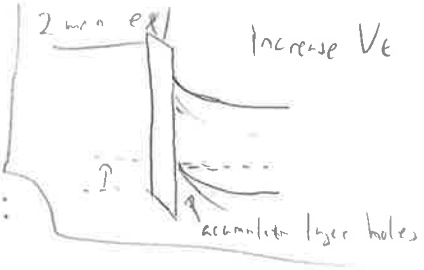
L4 MOSFET/

$I_D(V_{DS})$ for small V_{DS} (page 48-43)



.. At position y along channel:

$$I_D = W \cdot Q_I(y) \cdot v_d(y) = W \cdot Q_I(y) \cdot \mu_n \cdot \mathcal{E}(y) =$$

width charge/area velocity electron "surface" mobility ($< \mu_{bulk}$) electric field in y-direction

capacitance/area

$$= \left\{ \begin{array}{l} Q_I(y) = C_{ox}\left(V_{GS} - V(y) - V_t\right) \quad \left(\text{assume } V_{GS} - V(y) > V_t\right) \\ \\ \mathcal{E}(y) = \frac{dV}{dy} \end{array} \right\} =$$

voltage at pos. y w.r.t. source

need to overcome V_t to get any inversion charge

$$= W C_{ox}\left(V_{GS} - V(y) - V_t\right) \cdot \mu_n \cdot \frac{dV}{dy}$$

integrate

$$\int_0^L I_D \, dy = \int_0^{V_{DS}} W \mu_n \cdot C_{ox} \left(V_{GS} - V(y) - V_t\right) dV$$

$$I_D \cdot L = W \cdot \mu_n \cdot C_{ox} \cdot \left[V_{GS} \cdot V - \frac{V^2}{2} - V_t \cdot V \right]_0^{V_{DS}}$$

$$I_D = \frac{W}{L} \cdot \frac{\overbrace{\mu_n \cdot C_{ox}}^{k'}}{2} \cdot \left(2\left(V_{GS} - V_t\right) \cdot V_{DS} - V_{DS}^2\right) \qquad (*)$$

I_d (high V_{DS}) pinch-off

(*) not for $V_{DS} > V_{GS} - V_t$

Channel voltage at pinch-off point is $V_{GS} - V_t$

Integrate r.h.s. $0 \to V_{GS} - V_t$ instead

$$I_D = \frac{W}{L} \frac{\mu_n \cdot C_{ox}}{2} \left(2 (V_{GS} - V_t)(V_{GS} - V_t) - (V_{GS} - V_t)^2 \right) =$$

$$= \frac{W}{L} \frac{\mu_n C_{ox}}{2} (V_{GS} - V_t)^2 \quad (**)$$ independent of V_{DS}!

I_D with channel length modulation

Replace L with $L_{eff} = L - X_d$, X_{eff} depends on V_{DS}

$$I_D = \frac{W}{L_{eff}(V_{DS})} \cdot \frac{\mu_n C_{ox}}{2} (V_{GS} - V_t)^2$$

(inner derivative)

$$\frac{\partial I_D}{\partial V_{DS}} = - \frac{W}{L_{eff}^2} \frac{\mu_n \cdot C_{ox}}{2} (V_{GS} - V_t)^2 \cdot \frac{d L_{eff}}{d V_{DS}} =$$

$$= - \frac{I_D}{L_{eff}} \cdot \frac{d(L - X_d)}{d V_{DS}} = \frac{I_D}{L_{eff}} \cdot \frac{d X_d}{d V_{DS}}$$

define

$$\lambda = \frac{\frac{\partial I_D}{\partial V_{DS}}}{I_D} = \frac{1}{L_{eff}} \cdot \frac{d X_d}{d V_{DS}}$$

how large part of I_D change with increasing V_{DS}

modify (**) $$I_D = \frac{W}{L} \cdot \frac{\mu_n C_{ox}}{2} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$ → output resistance $r_o = \frac{dI_D}{dV_{DS}} = \frac{1}{\lambda I_D}$

additional contribution

Example: MOSFET with channel length modulation

~~Linear region~~ $$I_D = \overbrace{\frac{0{,}135\cdot 3\cdot 10^{-3}}{2}\cdot\frac{50\cdot 10^{-6}}{5\cdot 10^{-6}}}^{0{,}0020}\cancel{\left(2\left(V_{GS}-V_t\right)V_{DS}-V_{DS}^2\right)}$$

$$= 0{,}0020\cdot\cancel{\left(2\left(V_{GS}-V_t\right)V_{DS}-V_{DS}\right)} =$$

1) $V_{GS} < V_t \rightarrow I_D = 0$ for all V_{DS}

2) $V_{GS} \geq V_t$ ⓞk

$V_{GS} = 2{,}5\,V$

$\underline{V_{DS} = 0{,}5\,V} \Rightarrow 0{,}5 < \overbrace{V_{GS}-V_t}^{2{,}5-1} \rightarrow$ linear region

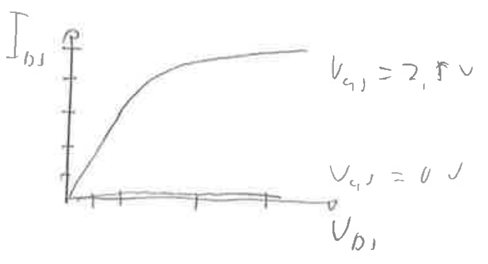
$I_D = 0{,}002\cdot\left(2\cdot(2{,}5-1)\,0{,}5-0{,}5^2\right) = 2{,}5\,mA$

$\underline{V_{DS} = 1\,V}$. $1 < 2{,}5-1 \rightarrow$ linear $I_D = 4\,mA$

$\underline{V_{DS} = 2\,V}$ $2 > 2{,}5-1 \rightarrow$ saturation

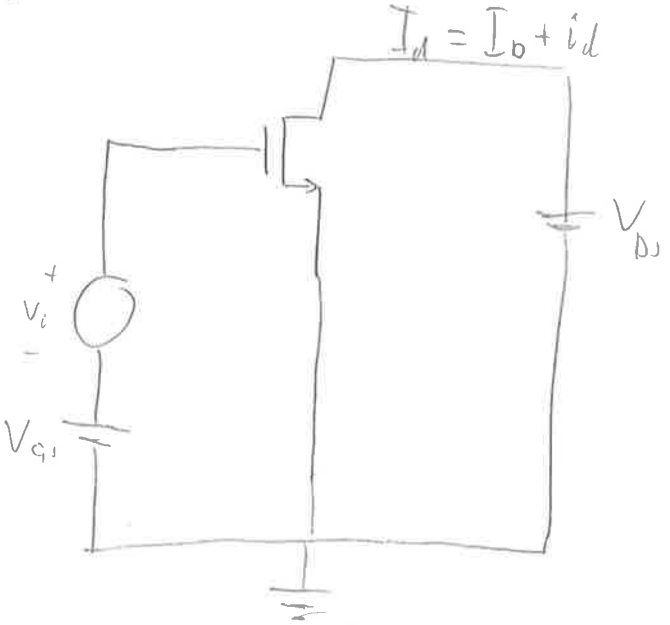
$I_D = 0{,}002\,(2{,}5-1)^2(1+0{,}02\cdot 2) = 4{,}7\,mA$

$\underline{V_{DS} = 3V}$ $3 > 2{,}5-1 \rightarrow$ saturation $I_D = 4{,}8\,mA$



③

Small signal model - transconductance



Transconductance

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{\partial}{\partial V_{GS}} \left(\frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \left(V_{GS} - V_t\right)^2 \left(1 + \lambda V_{DS}\right) \right) =$$

$$= \mu_n C_{ox} \cdot \frac{W}{L} \left(V_{GS} - V_t\right)\left(1 + \lambda V_{DS}\right) = \left\{ \text{small } \lambda \right\} = \mu_n C_{ox} \frac{W}{L}\left(V_{GS} - V_t\right)$$

$$= \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \Rightarrow g_m \propto \sqrt{I_D} \quad \left(\begin{matrix} \text{for} \\ \text{BJT} \end{matrix} \quad g_m \propto I_D \right)$$

To get high g_m $\left\{ \begin{array}{l} \text{thin oxide} \\ \text{short gate length} \\ \text{large width} \end{array} \right.$ increase μ_n (III-V, strain)
increase ε_{ox} (high-k)

Input resistance

∞ due to oxide $\to$ $I_G = 0$ for low frequency

output resistance

$$\frac{\partial I_D}{\partial V_{DS}} = \frac{1}{\lambda I_D} = r_o$$