F10 – Frequency Response

Outline

- Amplifier gain function
- Low frequency coupling/ bypass of a discrete CS amplifier
- High frequency transistor internal capacitive effects
- High frequency MOS and BJT model
- Current gain transition frequency
- Miller's voltage theorem
- Frequency response analysis
    - Gain, dominant pole, short/ open circuit time constants
    - CS amplifier
    - CG amplifier and Cascode amplifier

Reading Guide *Sedra/Smith 7ed int*

- Chapter 9.1-9.5, (9.6-9.8)
- Appendix F (Bode plot rules)

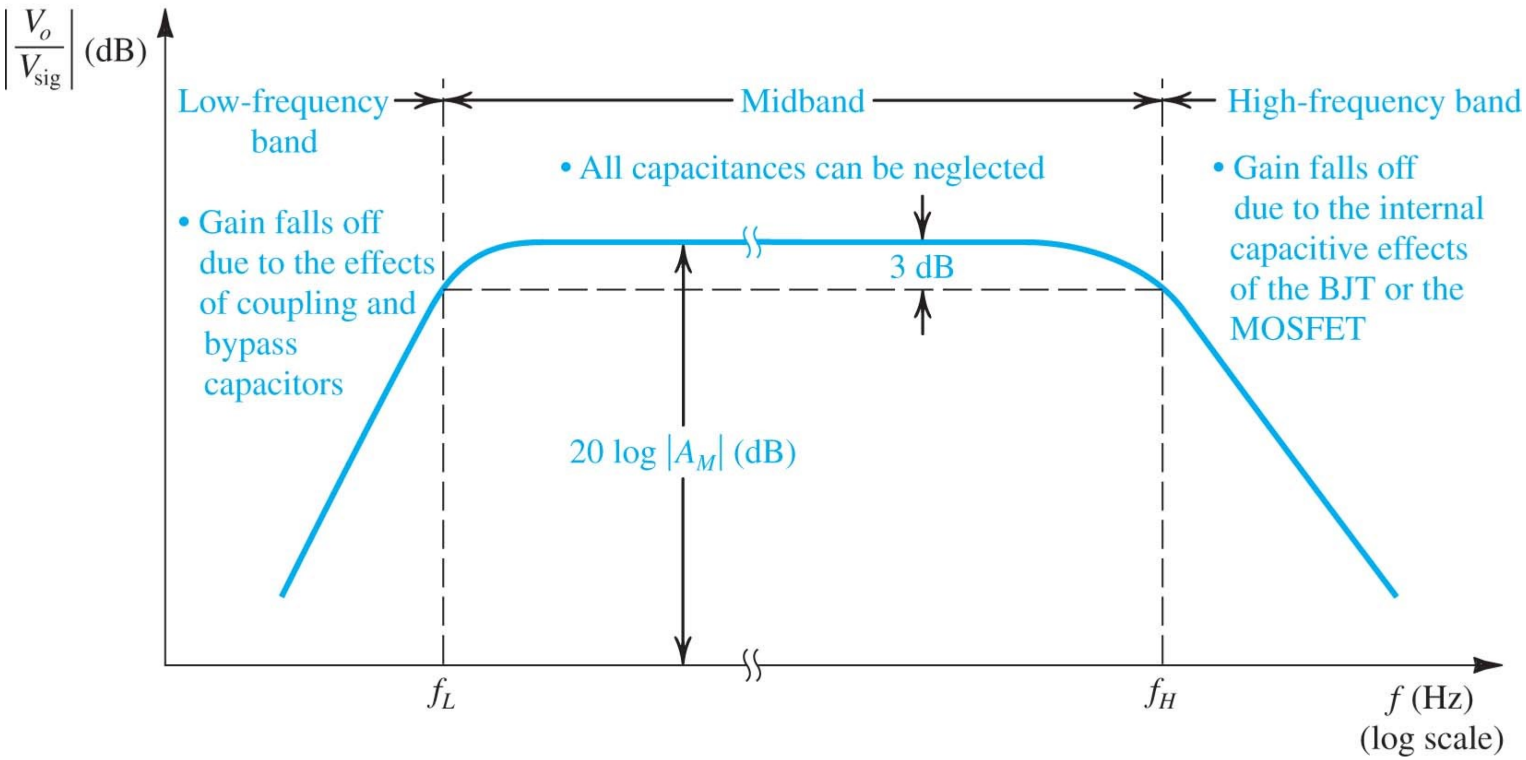
Problems *Sedra/Smith 7ed int*

- P9.14, 9.15, 9.24, 9.30, 9.67

Frequency Response of Amplifiers

- Low frequency band, $f < f_L$ (not in dc coupled IC amplifiers)
  - Low corner frequency, f_L
  - Coupling and bypass capacitances
- Midband, $f_L < f < f_H$
  - Bandwidth, $BW = f_H - f_L$
  - Midband gain, A_M
  - Neglect capacitances
- High frequency band, $f > f_H$
  - High corner frequency, f_H
  - Intrinsic capacitances

$$GB = |A_M| BW = |A_M|(f_H - f_L)$$



Gain bandwidth product, GB, is a key figure of merit.

Amplifier Gain Function

- Amplifier gain function, $A(s)$, where $s = j\omega = j2\pi f$ is the complex angular frequency
  - Low frequency transfer function, $F_L(s)$
  - Midband gain (no frequency dependence), A_M
  - High frequency transfer function, $F_H(s)$

$$A(s) = F_L(s) A_M F_H(s)$$

- Transfer functions, $F(s)$, can be found from circuit analysis in the s-plane (using immittances)
  - Rational polynomial in s, with m numerator coefficients a_i and $n \geq m$ denominator coefficients b_j
  - Physical (R, G, C, L) networks produce real coefficients, which yield real or conjugate paired roots
  - Numerator roots, $Z_i = -\omega_{Zi}$, a.k.a. transmission zeros ($+20\ \mathrm{dB/dec}$ and $+90°$ about $|s| \approx \omega_{Zi}$)
  - Denominator roots, $P_j = -\omega_{Pj}$, a.k.a. poles, natural modes ($-20\ \mathrm{dB/dec}$ and $-90°$ about $|s| \approx \omega_{Pj}$)

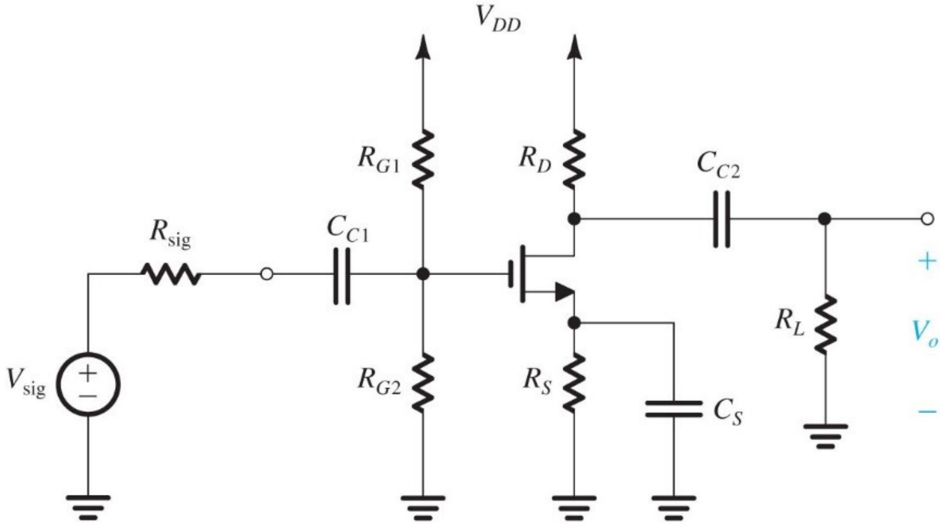
$$F(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{j=0}^{n} b_j s^j} = a_m \frac{\prod_{i=1}^{m}(s - Z_i)}{\prod_{j=1}^{n}(s - P_j)} = A \frac{\prod_{i=1}^{m}\left(1 + \frac{s}{\omega_{Zi}}\right)}{\prod_{j=1}^{n}\left(1 + \frac{s}{\omega_{Pj}}\right)}$$

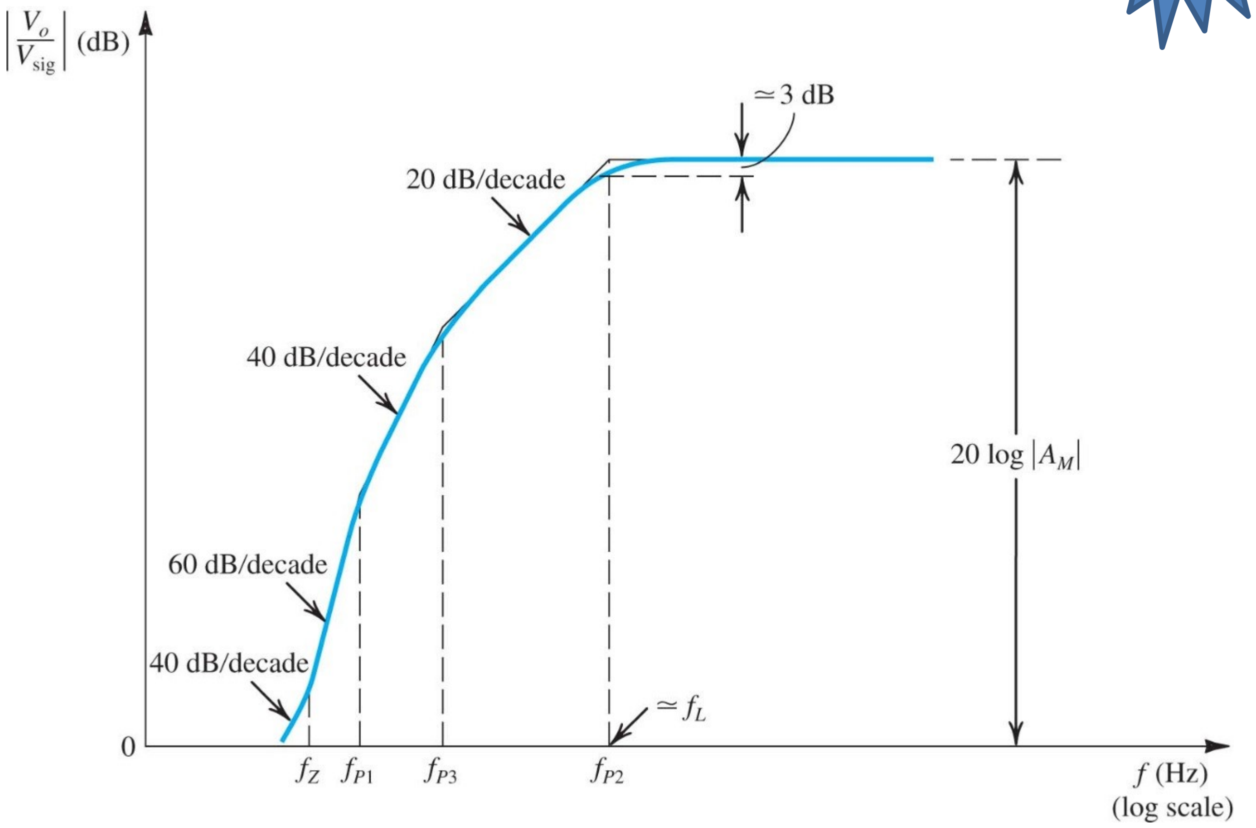
Poles may come with up to an equal number of zeros.

Low Frequency Coupling/ Bypass of Discrete CS Amplifier

- Signal coupling capacitances, isolates bias and pass signals to/ from input/ output
  - Gate input
  - Drain output

- Bypass capacitance, cancels parallel component
  - Source bias network resistance





$$Z_C = \frac{1}{Y_C} = \frac{1}{j\omega C}$$

Coupling and bypass impedances “disappear” at high frequency.

MOSFET Internal Capacitive Effects

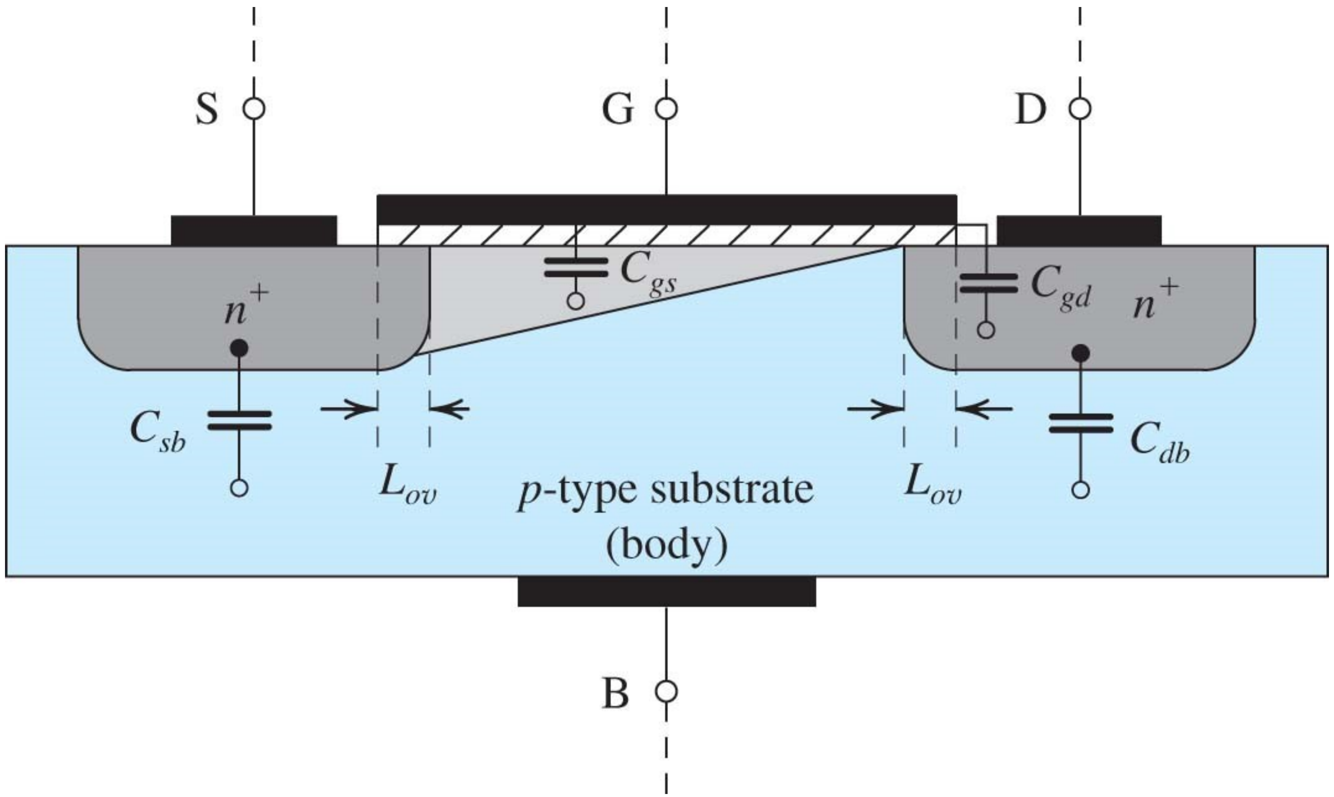
- Channel capacitance
  - Gate-source

$$C_{ch} = \frac{2}{3} W L C_{ox}$$

- Overlap capacitances
  - Gate-source
  - Drain-source

$$C_{ov} = W L_{ov} C_{ox}$$

- Junction capacitances
  - Source-body
  - Drain-body



High Frequency MOSFET Model (recap)

- Transconductance

$$g_m = k_n'\left(\frac{W}{L}\right)V_{OV} = \sqrt{2k_n'\left(\frac{W}{L}\right)I_D} = \frac{2I_D}{V_{OV}}$$

- Output resistance

$$r_o = \frac{1}{\lambda I_D'} = \frac{L}{\lambda' I_D'} = \frac{LV_A'}{\lambda' I_D'} = \frac{V_A}{I_D'}$$

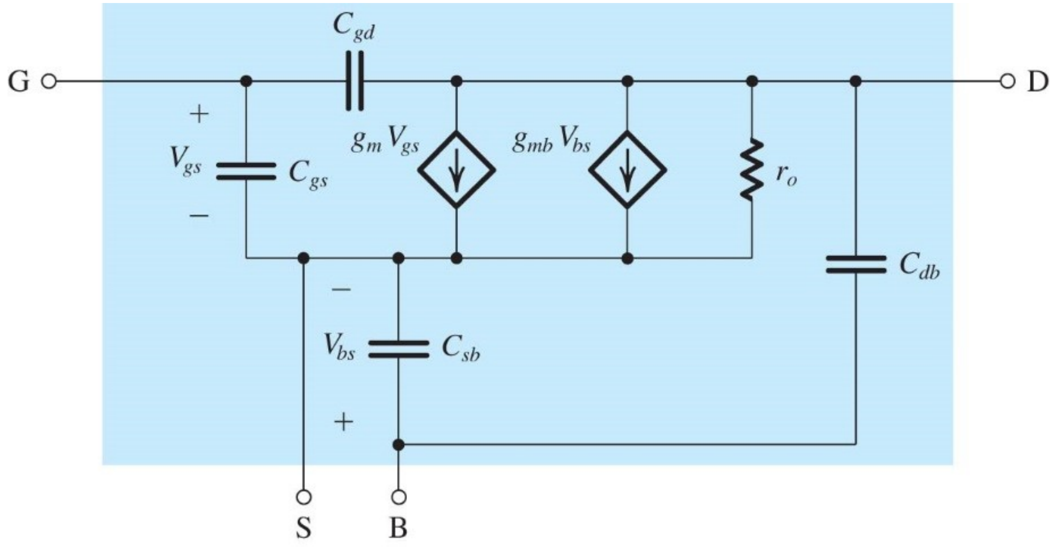
- Channel and overlap capacitance

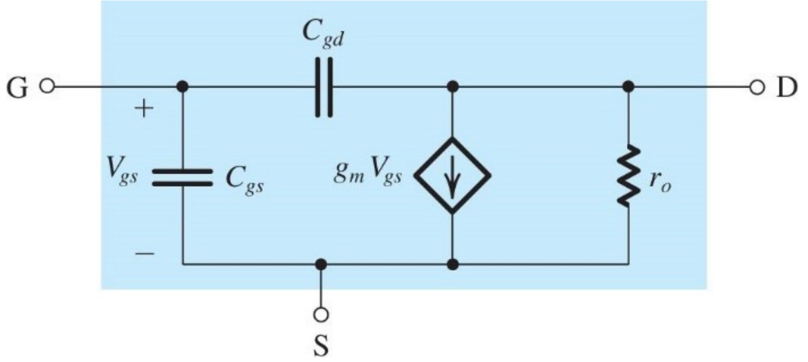
$$C_{gs} = C_{ch} + C_{ov} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

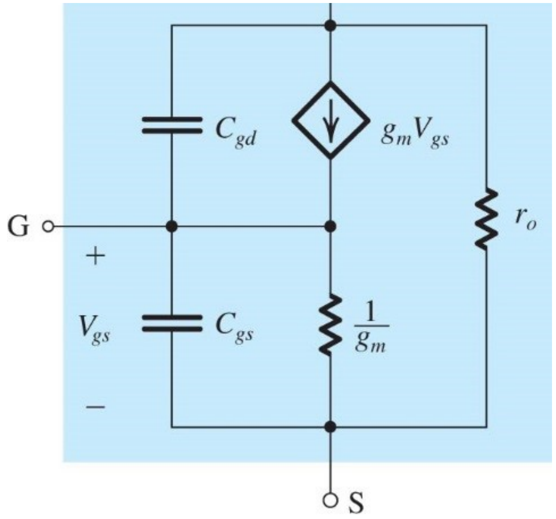
$$C_{ds} = C_{ov} = WL_{ov}C_{ox} < C_{gs}$$

- (Body)
  - Transconductance $g_{mb} = \chi g_m, 0.1 < \chi < 0.2$
  - Junction capacitance

$$C_{xb} = C_{xb0} \Big/ \sqrt{1 + \frac{V_{XB}}{V_0}}$$







High Frequency BJT Model (recap)

- Transconductance

$$g_m = \frac{I_C}{V_T}$$

- Output resistance

$$r_o = \frac{V_A}{I_C'}$$

- Base/ emitter resistance

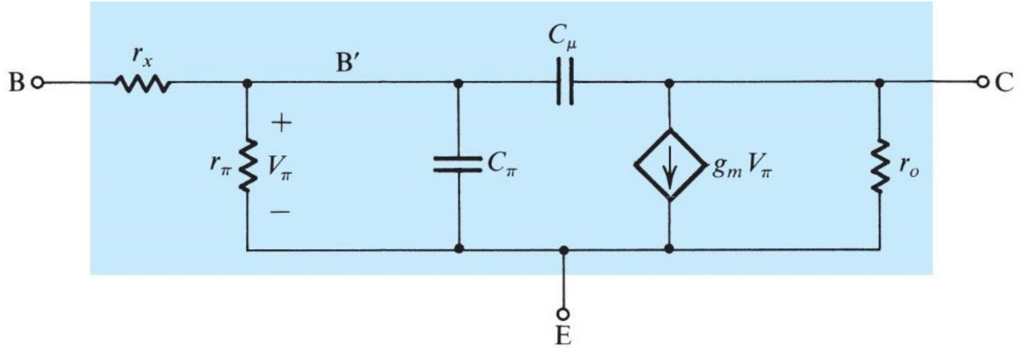
$$r_\pi = \frac{\beta}{g_m} \qquad r_e = \frac{r_\pi}{(1+\beta)}$$

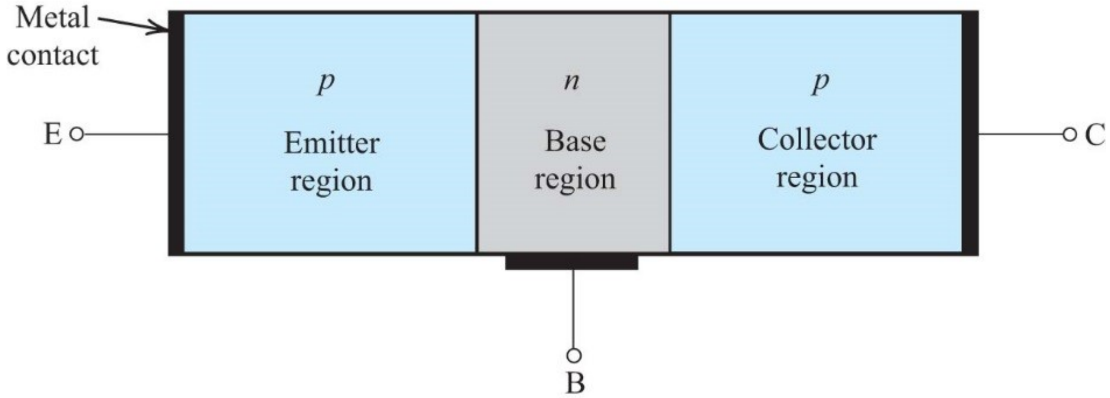
- Base access resistance

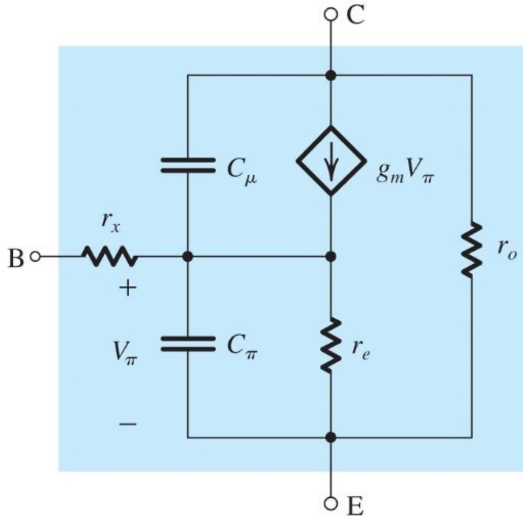
$$r_x$$

- Base transit and junction capacitances

$$C_\pi = C_{de} + C_{je} \approx \tau_F g_m + 2C_{je0} \qquad C_\mu = C_{jc} = C_{jc0} \Big/ \left(1 + \frac{V_{CB}}{V_{0c}}\right)^m, 0.3 < m < 0.5$$



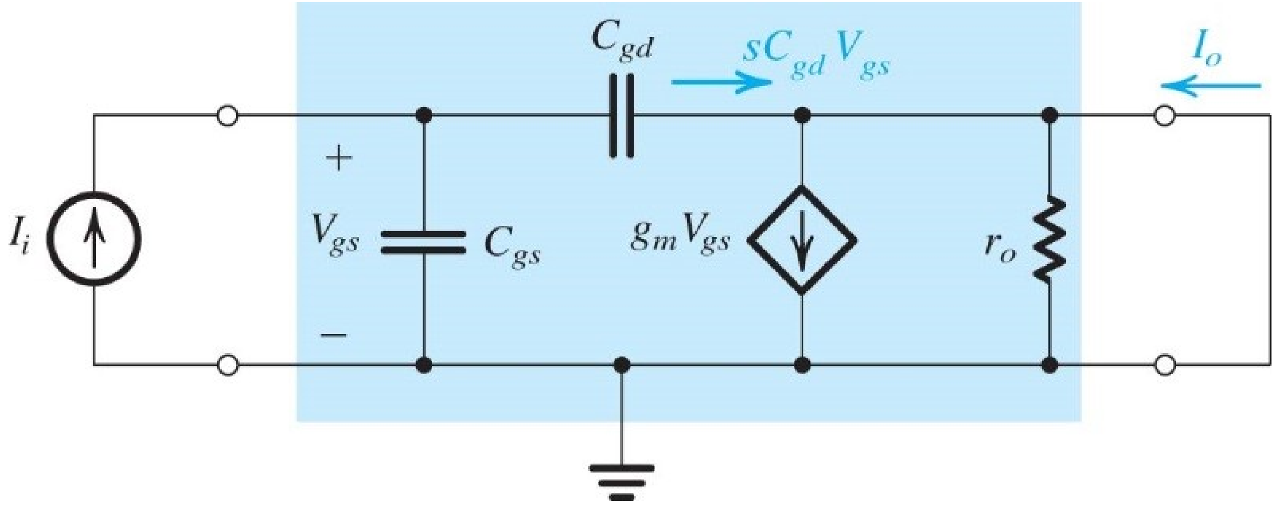




CS(/ CE) Amplifier Cutoff Frequency

- The frequency where the transistor current gain transitions from above unity to below unity, i.e. where the gain cuts off
  - Current gain transition frequency, f_T

$$A_i(f_T) = \frac{I_o}{I_i} = 1$$



- MOSFET

$$f_T \approx \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

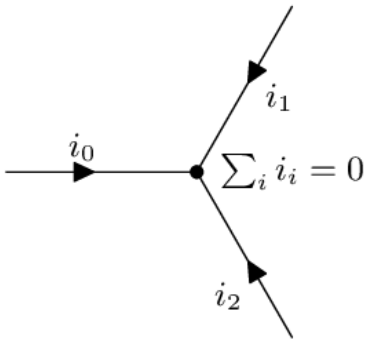
- BJT

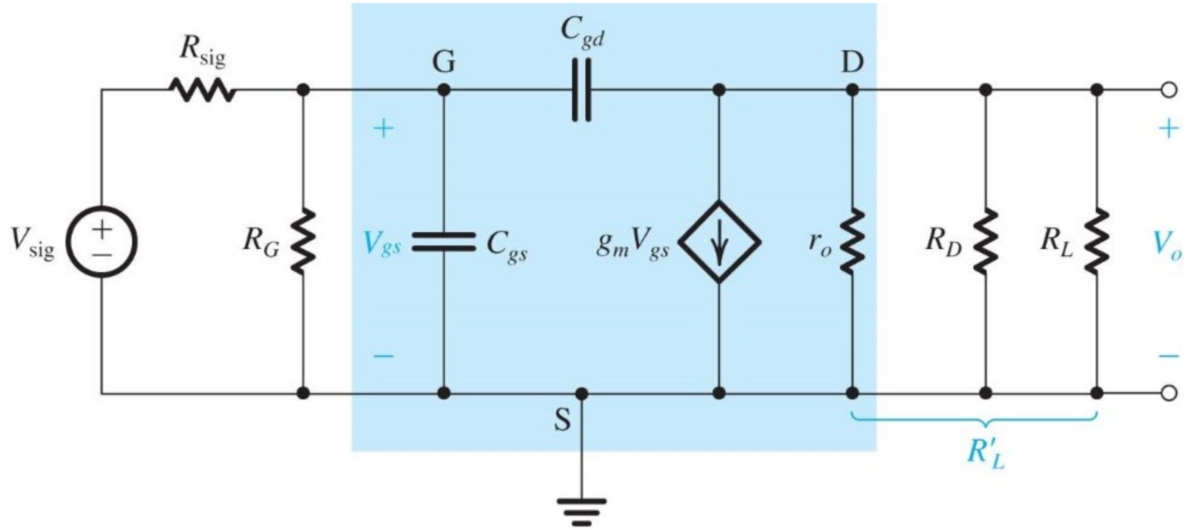
$$f_T \approx \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

Current gain transition frequency, cutoff frequency, unity gain frequency…
An important metric for MOSFETs and BJTs.

Gain Function by Nodal Analysis: CS Amplifier

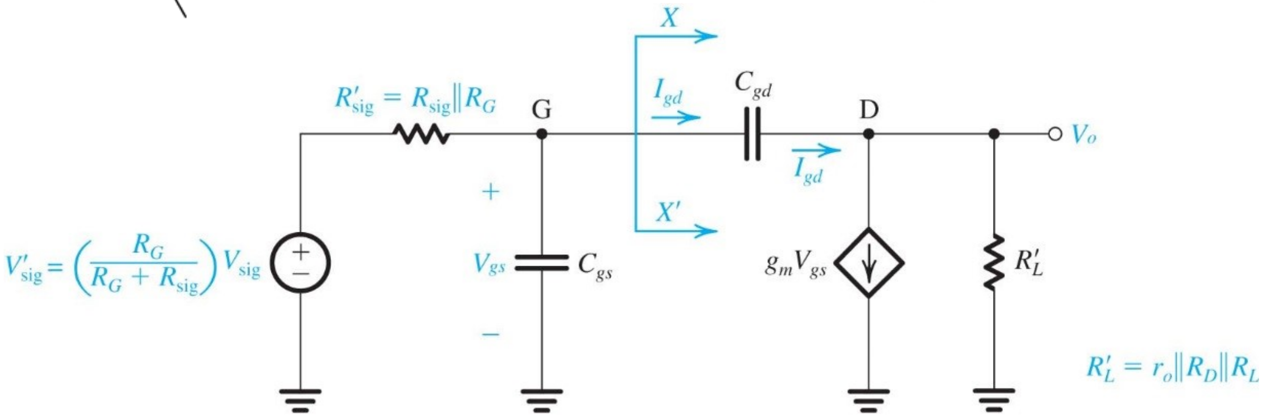
- Simplify with source theorems
- Formulate KCL node equations
  - Equations system with immittances





- Gauss elimination of internal node voltages
  - Large rational expressions in complex numbers

$$F(s) = A \frac{\prod_{i=1}^{m}\left(1 + \frac{s}{\omega_{Zi}}\right)}{\prod_{j=1}^{n}\left(1 + \frac{s}{\omega_{Pj}}\right)}$$



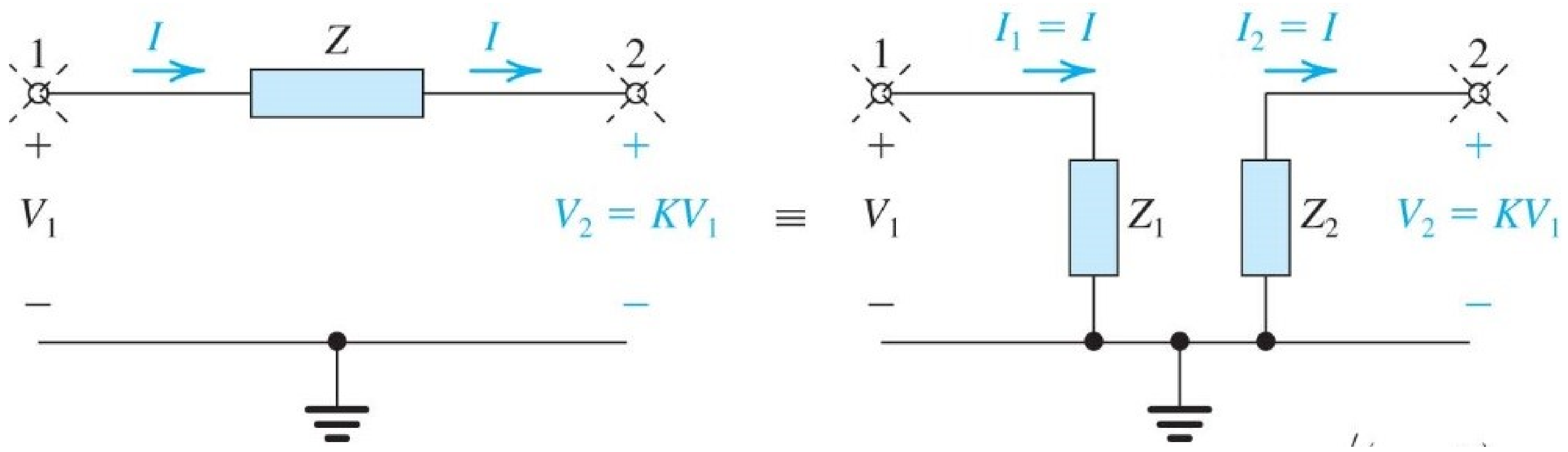
- Is there not a simpler way???

Nodal analysis works, but it is tedious and gives a “complex” result.

BREAK

Miller's Voltage Theorem

- Impedance, Z, connected between dependent nodes 1 and 2, where voltages are $V_2 = KV_1$ related to a common node
- Equivalent to impedances, Z_1 and Z_2, to common terminal



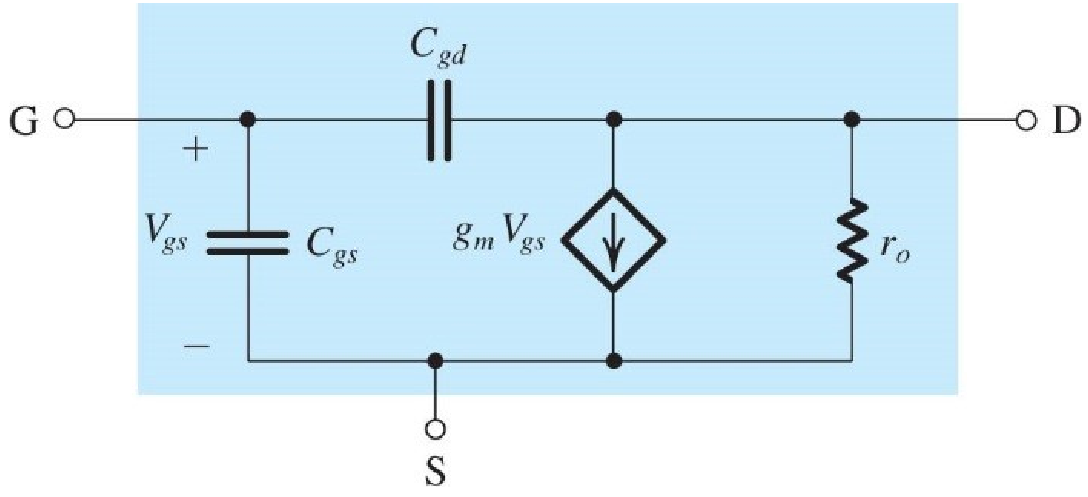
$$Z_1 = \frac{Z}{1 - K}$$

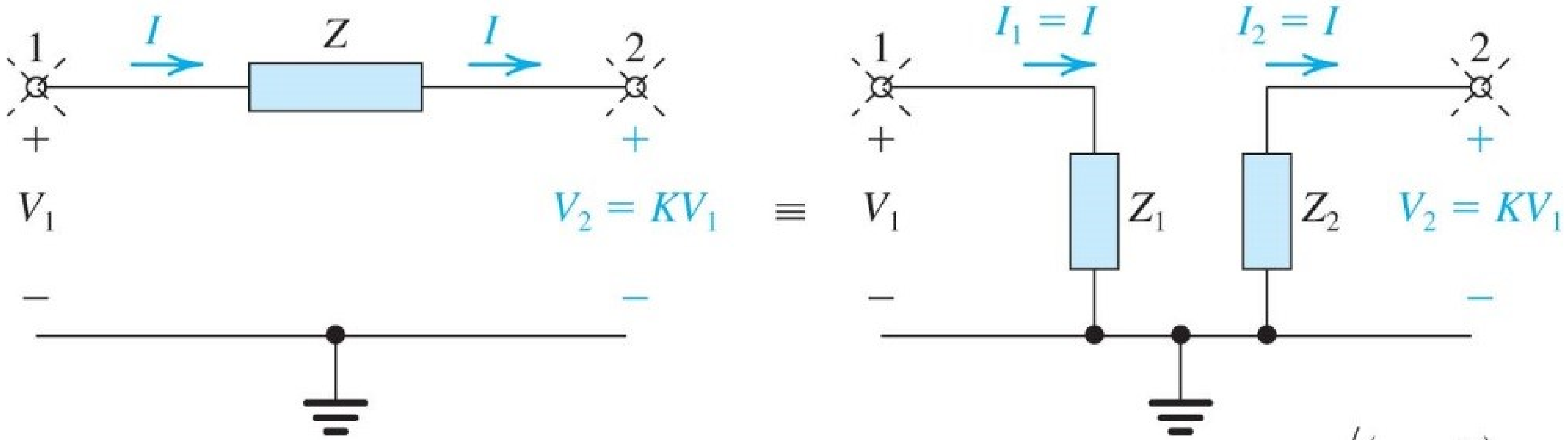
$$Z_2 = \frac{Z}{1 - \frac{1}{K}}$$

Only valid for finding input impedance, since external circuit must remain invariant.

How can Miller's theorem be adapted to transistor analysis?

- Z?
- K?





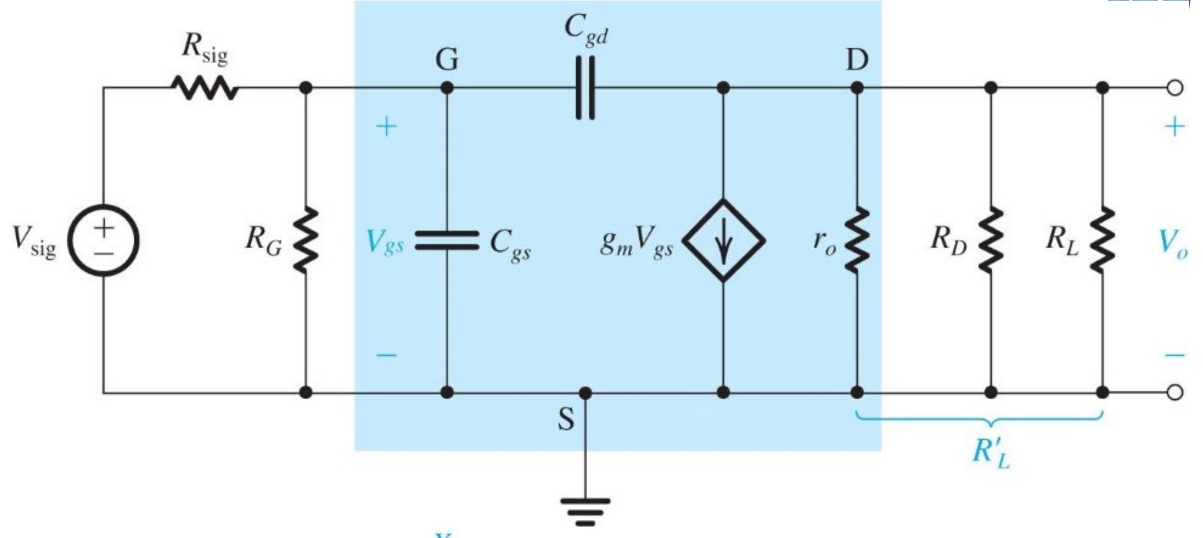
Gain Function by Miller Approximation: CS Amplifier

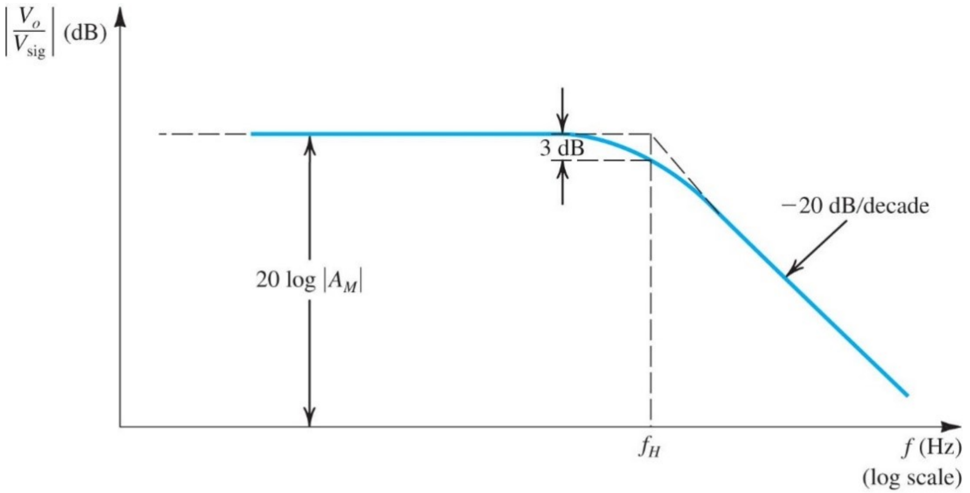
- Midband gain (same as before)

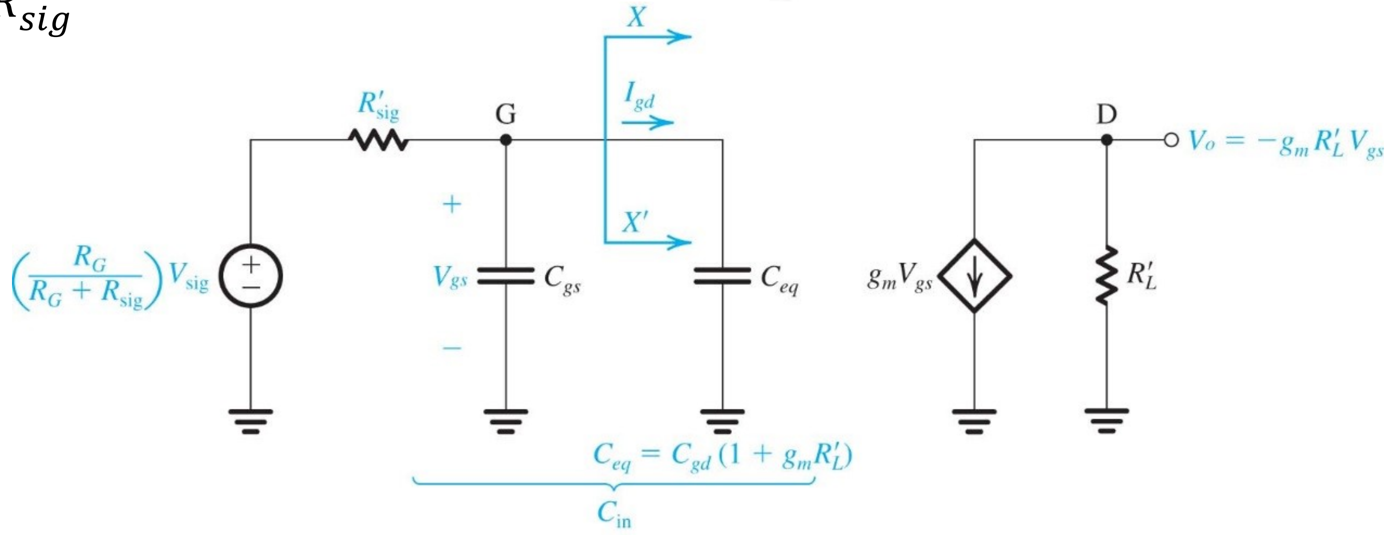
$$A_M = -g_m R_L'$$

- High corner frequency
  - Miller multiplier, $(1 + g_m R_L')$
  - Input time constant

$$f_H = \frac{1}{2\pi\tau_H} \approx \frac{1}{2\pi\left[C_{gs} + (1 + g_m R_L')C_{gd}\right]R_{sig}'}$$





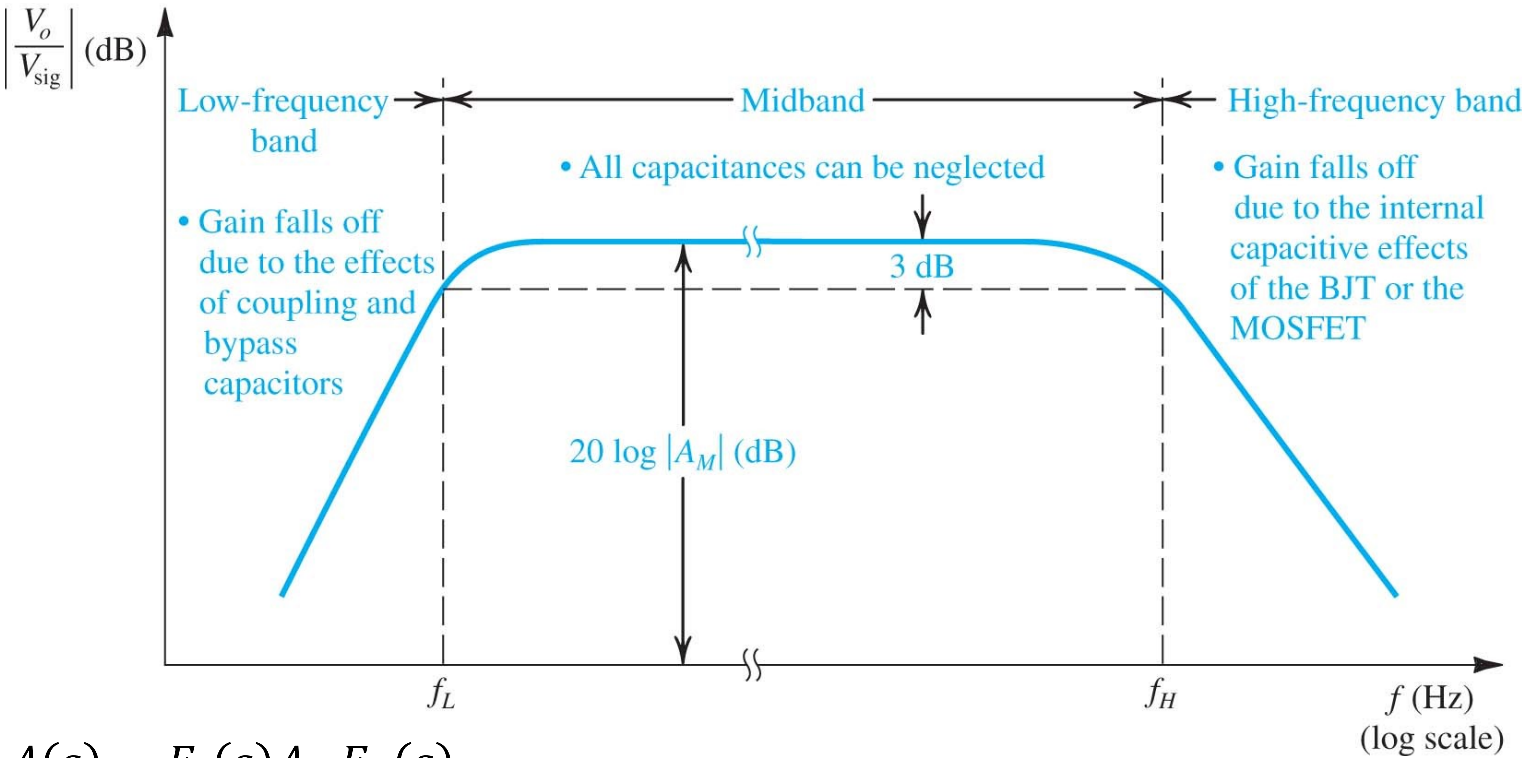


Frequency Response – Gain Function Zeros and Poles

- Low frequency band, $f < f_L$ (not in dc coupled IC amplifiers)
  - Low corner frequency, f_L
  - Coupling and bypass capacitances

- Midband, $f_L < f < f_H$
  - Bandwidth, $BW = f_H - f_L$
  - Midband gain, A_M
  - Neglect capacitances

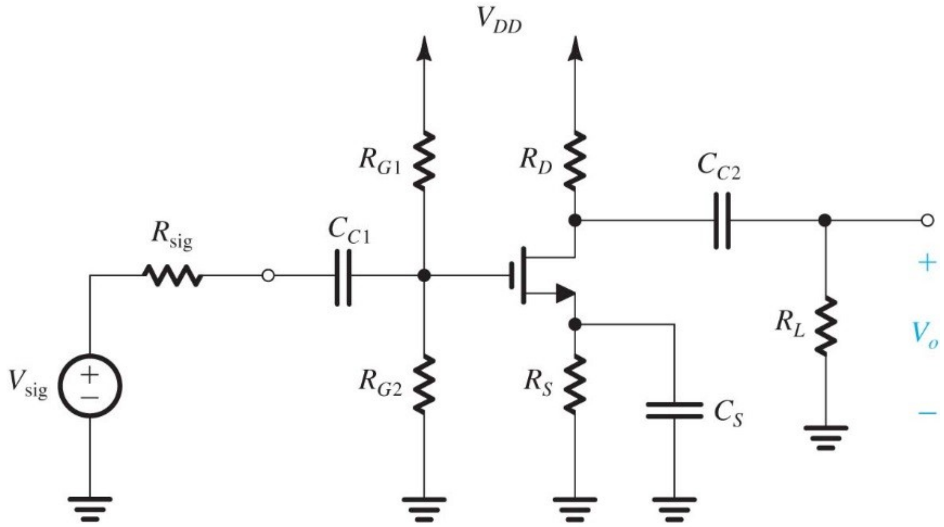
- High frequency band, $f > f_H$
  - High corner frequency, f_H
  - Intrinsic capacitances

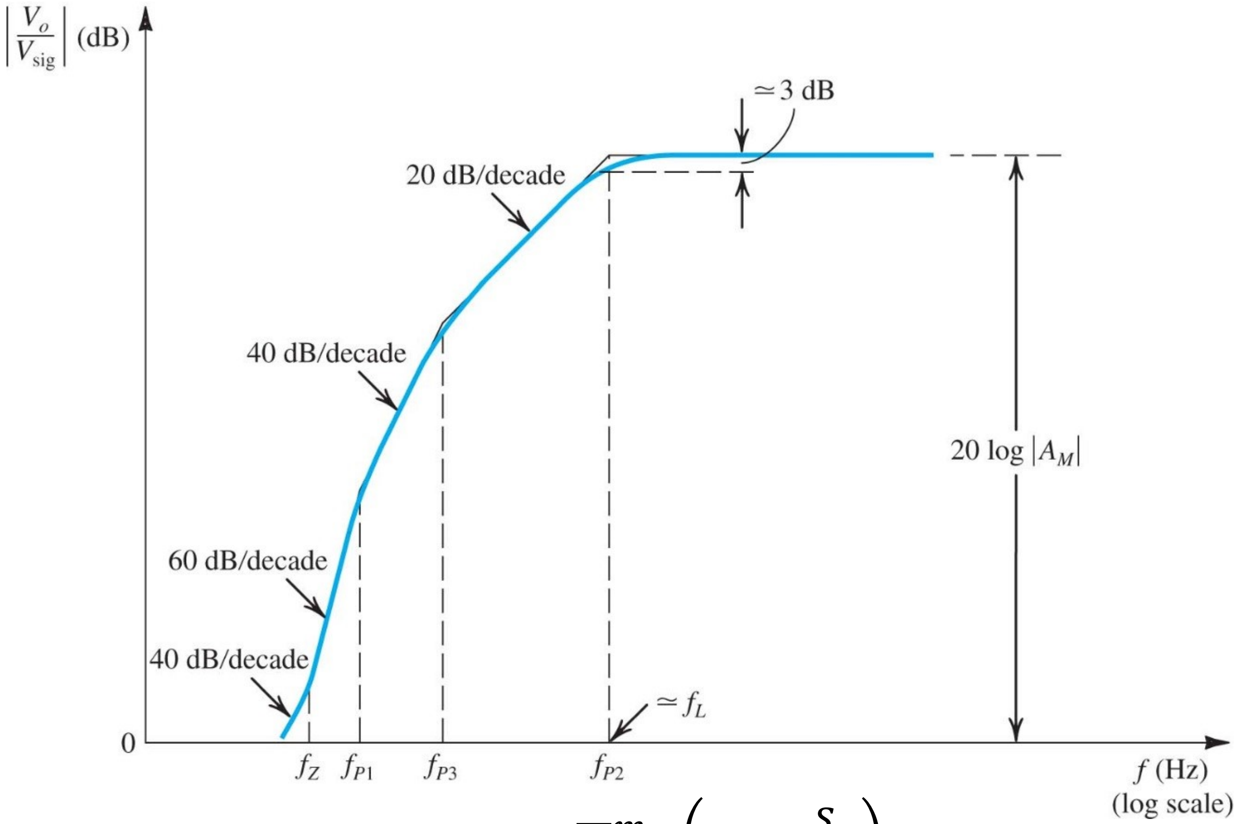


$$A(s) = F_L(s)A_M F_H(s)$$

$$F(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{j=0}^{n} b_j s^j} = a_m \frac{\prod_{i=1}^{m}(s - Z_i)}{\prod_{j=1}^{n}(s - P_j)} = A \frac{\prod_{i=1}^{m}\left(1 + \frac{s}{\omega_{Zi}}\right)}{\prod_{j=1}^{n}\left(1 + \frac{s}{\omega_{Pj}}\right)}$$

Of all the zero and pole frequencies, which is important?





$$F(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{j=0}^{n} b_j s^j} = a_m \frac{\prod_{i=1}^{m}(s - Z_i)}{\prod_{j=1}^{n}(s - P_j)} = A \frac{\prod_{i=1}^{m}\left(1 + \frac{s}{\omega_{Zi}}\right)}{\prod_{j=1}^{n}\left(1 + \frac{s}{\omega_{Pj}}\right)}$$

Dominant Poles

- Estimate low and high corner frequencies of midband from highest and lowest respective pole frequency

- Low corner dominant pole, P_L
  - Defines the "turn" towards midband
  - Pole related to coupling and bypass capacitors, disconnects the input/output at low frequency
  - Higher frequency than all else by some margin…

- High corner dominant pole, P_H
  - Defines the "turn" towards cutoff
  - A pole related to intrinsic input and output capacitors, short circuits the input/output at high frequency
  - Lower frequency than all else by some margin…

$$F_L(s) \approx \frac{\frac{s}{\omega_L}}{1+\frac{s}{\omega_L}} \text{ where } Z_L = 0 \text{ and } P_L = -\omega_L$$

$$\omega_L = \frac{1}{\tau_L} \approx \sqrt{\sum_{j=1}^{n} \omega_{Pj} - 2\sum_{i=1}^{m} \omega_{Zi}} \approx \omega_{P,max}$$

$$F_H(s) \approx \frac{1}{1+\frac{s}{\omega_H}} \text{ where } P_H = -\omega_H$$

$$\omega_H = \frac{1}{\tau_H} \approx 1\Bigg/\sqrt{\sum_{j=1}^{n} \frac{1}{\omega_{Pj}} - 2\sum_{i=1}^{m} \frac{1}{\omega_{Zi}}} \approx \omega_{P,min}$$

Two octaves (x4) of separation is the rule of thumb for the dominant approximation.

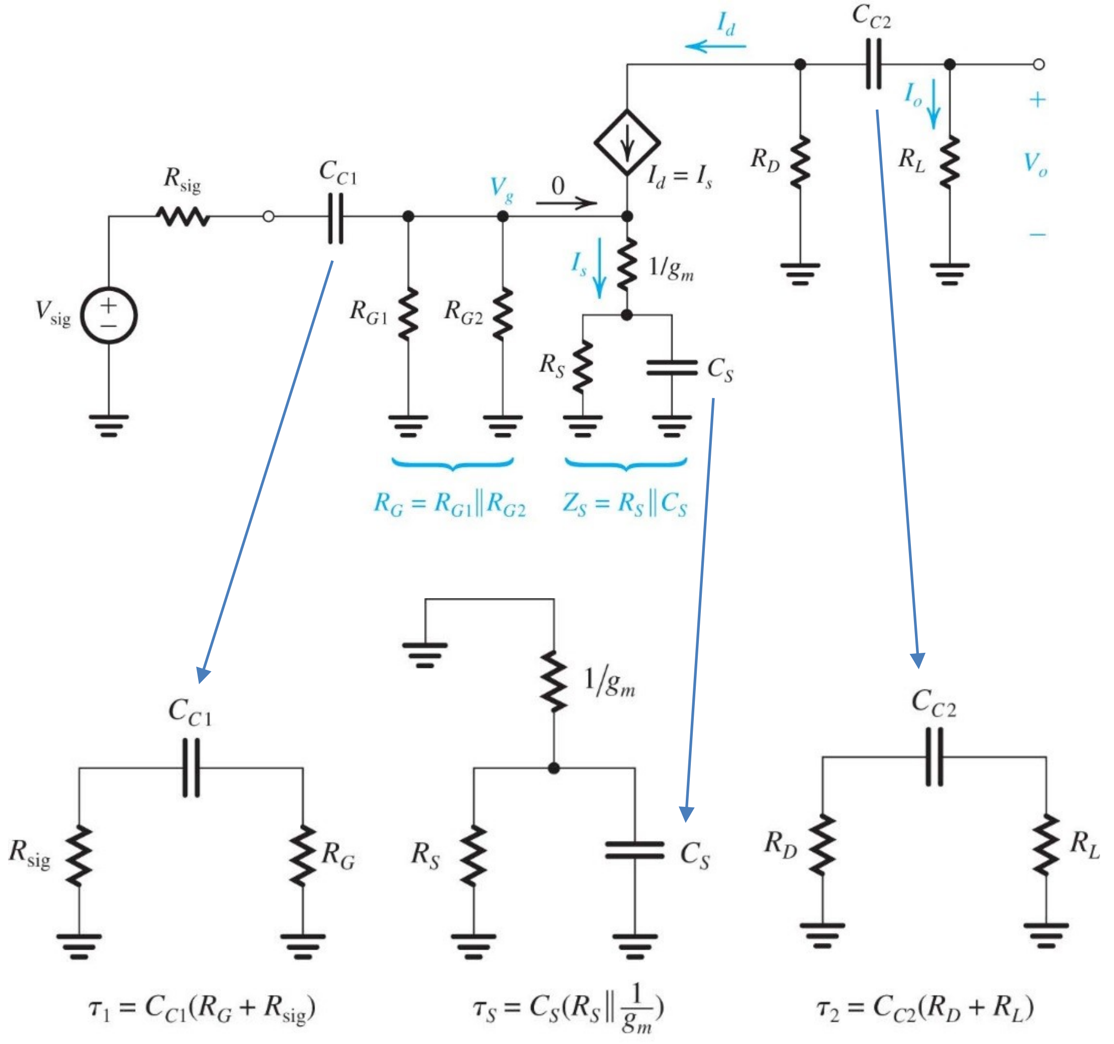
Low Corner: Method of Short Circuit Time Constants (SCTCs)

- Assume that one capacitor generates a dominant pole in the low frequency transfer function
  - Implies other capacitors are short circuited at the low corner frequency

$$F_L(j\omega) \approx \frac{\frac{j\omega}{\omega_L}}{1 + \frac{j\omega}{\omega_L}} = \frac{j\omega\tau_L}{1 + j\omega\tau_L}$$

- Method of SCTCs
  - Turn off external sources
  - For each capacitor, C_i
    - Short circuit all other capacitors
    - Analyse resistance seen by capacitor
    - Calculate individual time constant, τ_i
  - Sum up the inverse time constants

$$f_L = \frac{1}{2\pi\tau_L} \approx \frac{1}{2\pi}\sum_i \frac{1}{\tau_i} = \frac{1}{2\pi}\sum_i \frac{1}{C_i R_i}$$



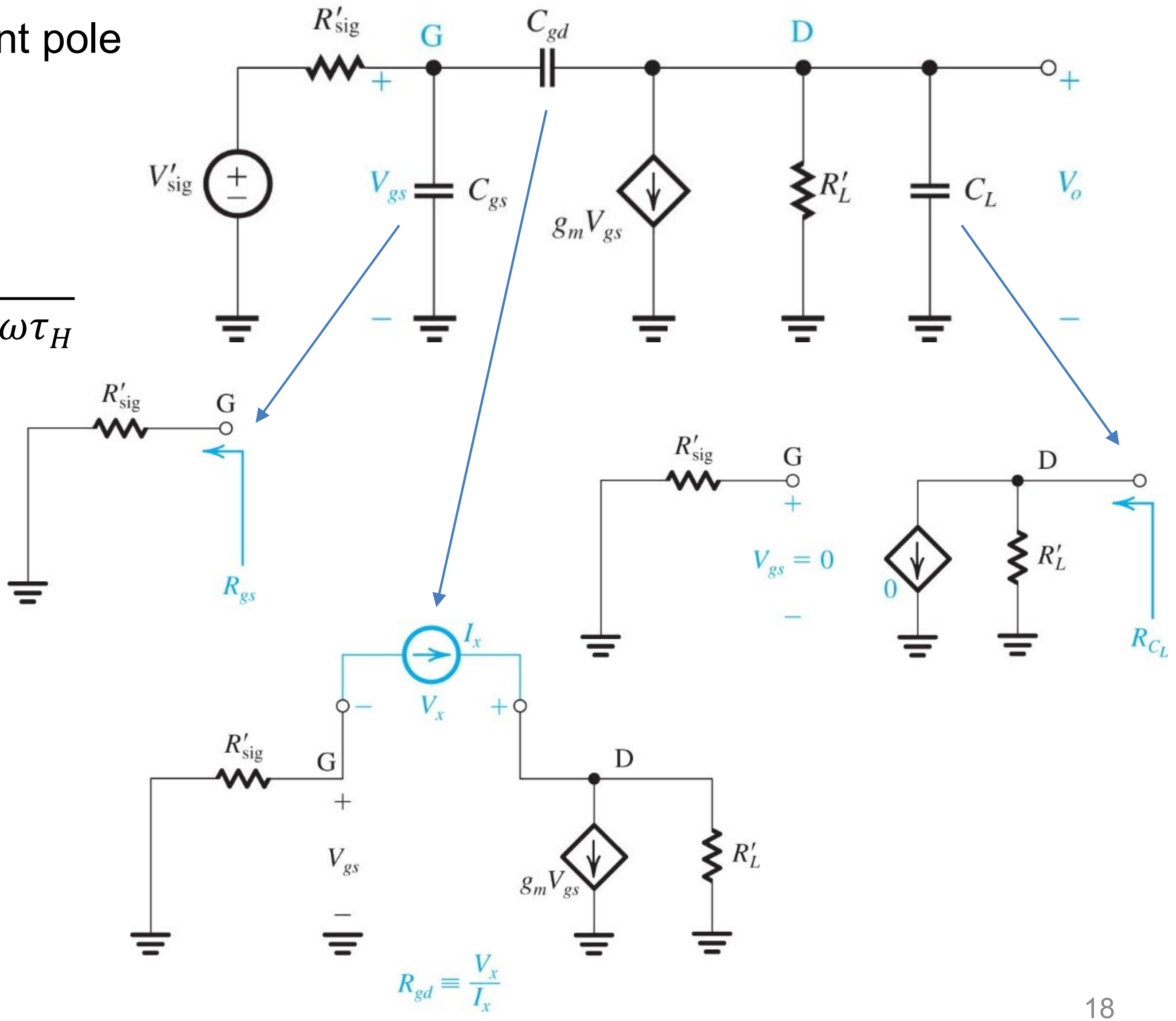
High Corner: Method of Open Circuit Time Constants (OCTCs)

- Assume that one capacitor generates a dominant pole in the high frequency transfer function
  - Implies other capacitors are open circuited at the high corner frequency

$$F_H(j\omega) \approx \frac{1}{1+\frac{j\omega}{\omega_H}} = \frac{1}{1+j\omega\tau_H}$$

- Method of OCTCs
  - Turn off external sources
  - For each capacitor, C_i
    - Open circuit all other capacitors
    - Analyse resistance seen by capacitor
    - Calculate individual time constant, τ_i
  - Sum up the time constants

$$f_H = \frac{1}{2\pi\tau_H} \approx \frac{1}{2\pi} \Big/ \sum_{i=1}^{n} \tau_i = \frac{1}{2\pi} \Big/ \sum_{i=1}^{n} C_i R_i$$



OCTC: CS Amplifier

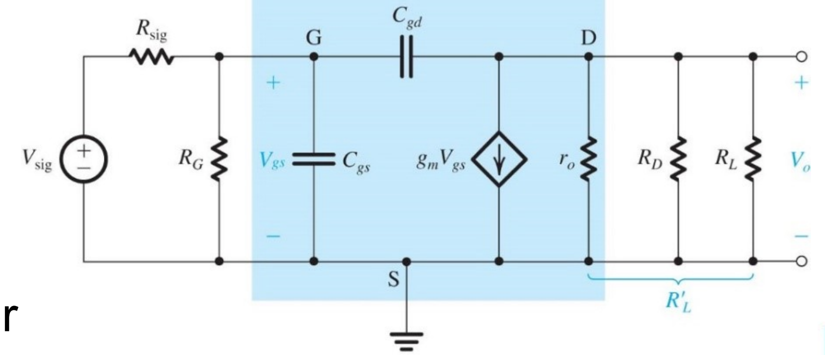
- Method of OCTCs
  - Turn off external sources
  - For each capacitor, C_i
    - Open circuit all other capacitors
    - Analyse resistance seen by capacitor
    - Calculate individual time constant, τ_i
  - Sum up the time constants

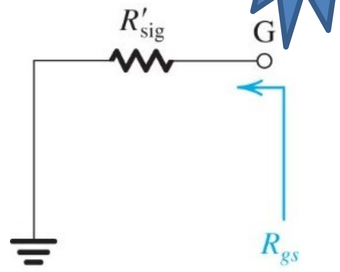
$$\tau_{gs} = C_{gs}R_{gs} = C_{gs}R'_{sig}$$

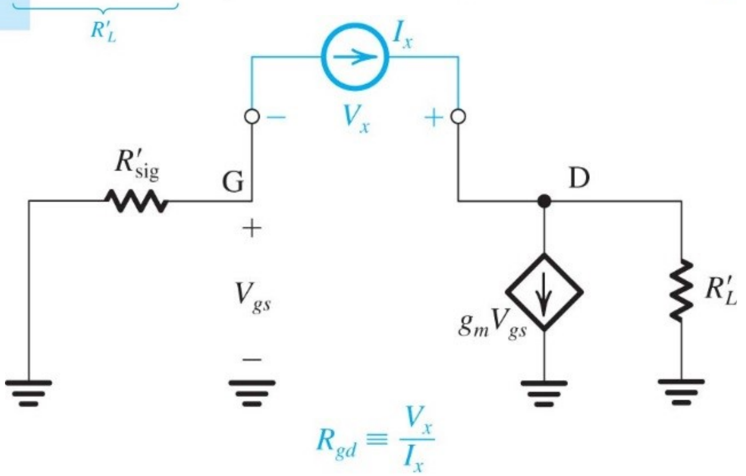
$$\tau_{gd} = C_{gd}R_{gd} = C_{gd}\left([1 + g_m R'_L]R'_{sig} + R'_L\right)$$

$$\tau_L = C_L R_{CL} = C_L R'_L$$

- If signal source resistance high
  - Miller effect dominates corner frequency

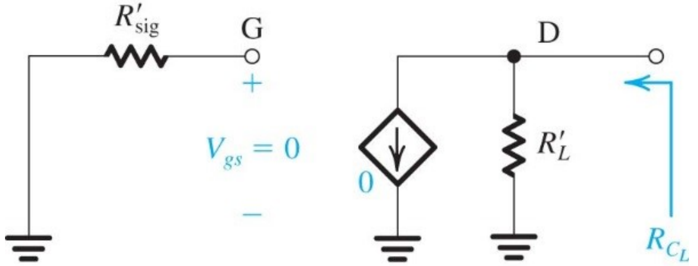






$$\tau_H \approx \tau_{gs} + \tau_{gd} + \tau_L$$

$$f_H = \frac{1}{2\pi\tau_H}$$



OCTC: CG Amplifier

- Method of OCTCs
  - Turn off external sources
  - For each capacitor, C_i
    - Open circuit all other capacitors
    - Analyse resistance seen by capacitor
    - Calculate individual time constant, τ_i
  - Sum up the time constants

$$\tau_{gs} = C_{gs}\left(R_{sig}||R_i\right)$$

$$\tau_{gd} = \left(C_{gd} + C_L\right)(R_L||R_o)$$

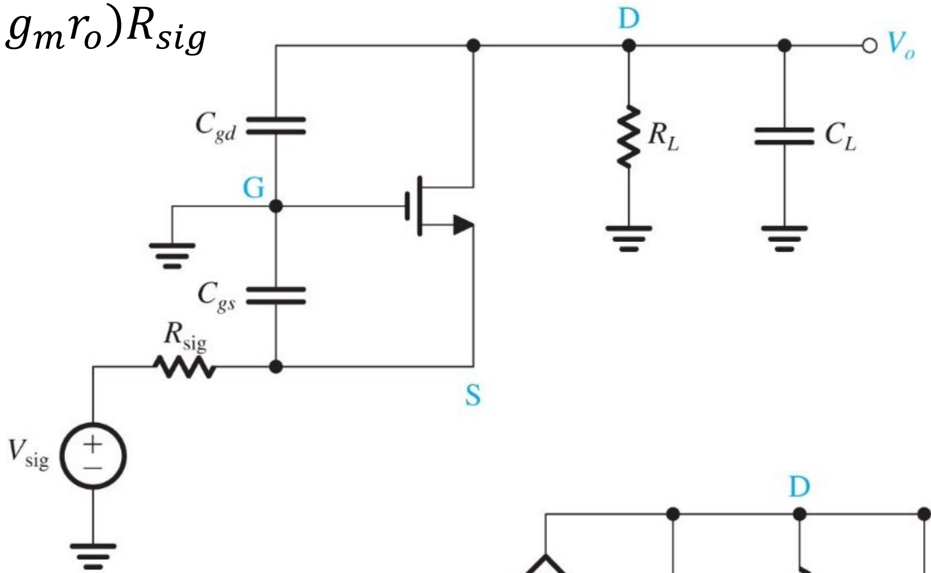
- Capacitances to grounded gate
  - Effectively "moved" to ground
  - Analysis simplified

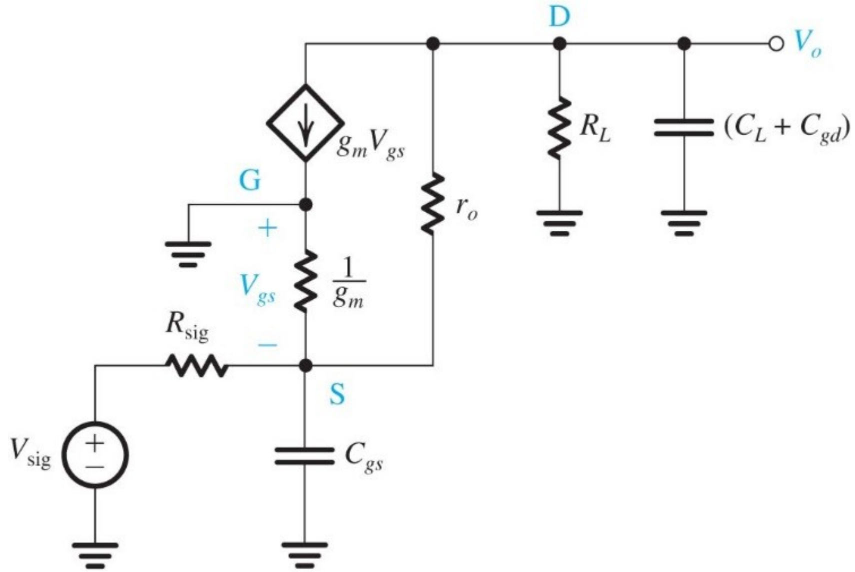
$$R_o = r_o + (1 + g_m r_o)R_{sig}$$

$$R_i = \frac{r_o + R_L}{1 + g_m r_o}$$

$$\tau_H \approx \tau_{gs} + \tau_{gd}$$

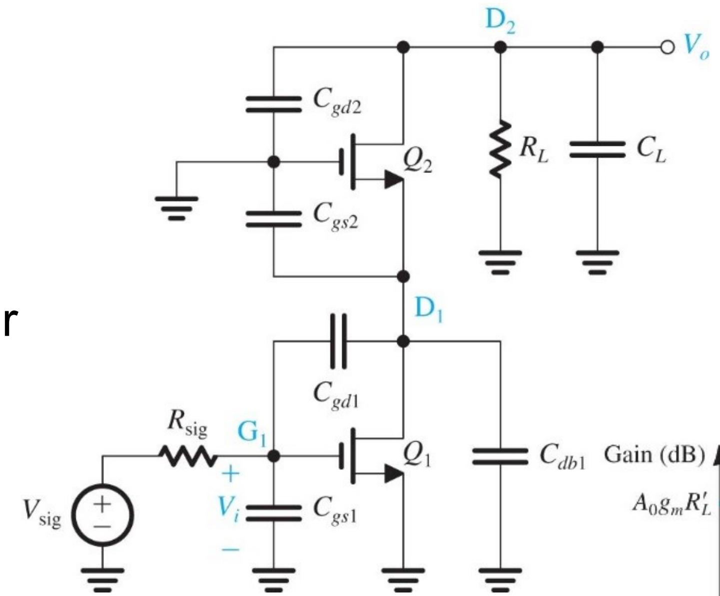
$$f_H = \frac{1}{2\pi\tau_H}$$





OCTC: Cascode Amplifier

- Method of OCTCs
  - Turn off external sources
  - For each capacitor, C_i
    - Open circuit all other capacitors
    - Analyse resistance seen by capacitor
    - Calculate individual time constant, τ_i
  - Sum up the time constants



$$R_o = r_{o2} + (1 + g_{m2}r_{o2})r_{o1}$$

$$R_{in2} = \frac{r_{o2} + R_L}{1 + g_{m2}r_{o2}}$$

$$R_{d1} = r_{o1}||R_{in2}$$

$$\tau_{gs1} = C_{gs1}R_{sig}$$

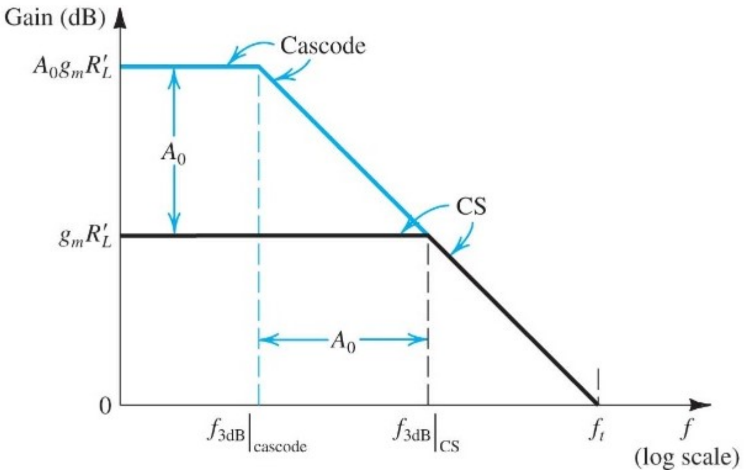
$$\tau_{gd1} = C_{gd1}[(1 + g_{m1}R_{d1})R_{sig} + R_{d1}]$$

$$\tau_{gs2} = (C_{gs2} + C_{db1})R_{d1}$$

$$\tau_{gd2} = (C_{gd2} + C_L)(R_o||R_L)$$

$$\tau_H \approx \tau_{gs1} + \tau_{gd1} + \tau_{gs2} + \tau_{gd2}$$

$$f_H = \frac{1}{2\pi\tau_H}$$



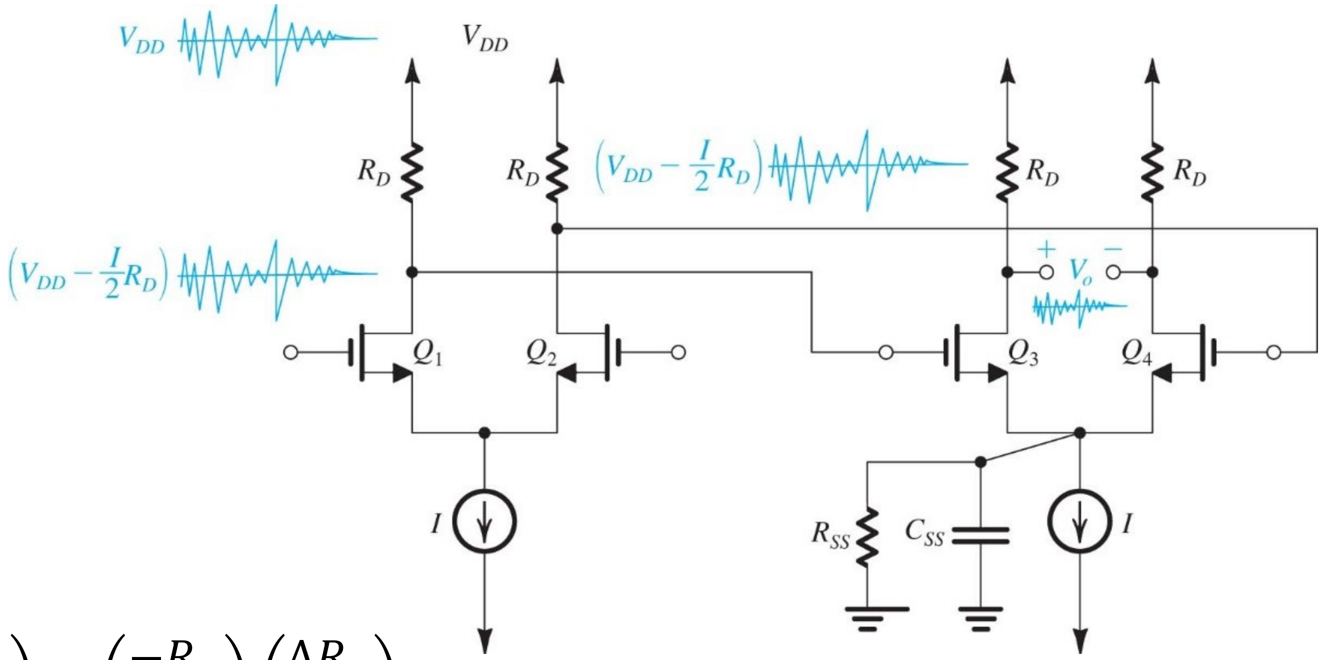
- If signal source resistance negligible
  - Increased dc gain, reduced BW, same cutoff w.r.t. CS

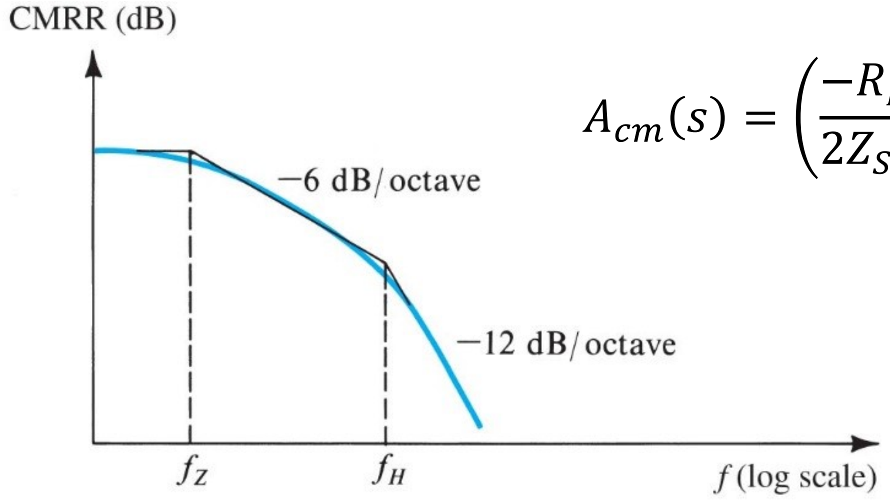
Cascode input, middle node (D1,S2), or output may dominate…

(Differential Amplifier: Common Mode Gain and CMRR)

- Noise injected from suppy in first stage
  - Common mode signal

- Second stage relied on to supress CM noise
  - CMRR proportional to sink resistance
  - Capacitance in parallel to sink resistance





$$A_{cm}(s) = \left(\frac{-R_D}{2Z_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right) = \left(\frac{-R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right)(1 + sC_{SS}R_{SS})$$

$$Z = \frac{-1}{C_{SS}R_{SS}}$$

Already at low frequency (large RC time constant), CMRR degraded by sink bypass.