Exercise 1: Semiconductors / pn-junctions

Carrier concentrations

1. Consider undoped GaAs (N_c=4.7x10^{17} cm^{-3}, N_v=7.0x10^{18} cm^{-3}, E_g=1.424 eV) and Si (N_c=2.8x10^{19} cm^{-3}, N_v=1.04x10^{19}, cm^{-3}, E_g=1.12 eV). At what temperature is the intrinsic carrier density of GaAs equal to that of Si at 200K?

***Ans**: The intrinsic carrier concentration is*

$$n_i = \sqrt{n \cdot p} = \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2kT}\right)$$

which gives n_i(Si, 200K) = 1.33e5 cm^{-3}. Solving for T gives

$$T = -\frac{\frac{E_g}{2k}}{\ln\left(\frac{n_i}{\sqrt{N_c N_v}}\right)}$$

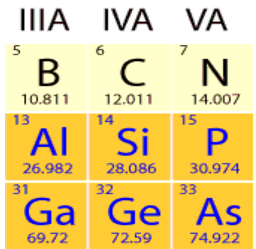
and inserting the result for n_i(Si, 200K) gives T= 273 K.

Due the larger band gap of GaAs compared to Si there are fewer carriers excited to the conduction band (i.e. smaller part of the tail of the Fermi-Dirac distribution is above (below) the conduction (valence) band edge) at a specific temperature. Raising the temperature allows more carriers to be excited.

Transport

2. Find the resistivity at 300 K for a Si sample doped with

Phosphorus (P): 1.0x10^{14} cm^{-3}
Arsenic (As): 8.5x10^{12} cm^{-3}
Boron (B): 1.2x10^{13} cm^{-3}



The electron mobility of Si is μ_n=1500 cm^2/Vs and the hole mobility μ_p=500 cm^2/Vs. Hint: the dopants can be either acceptors or donors depending on the group in the periodic table of elements.

***Ans**: P and As are in group V i.e. they have one extra valence electron compared to Si and therefore act as donors of electrons which gives a total donor concentration of N_D=1x10^{14} + 8.5x10^{12} = 1.085x10^{14} cm^{-3}. B is in group III with one fewer electron in the valence band compared to Si i.e. it's an acceptor giving a density N_A=1.2x10^{13} cm^{-3}. The donors and acceptors compensate (cancel out) each other giving an electron concentration of $n=N_D-N_A$=9.65x10^{13} cm^{-3}. According the mass action law $n_i^2 = n*p$ where the intrinsic carrier concentration is given by*

$$n_i = \sqrt{n \cdot p} = \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2kT}\right) = 6.67\text{e}10^9 \text{ cm-3}$$

which gives a hole concentration

$$p = \frac{n_i^2}{n} = \frac{(6.67 \cdot 10^9)^2}{9.65 \cdot 10^{13}} = 4.61 \cdot 10^5 \ cm^{-3}$$

Both electrons and hole contribute to the conductivity

$$\sigma = q(\mu_n n + \mu_p p) = 1.6022 \cdot 10^{19}(1500 \cdot 9.65 \cdot 10^{13} + 500 \cdot 4.61 \cdot 10^5) = 0.0232 \ (\Omega \cdot cm)^{-1}$$

Giving a resistivity of

$$\rho = \frac{1}{\sigma} = 43.1 \ \ \Omega \cdot cm$$

As can be seen, the hole concentration is so small (and the mobility is lower as well) so it can safely be ignored and the resistivity is set by the electron transport.

3. Calculate the electron and hole drift velocities through a 10-µm thick layer of intrinsic silicon across which a voltage of 1V is applied. Let μ_n = 1350 cm^2/Vs and μ_p = 480 cm^2/Vs.

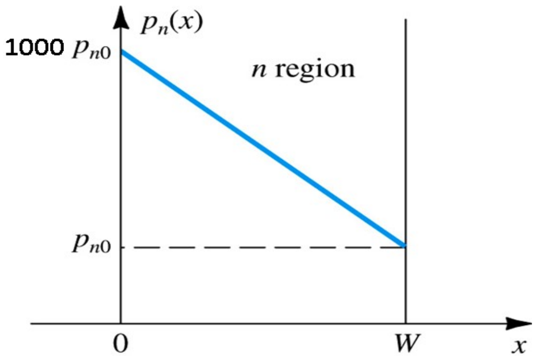
Ans: The electric field across the layer is $\varepsilon = V / L = 1\ (V) / 10^{-5}\ (m) = 10^5\ V/m$.

The electron velocity is given by $v_n = \varepsilon \cdot \mu_n = 10^5\ (V/m) \cdot 1350 \cdot 10^{-4}\ (m^2/Vs) = 13500\ m/s$

The hole velocity is given by $v_p = \varepsilon \cdot \mu_p = 10^5\ (V/m) \cdot 480 \cdot 10^{-4}\ (m^2/Vs) = 4800\ m/s$

Note that the electric field is lower than the critical field for the onset of velocity saturation (see lecture slides). If the voltage would be increased >100V the carrier velocity would saturate and become independent of bias.

4. Holes (minority carriers) are steadily injected into a piece of n-doped Si (N_D=10^{16} cm^{-3}) at x=0 and extracted at x=W resulting in the hole concentration shown in the image below, where p_{n0} is the hole carrier concentration at equilibrium (no injection i.e. no bias). Use an intrinsic carrier concentration of n_i=1.5*10^{10} cm^{-3} and W=5 µm. The hole mobility is μ_p = 480 cm^2/Vs. Calculate the current density that flows in the x-direction.



Ans: *The electron concentration is determined by the doping i.e. $n \approx N_D$.*
The equilibrium (without injection) hole (minority carriers) concentration is then given by the mass action law: $p_{no} = n_i^2/N_D = (1.5 \cdot 10^{10})^2 / 10^{16} = 22500\ cm^{-3}$.
The current density (due to diffusion of holes from 0 to W) is determined by the concentration gradient as

$$J_p = qD_p \frac{dp(x)}{dx} = q\left(\frac{kT}{q}\mu_p\right)\frac{dp(x)}{dx} = kT\mu_p \frac{p_{n0} - 1000 \cdot p_{n0}}{0 - W} =$$
$$= \frac{1.38 \cdot 10^{-23}[J/K] \cdot 300\,[K] \cdot 480\,[cm^2/Vs] \cdot (1-1000) \cdot 22500\,[cm^{-3}]}{0 - 5 \cdot 10^{-4}\,[cm]} = 8.93 \cdot 10^{-8}\ A/cm^2$$

The situation in the problem resembles that of the n-side of a forward biased pn-junction where minority carriers are steadily injected at the depletion region edge (x=0) and extracted at the contact (x=W) or the base of a bipolar junction transistor. Since the minority carrier concentration decreases linearly there is no recombination in the region and the current is constant at each point.

pn-junctions / diodes

5. (1.84 in Sedra/Smith 7e) Calculate the built-in voltage of a junction in which the p and n regions are doped equally with 5e16 cm^{-3}. Assume n_i=1.5e10 cm^{-3} (Si at room temp). With the terminals left open (no bias applied), what is the width of the depletion region, and how far does it extend into the p and n regions? If the cross sectional area of the junction is 20 μm^{-2}, find the magnitude of the charge stored on either side of the junction.

1.84 Using Eq. (1.46) and $N_A = N_D$
$= 5 \times 10^{16}\ \text{cm}^{-3}$ and $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$,

we have $V_0 = 778$ mV.

Using Eq. (1.50) and
$\epsilon_s = 11.7 \times 8.854 \times 10^{-14}$ F/cm, we have
$W = 2 \times 10^{-5}$ cm $= 0.2$ μm. The extension of the depletion width into the n and p regions is given in Eqs. (1.51) and (1.52), respectively:

$$x_n = W \cdot \frac{N_A}{N_A + N_D} = 0.1\ \mu\text{m}$$

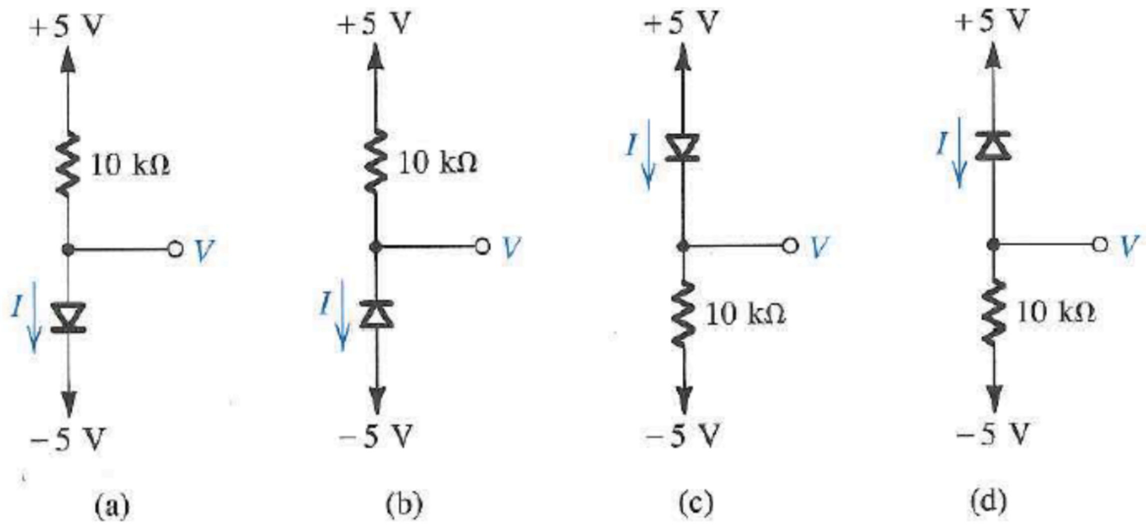
$$x_p = W \cdot \frac{N_D}{N_A + N_D} = 0.1\ \mu\text{m}$$

Since both regions are doped equally, the depletion region is symmetric.

Using Eq. (1.53) and
$A = 20\ \mu\text{m}^2 = 20 \times 10^{-8}\ \text{cm}^2$, the charge magnitude on each side of the junction is

$Q_J = 1.6 \times 10^{-14}$ C.

6. For the circuits shown below using ideal diodes, find the values of the voltages and currents indicated. Hint: 5 V is much higher than the junction voltage drop (see 3.3.5).



Ans

a) *The diode is forward biased i.e. its resistance is negligible and it can be treated as a short between the middle and lower terminals i.e. V = -5 V. The current is I=U/R= 5-(-5) [V] / 10 [kOhm] = 1 mA*
b) *The diode is reverse biases i.e. its resistance is very high and it can be treated as an open circuit between the middle and lower terminals i.e. V = +5 V. The current I = 0 A.*
c) *Diode is forward biased. V=5 V and I = 1 mA.*
d) *Diode is reverse biased V=-5 V and I = 0 A.*

7. a) At what forward voltage does a diode for which n=2 conduct a current equal to $1000I_s$ (I_s= saturation current at reverse bias)? b) Expressed in terms of I_s, what current flows in the same diode when its forward voltage is 0.7V?

Ans:

a) Use the ideal diode equation

$$I = I_S(e^{qV/nkT} - 1)$$

Set I=$1000I_S$ and solve for V to get

$$V = \ln\left(\frac{1000\,I_S}{I_S} + 1\right) \cdot \frac{2kT}{q} = 0.355\ V$$

b) The ideal diode equation gives

$$I = I_S(e^{qV/nkT} - 1) = (e^{q0.7/nkT} - 1) = 749 \cdot 10^3\ I_s$$

8. A diode for which the forward voltage drop is 0.7 V at 1.0 mA and for which n=1 is operated at 0.5V. What is the value of the current?

Ans:

Use the ideal diode equation

$$I = I_S(e^{qV/nkT} - 1)$$

and solve for I_S to get

$$I_S = \frac{I}{(e^{qV/nkT} - 1)} = \frac{10^{-3}}{(e^{1.6022 \cdot 10^{-19} \cdot 0.7/1.38 \cdot 10^{-23} \cdot 300} - 1)} = 1.78 \cdot 10^{-12} \ mA$$

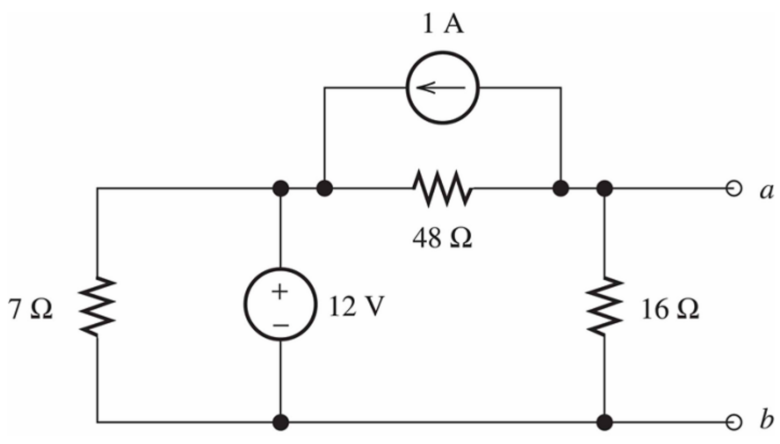
Inserted in the diode equation this gives

$$I = I_S(e^{qV/nkT} - 1) = 1.78 \cdot 10^{-15}(e^{1.6022 \cdot 10^{-19} \cdot 0.5/1.38 \cdot 10^{-23} \cdot 300} - 1) = 4.40 \cdot 10^{-4} \ mA$$

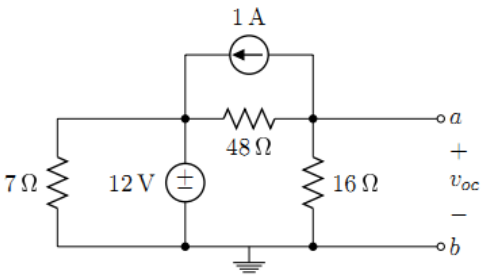
Note the very large difference in current of four orders of magnitude in the forward direction for only a change of 0.2 V.

Equivalent circuits

9. Find the Thevenin and Norton equivalent circuits for the circuit shown below. Take care that you orient the polarity of the voltage source and the direction of the current source correctly relative to the terminals a and b.



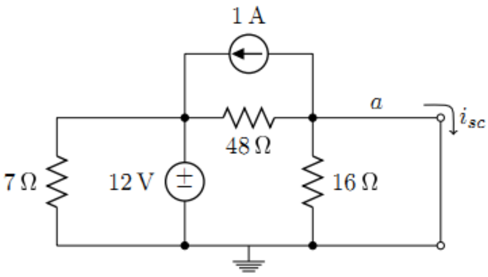
Solution



1 Open Circuit Voltage
KCL @ a with b ground $\Rightarrow v_{oc} = v_a$

$$\frac{v_a}{16} + \frac{v_a - 12}{48} + 1 = 0$$
$$v_a\left(\frac{1}{16} + \frac{1}{48}\right) = \frac{12}{48} - 1$$

$$v_a = \left(\frac{48}{4}\right)\left[\frac{12}{48} - 1\right]$$
$$= -9\,\mathrm{V}$$



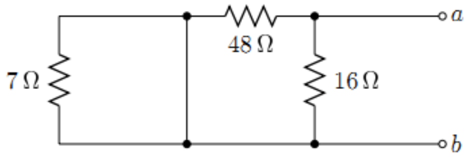
2 Short Circuit Current
KCL @ a

$$i_{sc} + \frac{-12}{48} + 1 = 0$$
$$i_{sc} = -0.75\,\mathrm{A}$$

3 Equivalent Resistance

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{-9}{-0.75} = 12\,\Omega$$

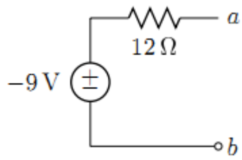
Alternatively, becuase there are only dependent sources in the circuit, this can be solved by zeroing the dependent sources and finding the equivalent resistance seen from the terminals. Zeroing the sources means voltage sources are short circuits and current sources are open circuits.



In this case,

$$R_t = 48 \parallel 16 = 12\,\Omega$$

The Thevenin circuit is shown below.



The Norton circuit is shown below.