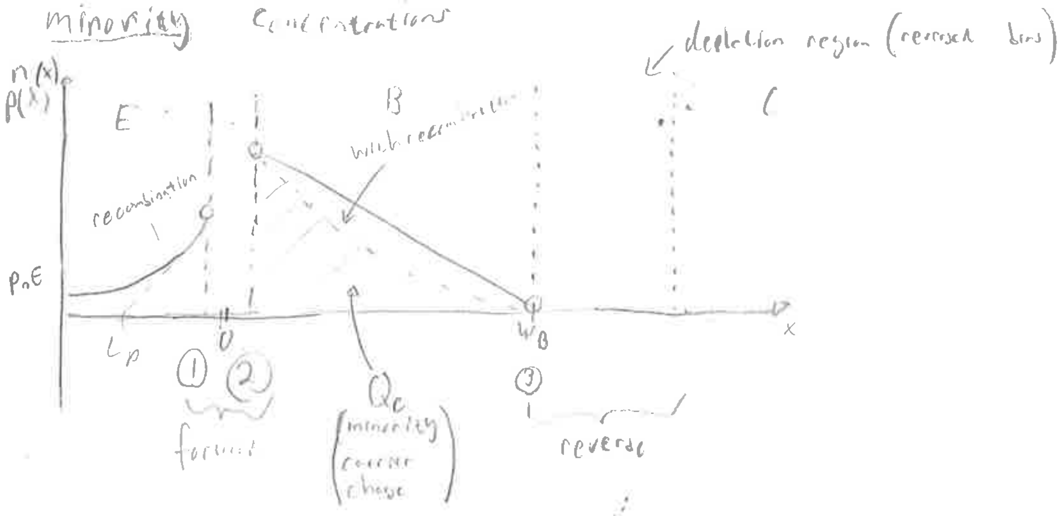
F3 Bipolar junction transistors

Calculate I_c and I_B of NPN in active mode

$\Big\{ N_D(\text{emitter}) > N_A(\text{base})$

minority concentrations



Concentration at

$$\begin{cases} ① & p_{nE}(0) = p_{nE}\, e^{qV_{BE}/kT} \\ ② & n_p(0) = n_{p0}\, e^{qV_{BE}/kT} \\ ③ & n_p(W_B) = n_{p0} \cdot e^{qV_{BC}/kT} \approx 0 \end{cases}$$

negative reversed biased

Since $N_D(\text{emitter}) > N_A(\text{base})$

$p_{nE}(\text{emitter}) < n_{p0}(\text{base})$

$\left(n = \frac{n_i^2}{N_A} \;/\; p = \frac{n_i^2}{N_D} \right)$

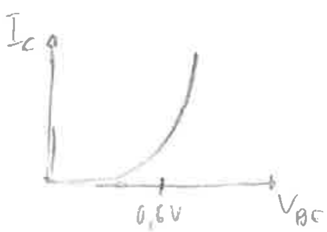
Collector current

Linear concentration gradient → Constant diffusion current (reminder from F1)

$$I_c = A \cdot q \cdot D_n \cdot \frac{dn}{dx} = A q D_n \frac{n_p(0) - n_p(W_B)}{W_B} = \frac{A q D_n n_{p0} e^{qV_{BE}/kT}}{W_B} = \left\{ n_{p0} = \frac{n_i^2}{N_A} \right\}$$

$$= \frac{A q D_n n_i^2}{W_B N_A} e^{qV_{BE}/kT} = I_s e^{qV_{BE}/kT}$$

(explain i_C)



Base current (two parts)

1) Back injection into emitter (forward biased pn-junction)

$$I_{B_2} = \frac{q A D_p \cdot n_i^2}{L_p N_D} e^{qV_{BE}/kT}$$

$\Big\{ D_p$ = diffusion constant, L_p = diffusion length $\Big\}$

2) Recombination in base (need refill)

minority carrier charge in base

$$I_{B_1} = \frac{Q_e}{\tau_b} = \frac{W_B \cdot n_p(0)}{2 \tau_b} \cdot q A = \frac{W_B \cdot n_{p0} e^{qV_{BE}/kT}}{2 \tau_b} \cdot q A =$$

minority carrier lifetime

$$= \frac{1}{2} \frac{n_i^2 W_B q A}{\tau_B \cdot N_A} e^{qV_{BE}/kT}$$

$I_B = I_{B1} + I_{B2}$ ①

Next ppt-slide

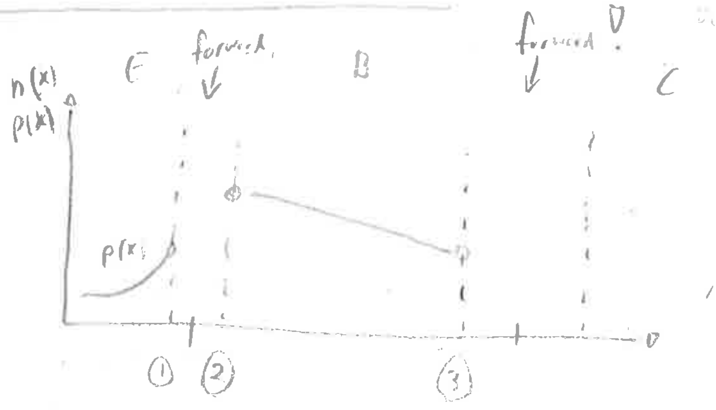
Want large gain $\beta_F = \frac{I_C}{I_B} = \frac{\cancel{A}\cancel{q} D_n n_i^2}{W_B N_A} \Big/ \left(\frac{n_i^2 W_B \cancel{q} \cancel{A}}{2 \tau_B N_A} + \frac{\cancel{q}\cancel{A} D_p n_i^2}{L_p N_D}\right)$

$$= \frac{1}{\underbrace{\frac{W_B^2}{2\tau_b D_n}}_{\text{recombination}} + \underbrace{\frac{D_p}{D_n}\cdot\frac{W_B}{L_p}\cdot\frac{N_A}{N_D}}_{\text{backinjection}}}$$

(limits β_F)

- Make W_B small → no recombination
- $N_A << N_D$ i.e. { lightly doped base / highly doped emitter }
- * NPN is better than PNP since $D_n > D_p$

2 min exercise − ~~Saturation~~



①② } Same V_{BE} → as before. $V_{BC} > 0$ i.e. forward biased gives

$$n_p(W_B) = n_{p0}\underbrace{e^{qV_{BC}/kT}}_{>0}$$

Since $I_C \propto \frac{dn}{dx}$ I_C decreases → β_F decreases

Example Si NPN transistor

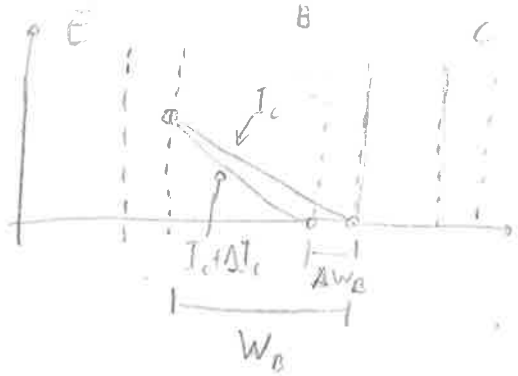
$$\beta_F = \frac{1}{\frac{(0,5\cdot10^{-4})^2}{2\cdot1\cdot10^{-6}\cdot15} + \frac{1}{15}\cdot\frac{0,5}{0,5}\cdot\frac{2\cdot10^{17}}{1\cdot10^{19}}} = \frac{1}{8,3\cdot10^{-3}+1,33\cdot10^{-3}} = 105$$

(from F1 $n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$)

$$I_S = \frac{A q D_n n_i^2}{W_B N_A} = \frac{100\cdot10^{-8}\cdot1,602\cdot10^{-19}\cdot15\cdot(10^{10})^2}{5\cdot10^{-4}\cdot2\cdot10^{17}} = 4,8\cdot10^{-18}\,A$$

Early effect (active mode)

Increasing V_{CE} $\rightarrow$



depletion width $\propto \sqrt{\psi_0 - V_{BC}}$

neg. in active

$\rightarrow$ I_c does not saturate with V_{CE}

from before

$\boxed{\text{KANSKE INTE HINNER V!}}$

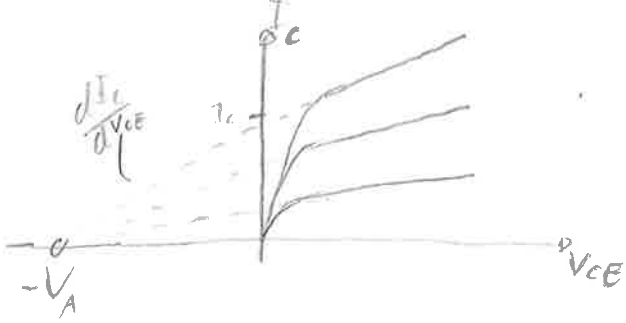
$$I_c = \frac{A \cdot q \cdot D_n \cdot n_i^2}{N_A} e^{qV_{BE}/kT} \cdot \frac{1}{W_B(V_{CE})}$$

differentiate

(inner derivative $\frac{1}{f(x)} \rightarrow -\frac{1}{f(x)^2} \cdot f'(x)$)

$$\frac{\partial I_c}{\partial V_{CE}} = -\frac{A \cdot q \cdot D_n n_i^2}{N_A} e^{qV_{BE}/kT} \cdot \frac{1}{W_B^2} \cdot \frac{dW_B}{dV_{CE}} = -\frac{I_c}{W_B} \cdot \underbrace{\frac{dW_B}{dV_{CE}}}_{\text{negative}} \quad (*)$$

- thin base (W_B) $\rightarrow$ large Early effect.
- Effect increases with I_c



Early voltage (without Early effect)

$$V_A = \frac{I_c}{\frac{\partial I_c}{\partial V_{CE}}}$$

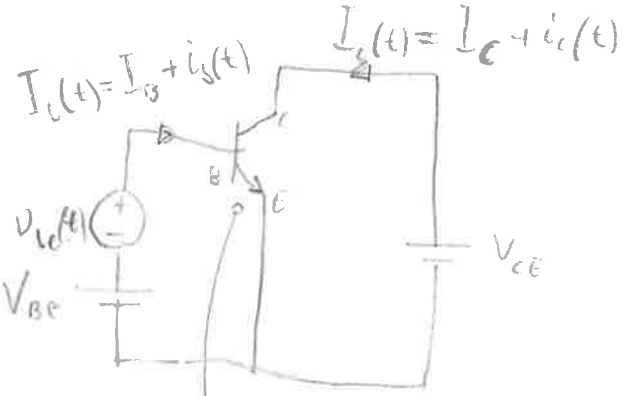
(Large base doping N_A $\rightarrow$ larger V_A)

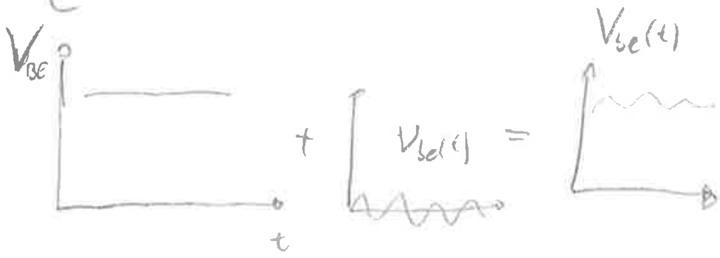
Insert $(*)$ $\quad V_A = -W_B \frac{dV_{CE}}{dW_B}$

Modify I_c expression $\quad I_c = I_s \left(1 + \frac{V_{CE}}{V_A}\right) e^{qV_{BE}/kT}$

Small - signal model

$$\begin{cases} V_{BE} : DC \\ v_{be}(t) : AC \\ V_{be}(t) : DC + AC \end{cases}$$





$$V_{be}(t) = V_{BE} + v_{be}(t)$$

collector current (function of V_{be})

(identify)
$x_0 + \delta x$

$$I_c(V_{be}(t)) = I_c(V_{BE} + v_{be}(t)) = I_s e^{q(V_{BE} + v_{be}(t))/kT} \overset{\text{Taylor}}{\approx}$$

$$\approx \underbrace{I_s e^{qV_{BE}/kT}}_{I_c(V_{BE})} + \underbrace{\frac{\partial I_c}{\partial V_{BE}}} \cdot v_{be}(t) = I_c(V_{BE}) + g_m \cdot v_{be}(t)$$

$$\frac{q}{kT} \underbrace{I_s e^{qV_{BE}/kT}}_{I_c} = \boxed{g_m = \frac{q}{kT} \cdot I_c}$$

Transconductance is proportional to collector current!

base current

$$\left[\beta_F = \frac{I_c}{I_B} \Rightarrow I_B = \beta_F \cdot I_c\right]$$

$$I_b(V_{be}(t)) =$$

$$\underset{x_0}{I_b(V_{BE}} + \underset{\delta x}{v_{be}(t))} \overset{\text{Taylor}}{\approx} \underset{f(x_0)}{I_B} + \underbrace{\frac{\partial I_B}{\partial V_{BE}}}_{1/r_\pi} \cdot v_{be}(t) = \begin{cases} I_B + \underbrace{\frac{\partial I_c}{\partial V_{BE}}}_{g_m} \cdot \beta_F = \\ I_B + \frac{1}{r_\pi} \cdot v_{be}(t) \end{cases}$$

input resistance
(since there is a base current when changing V_{BE})

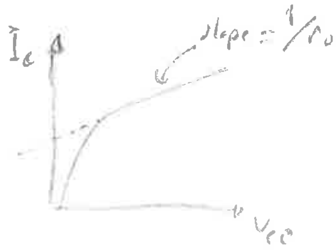
Collector Current (function of V_{CE})

result from Early effect

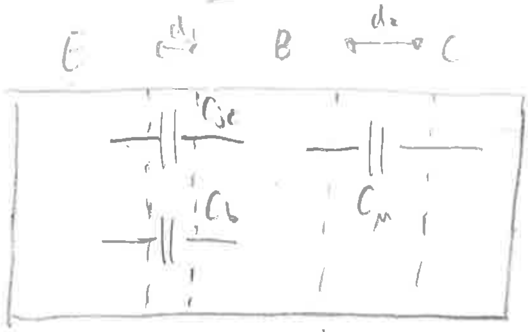
$$I_c\left(V_{CE} + v_{ce}(t)\right) \overset{\text{Taylor}}{=} I_c(V_{CE}) + \frac{\partial I_c}{\partial V_{CE}} \cdot v_{ce}(t) =$$

$$= I_c(V_{CE}) + \underbrace{\frac{I_c}{V_A}}_{1/r_o} \cdot v_{ce}(t)$$

$1/r_o$ ← output resistance



Capacitances (important at high freq. / open circuit at low freq.) $\boxed{\text{On OH}}$

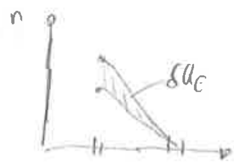


Three capacitances

depletion layer capacitance

$$C_\mu \approx \frac{A \varepsilon_r \varepsilon_0}{d_2} \qquad d_2 \propto \frac{1}{\sqrt{\Psi_0 - V_{BC}}}$$

$$C_{je} \approx \frac{A \varepsilon_r \varepsilon_0}{d_1} \qquad d_1 \propto \frac{1}{\sqrt{\Psi_0 + V_{BE}}}$$



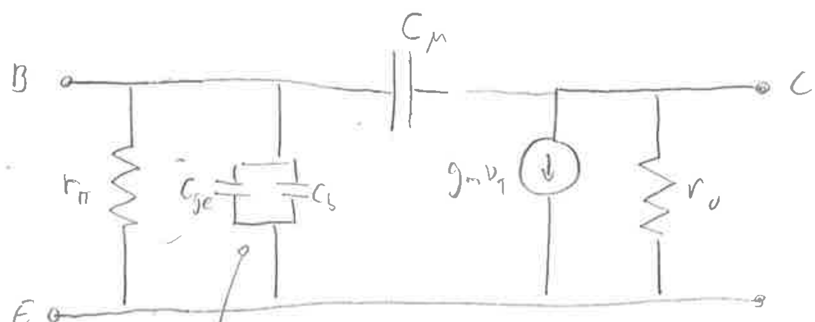
diffusion capacitance

$$C_b = \frac{\delta Q}{V_{be}} = \dots = \frac{W_B^2}{2 D_n} g_m = \tau_F \cdot g_m$$

τ_F: base transit time — Avg. time it takes a carrier to cross the base

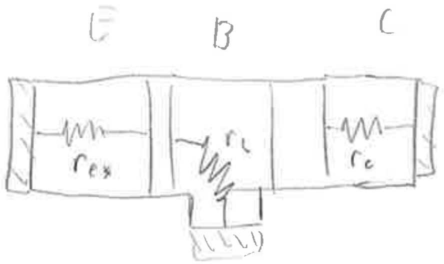
$C_b \gg C_{je}, C_\mu$ so it dominates

Final small-signal model (hybrid π)



$C_\pi = C_{je} + C_b$

Add resistances to model



* show on ppt-slide

Example low-f model

$$I_B = \frac{I_C}{\beta_F} = \frac{5 \cdot 10^{-3}}{500} = 10 \mu A$$

$$g_m = \frac{I_C}{V_T} = \frac{5 \cdot 10^{-3}}{25{,}9 \cdot 10^{-3}} = 0{,}19 \text{ S} \quad [\Omega^{-1}]$$

input resistance (current through base)

$$r_\pi = \frac{\beta_F}{g_m} = \frac{500}{0{,}19} = 2{,}6 \text{ k}\Omega$$

output resistance (Early effect)

$$r_0 = \frac{V_A}{I_C} = \frac{100}{5 \cdot 10^{-3}} = 20 \text{ k}\Omega$$