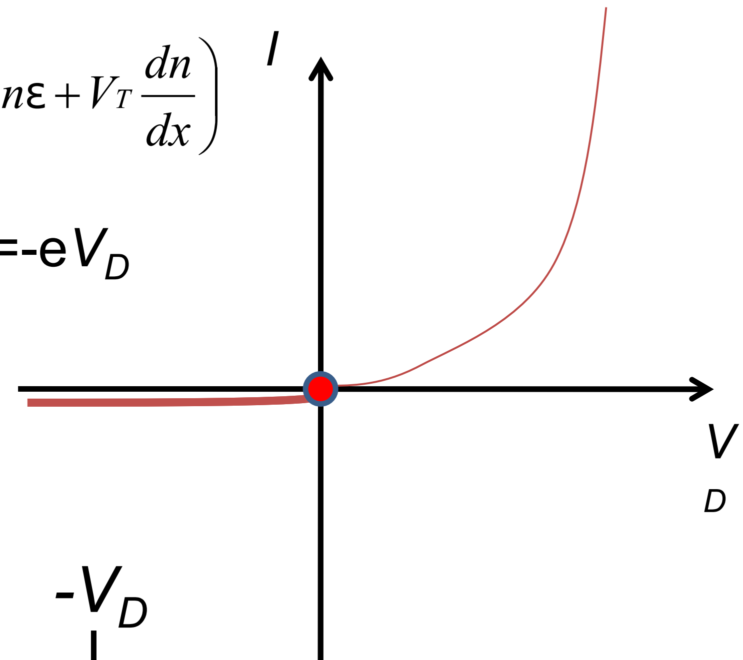
$$J_s = q\left(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{p0}\right)$$
$$= q\left(\frac{1}{L_p}\frac{kT}{q}\mu_p\frac{n_i^2}{N_D} + \frac{1}{L_n}\frac{kT}{q}\mu_n\frac{n_i^2}{N_A}\right)$$

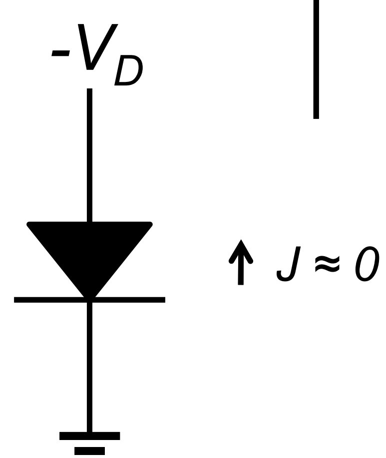
Diode – reverse bias



$$J_n = q\mu_n\left(n\varepsilon + V_T\frac{dn}{dx}\right)$$

ΔE_{pot}=-eV_D



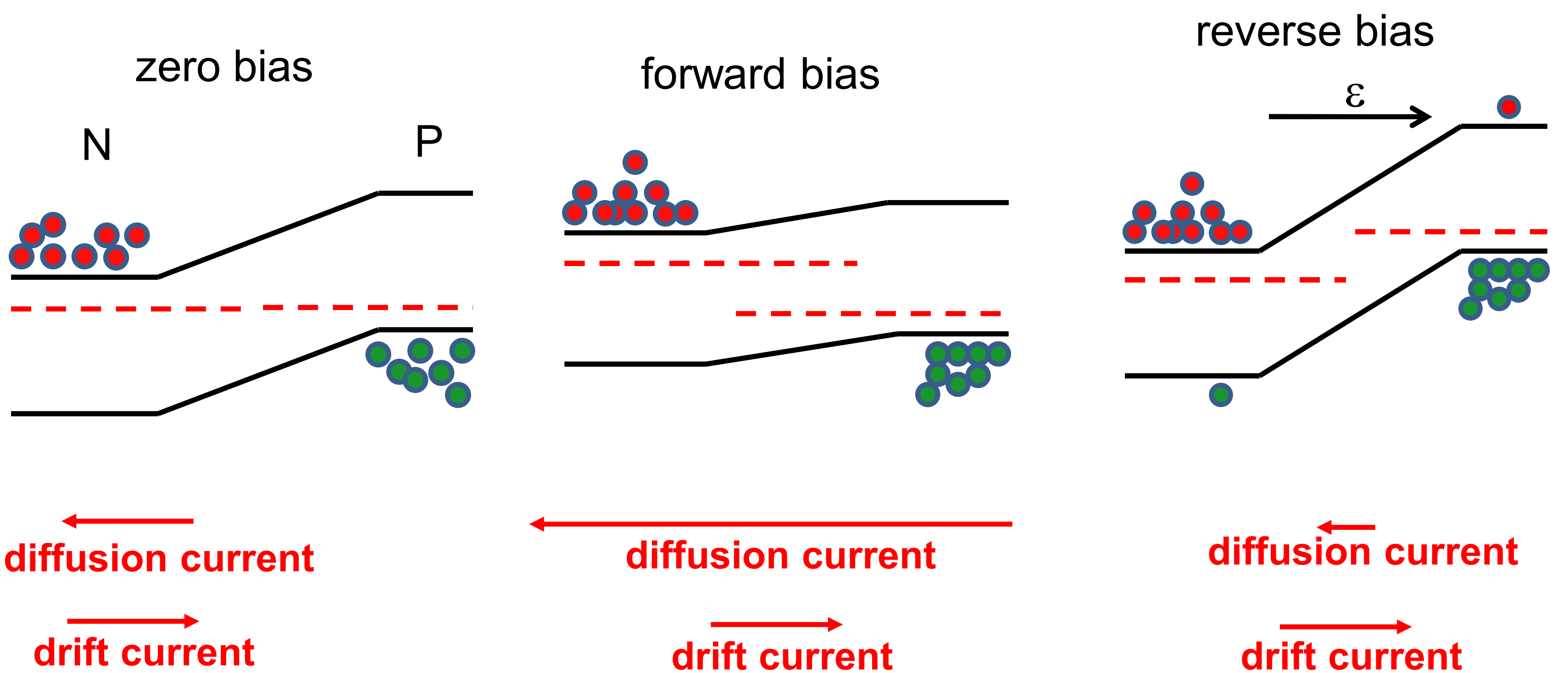


Diode – total current

$$J = J_s\left(e^{qV/nkT} - 1\right)$$

J_s = saturation current density
n = ideality factor (1-2)

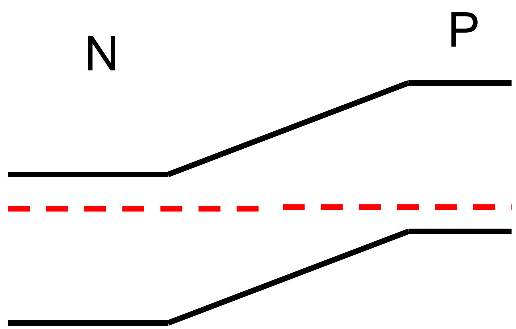
$$J_s = q\left(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{p0}\right)$$
$$= q\left(\frac{1}{L_p}\frac{kT}{q}\mu_p\frac{n_i^2}{N_D} + \frac{1}{L_n}\frac{kT}{q}\mu_n\frac{n_i^2}{N_A}\right)$$

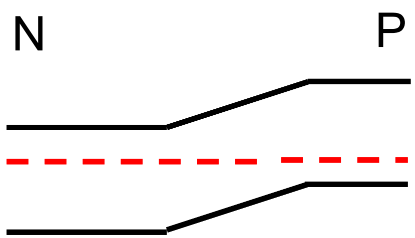


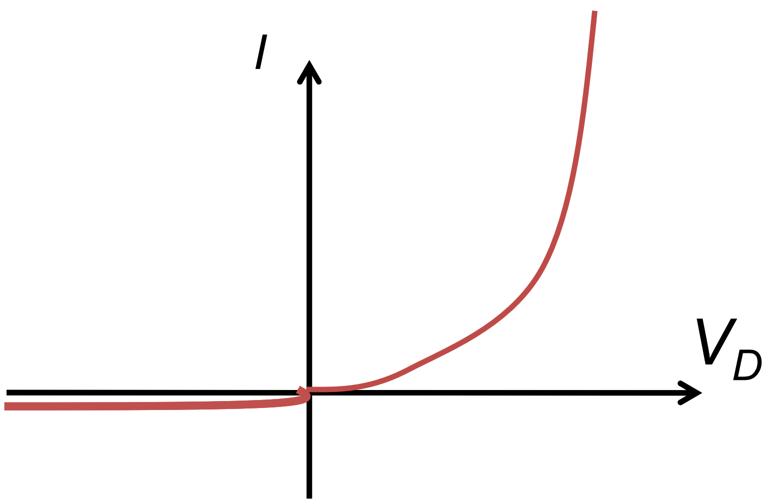
1 min excercise – Si vs Ge diode

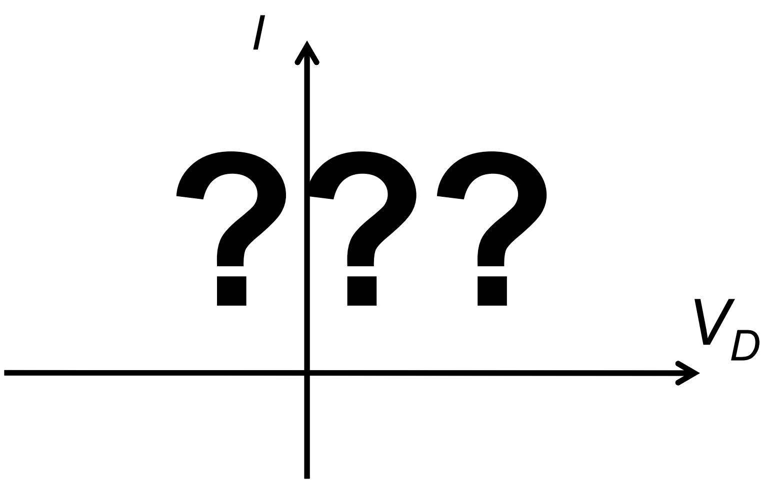
Si (E_g=1.11 eV) pn-junction

Ge (E_g=0.67 eV) pn-junction



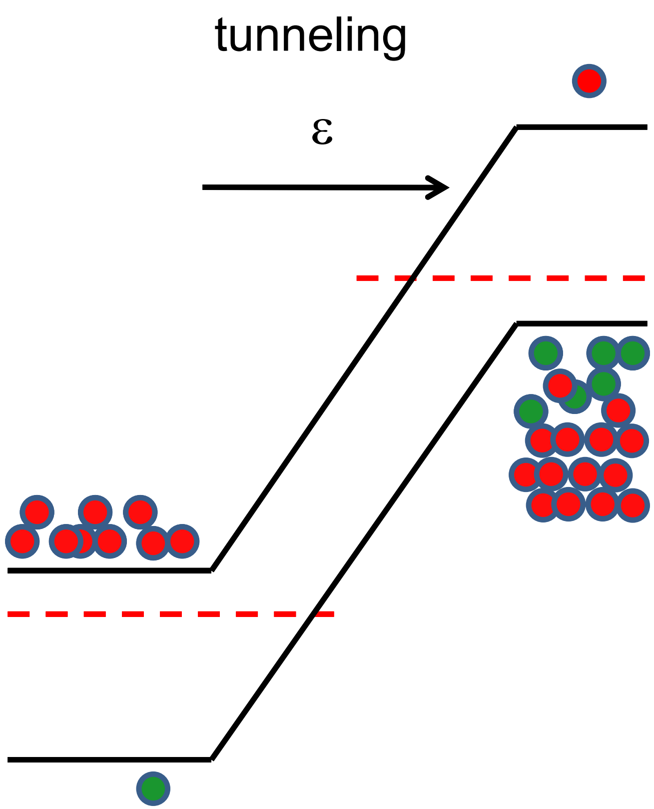


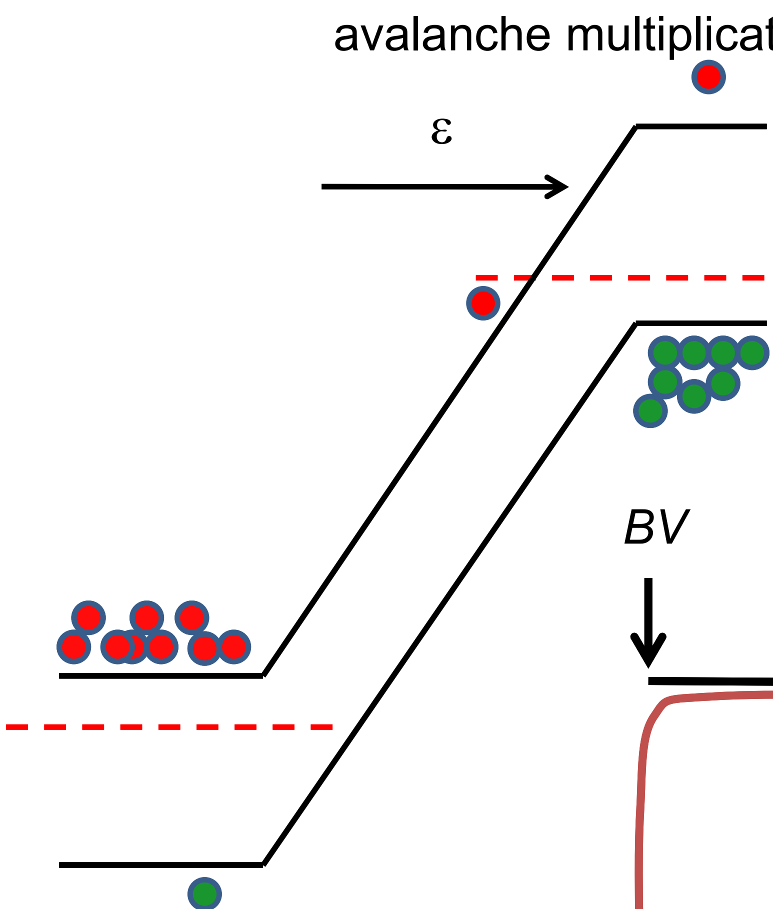


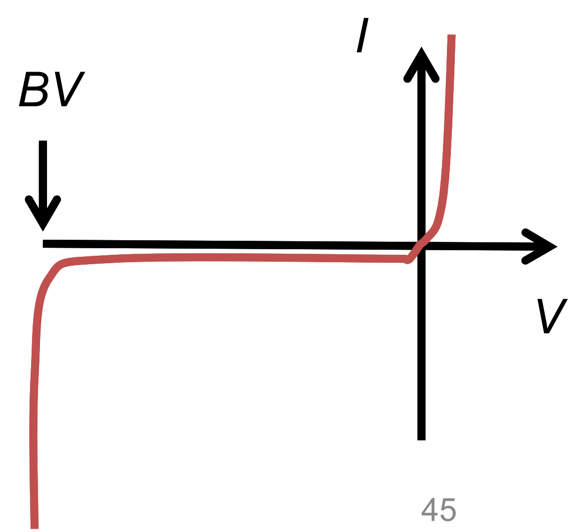


Zener tunneling / Avalanche Breakdown

- High reverse bias gives enough e-field to enable tunneling
- High energy of electron/hole can be lost by creating new e-h pairs through impact ionization



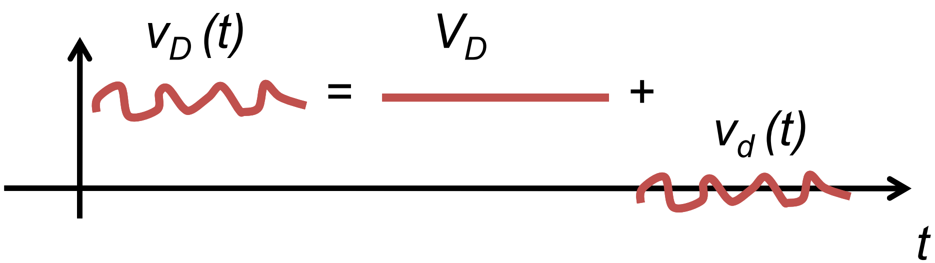


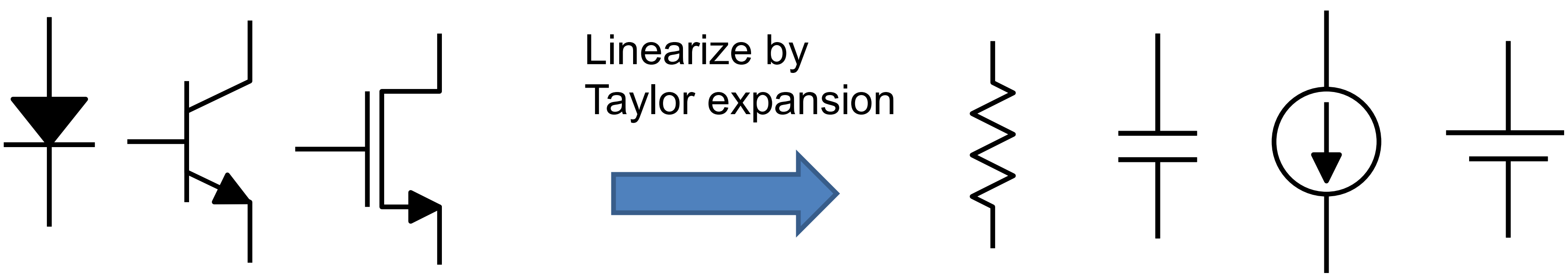


small-signal model

- Want to replace non-linear components with linear ones to simplify circuit calculations

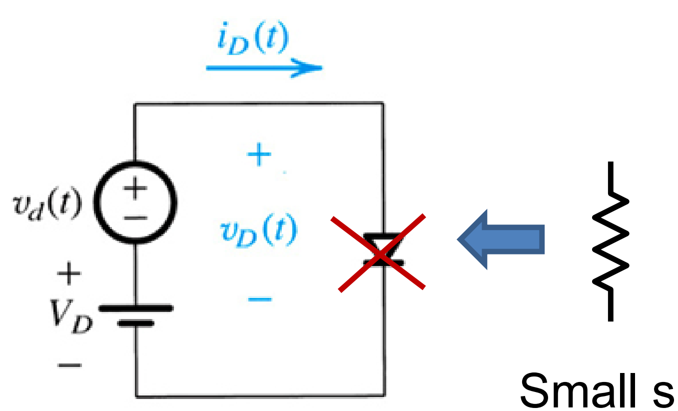
- Constant V_D + small varying $v_d(t)$ = total voltage $v_D(t)$.

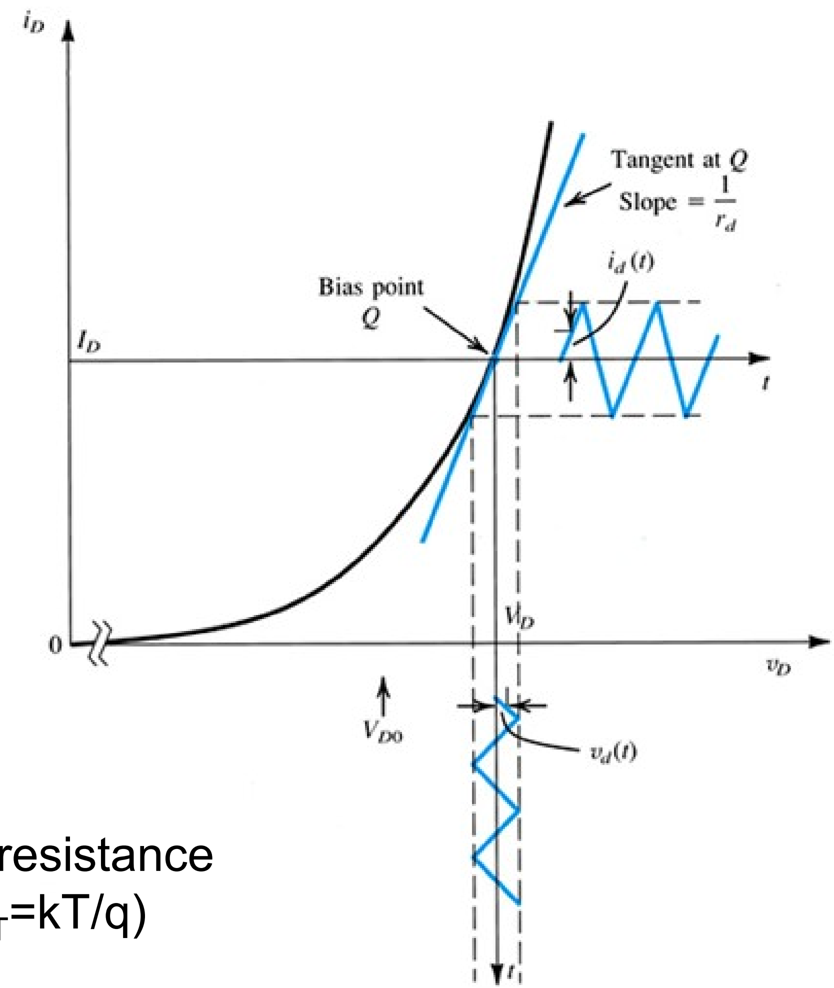




Small-signal model of diode (3.3.7)

- Diode IV is non-linear so difficult to do calculations
- Apply constant V_D and small varying $v_d(t)$
- Diode IV almost linear in a small region





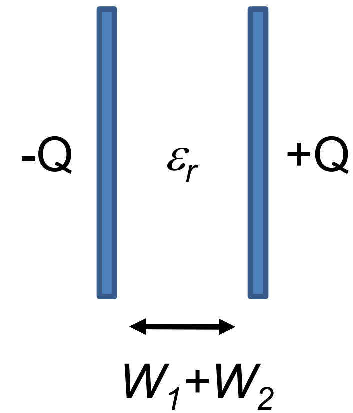
Small signal resistance
$r_d = V_T/I_D$ ($V_T = kT/q$)

$$J = J_s\left(e^{qV_D/kT} - 1\right) \approx J_s e^{qV_D/kT}$$

depletion-region / junction capacitance

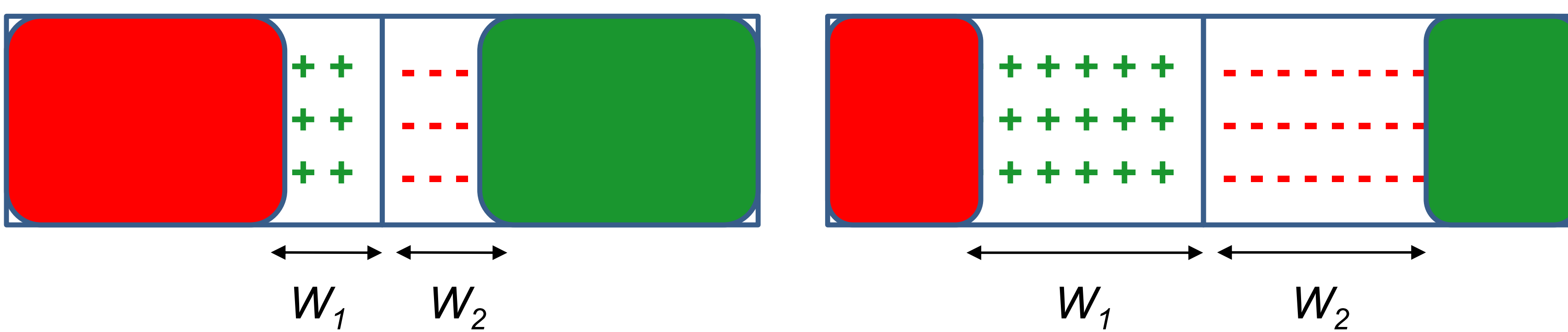
Example: paralell plate capacitor

Definition: $C = \dfrac{Q}{V}$ $\qquad C = \dfrac{\varepsilon_r \varepsilon_0}{W_1 + W_2} A$



$Q_J = A \cdot q \cdot N_D \, (W_1 + W_2)$ $\qquad W \propto \sqrt{V_0 + V_R}$

Non-linear relationship between V_R and Q -> $C(V_R)$



depletion region / junction capacitance

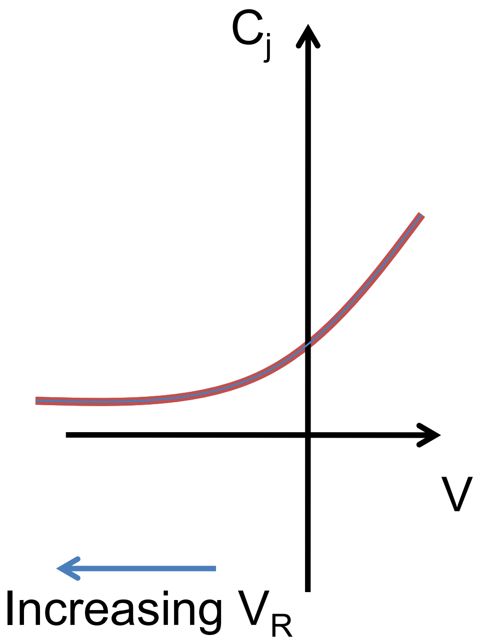
Definition $C_j = \dfrac{dQ_J}{dV_R}$

Applied bias (V_R) changes depletion width

Depletion width $W = \sqrt{\dfrac{2\epsilon_s}{q}\left(\dfrac{1}{N_A} + \dfrac{1}{N_D}\right)(V_0 + V_R)}$

Charge on either side $Q_J = AqN_DW_1 = Aq\dfrac{N_DN_A}{N_A+N_D}$ W

$$C_J = A\sqrt{2\epsilon_s q\left(\frac{N_DN_A}{N_A+N_D}\right)}\frac{1}{\sqrt{V_0 + V_R}}$$

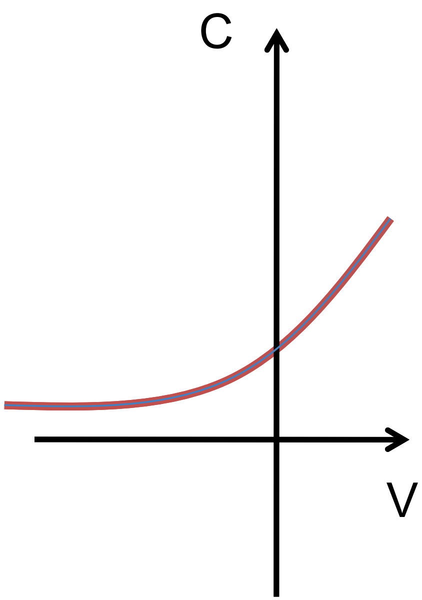


1 min exercise – pn-junction with forward bias

In forward bias there is a diffusion current flowing through the junction. How does the CV curve behave?

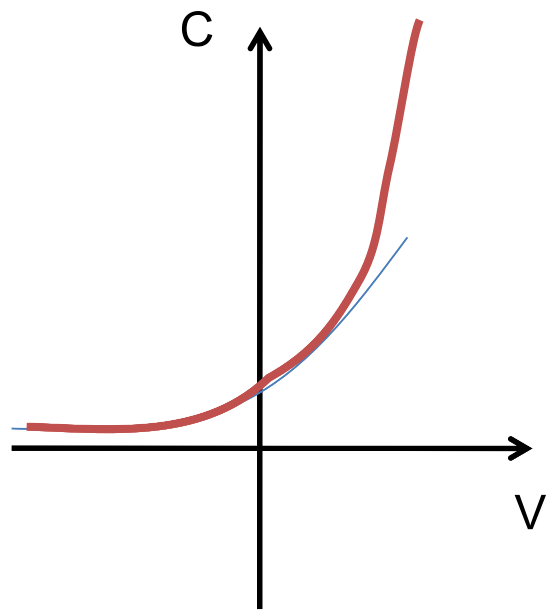
—— Only C_j —— $C = \frac{dQ}{dV}$

A



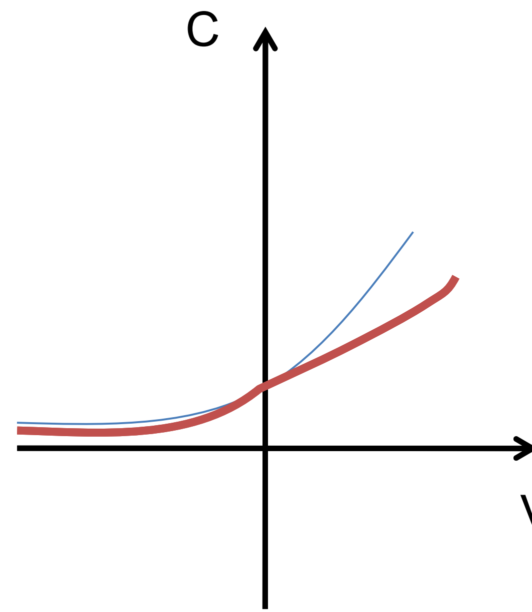
No change in capacitance

B



Larger capacitance for forward bias

C



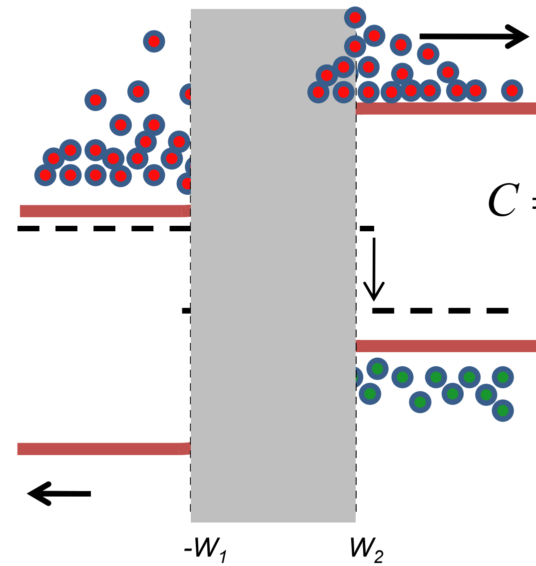
Lower capacitance for forward bias

diffusion capacitance

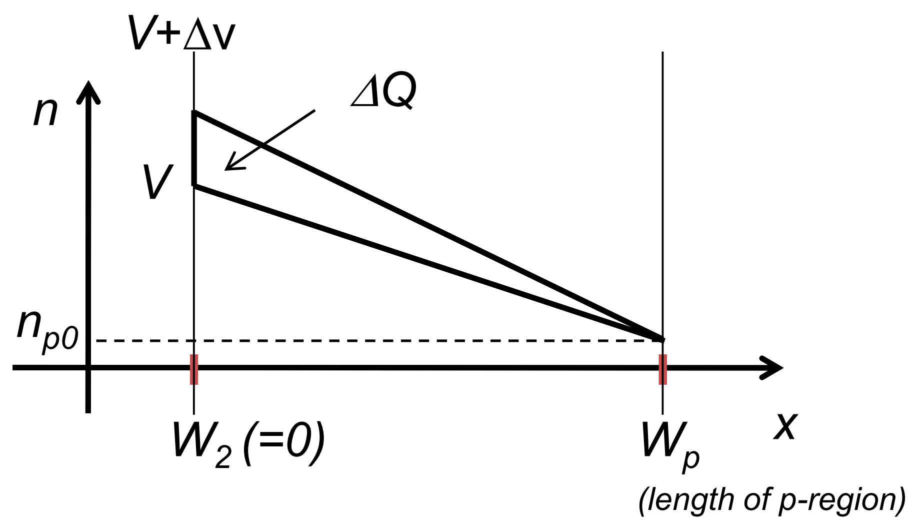
Forward bias -> inject minority carriers in neutral regions (electrons in p-region and holes in n-region)-> extra charge dQ.

$$n_p(W_2) = n_{p0} \exp\left(\frac{qV}{kT}\right)$$

$$n_p(W_p) = n_{p0}$$



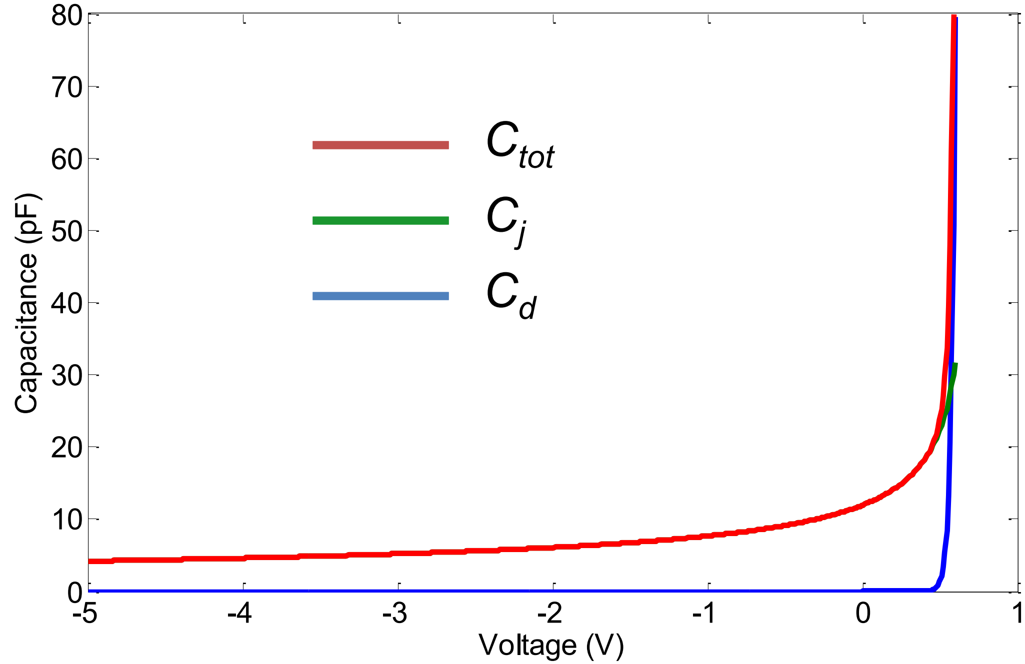
$$C = \frac{dQ}{dV}$$



Total capacitance

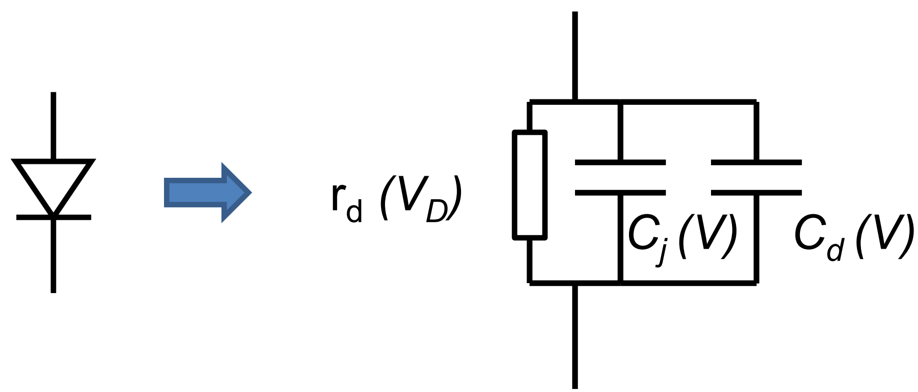
$C_{tot}=C_j+C_d$

Add capacitances in parallel



C_j: dominates for reverse bias

C_{diff}: dominates for forward bias.
$C_{diff} \approx 0$ for reverse bias.



Summary – pn-junctions

- pn-junctions used in LEDs, solar cells, BJT, MOSFETs
- Poissons equations: charge distribution -> electric field -> potential
- Drift is balanced by diffusion in unbiased pn-junction
- Current given by ideal diode equation:
  - Forward bias: current (diffusion) increases exponentially
  - Reverse bias: current saturates

$$J = J_s\left(e^{qV/nkT} - 1\right)$$

- Capacitances:
  - Junction capacitance due to change in depletion width (dominates reverse bias)
  - Diffusion capacitance due to change in charge in p/n region (dominates forward bias)
- Small-signal model (ok for $V \ll V_T$): replace diode with resistor + capacitances