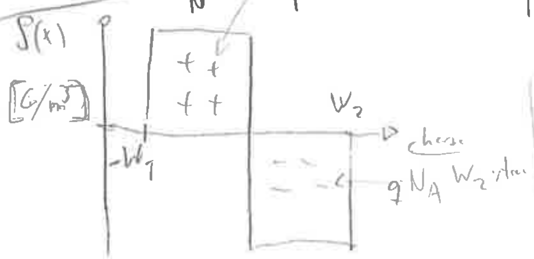
$\boxed{F2}$

pn-junction

Charge density

$qN_D W_1 \cdot$ Area



To analyse, use Poisson equations $\begin{cases} -\text{charge } (\rho)\ [C/m^3] \\ -\text{electric field } (\mathcal{E})\ [V/m] \\ -\text{potential } (V)\ [V] \end{cases}$

$$\boxed{\frac{\rho(x)}{\varepsilon_r \varepsilon_0} = \frac{d\,\mathcal{E}(x)}{dx} = -\frac{d^2 V(x)}{dx^2}}$$

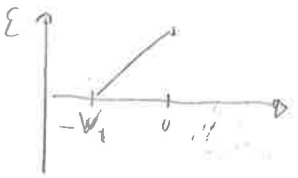
Maybe just graphically

Electric field (page 3 in book)

for $(-W_1 < x < 0)$

$$\mathcal{E}(x) = \frac{1}{\varepsilon_0 \varepsilon_r}\int_{-W_1}^{x} \rho(x)\,dx = \frac{1}{\varepsilon_0 \varepsilon_r}\int_{-W_1}^{x} q\Big(\overset{\text{const.}}{N_D(x)} \underbrace{-\cancel{n(x)} + p(x)}_{\text{no free carriers}} - \overset{=0 \text{ in N-region}}{\cancel{N_A(x)}}\Big)\,dx =$$

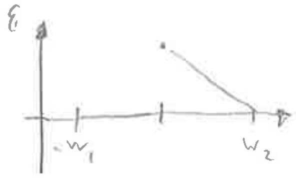
$$= \frac{1}{\varepsilon_0 \varepsilon_r} q \cdot N_D (x + W_1) \Rightarrow$$



for $(0 < x < W_2)$:

$$\mathcal{E}(x) = \frac{1}{\varepsilon_0 \varepsilon_r} qN_D W_1 + \frac{1}{\varepsilon_0 \varepsilon_r}\int_0^x -qN_A(x)\,dx = \frac{q}{\varepsilon_0 \varepsilon_r}(N_D W_1 - N_A \cdot x) = \begin{cases}\text{charge neutrality} \\ N_D W_1 = N_A \cdot W_2\end{cases}$$

$$= \frac{q}{\varepsilon_0 \varepsilon_r} N_A (W_2 - x) \Rightarrow$$



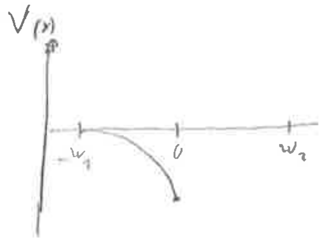
max e-field $\mathcal{E}(0) = \frac{q}{\varepsilon_0 \varepsilon_r} N_D W_1 = \frac{q}{\varepsilon_0 \varepsilon_r} N_A W_2$

Potential

$$V(x) = -\int \mathcal{E}(x)\, dx$$

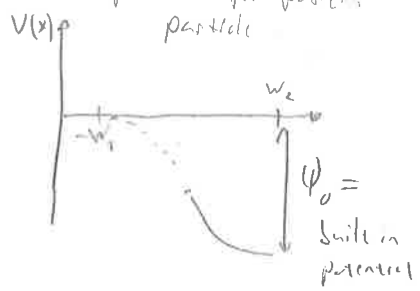
$\begin{pmatrix} \text{for} \\ -W_1 < x < 0 \end{pmatrix}$: $$V(x) = -\frac{qN_D}{\varepsilon_0 \varepsilon_r}\int_{-W_1}^{x} x + W_1 \, dx = -\frac{qN_D}{\varepsilon_0\varepsilon_r}\left[\frac{x^2}{2} + W_1 x\right]_{-W_1}^{x} =$$

$$= -\frac{qN_D}{\varepsilon_0\varepsilon_r}\left(\frac{x^2}{2} + W_1 x - \frac{W_1^2}{2} + W_1^2\right) = -\frac{qN_D}{2\varepsilon_0\varepsilon_r}\left(x + W_1\right)^2 \Rightarrow$$



$\begin{pmatrix} \text{for} \\ 0 < x < W_2 \end{pmatrix}$ $$V(x) = \underbrace{-\frac{qN_D}{2\varepsilon_0\varepsilon_r}\cdot W_1^2}_{\text{potential at } x=0} + \frac{qN_A}{\varepsilon_0\varepsilon_r}\int_0^x W_2 - x \, dx = -\frac{qN_D}{2\varepsilon_0\varepsilon_r}W_1^2 - \frac{qN_A}{\varepsilon_0\varepsilon_r}\left[W_2 \cdot x - \frac{x^2}{2}\right]_0^x$$

$$= -\frac{qN_D}{2\varepsilon_0\varepsilon_r}W_1^2 - \frac{qN_A}{\varepsilon_0\varepsilon_r}\left(W_2 x - \frac{x^2}{2}\right)$$



Built-in potential

$$\psi_0 = V(W_2) - V(-W_1) = -\frac{qN_D}{2\varepsilon_0\varepsilon_r}W_1^2 - \frac{qN_A}{\varepsilon_0\varepsilon_r}\overbrace{\left(W_2^2 - \frac{W_2^2}{2}\right)}^{\frac{W_2^2}{2}} - 0$$

$$= -\frac{q}{2\varepsilon_0\varepsilon_r}\left(N_D W_1^2 + N_A W_2^2\right) = \left\{ W_2 = \frac{N_D W_1}{N_A} \right\} = -\frac{q W_1^2 N_D}{2\varepsilon_0\varepsilon_r}\left(1 + \frac{N_D}{N_A}\right)$$

take absolute

Depletion widths (W_1, W_2)

$$W_1 = \sqrt{\frac{2\varepsilon_0\varepsilon_r(\psi_0 + V_R)}{qN_D\left(1 + \frac{N_D}{N_A}\right)}} \qquad W_2 = \sqrt{\frac{2\varepsilon_0\varepsilon_r\psi_0}{qN_A\left(1 + \frac{N_A}{N_D}\right)}}$$

- * depletion region width $\propto \frac{1}{\sqrt{\text{doping}}}$
- * Can add bias (V_R) $W \propto \sqrt{V_R}$

$N_D \gg N_A \Rightarrow W_1 \ll W_2$ short depletion region at highly doped side

(om det finns tid !!!)

2 Example: Depletion width with Si pn-junction $N_A = N_D = 10^{17}\ cm^{-3}$

Intrinsic carrier concentration

$$n_i(Si, 300K) = \sqrt{N_c N_v} \cdot \exp\left(-\frac{E_g}{2kT}\right) = 10^{10}\ cm^{-3} \quad \text{(from lecture 1)}$$

Built-in potential

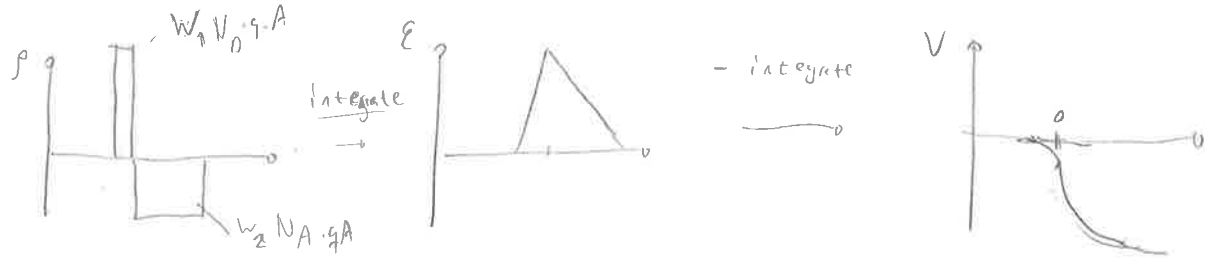
$$\Psi_0 = k_B T \left(\ln\left(\frac{10^{17} \cdot 10^{17}}{(10^{10})^2} \right) \right) = 0{,}025 \cdot \ln\left(10^{14}\right) = 0{,}025 \cdot 32 = 0{,}806\ V.$$

$$d_{tot} = 2 \cdot W_1 = 2 \cdot W_2 = 2 \cdot \sqrt{\frac{2\, \varepsilon_0 \cdot \varepsilon_r (\Psi_0 - V)}{q \cdot N_D \left(1 + \frac{N_D}{N_A}\right)}} = 2 \cdot \sqrt{\frac{2 \cdot 8{,}85 \cdot 10^{-14} [F/cm] \cdot 11{,}7 \cdot (0{,}806 - V)}{1{,}602 \cdot 10^{-19} \cdot 10^{17} \left(1 + \frac{10^{17}}{10^{17}}\right)}} =$$

$$= 2 \cdot \sqrt{6{,}46 \cdot 10^{-11} \cdot (0{,}806 - V)}\ \underset{\text{obs}}{cm} = \begin{cases} d_{tot} = 144\ nm & V = 0 \\ d_{tot} = 216\ nm & V = -1\ V \end{cases}$$

$d_{tot} \propto \sqrt{-V}$ for forward bias $V = \Psi_0$ (builtin pot) $\Rightarrow d_{tot} = 0$ i.e. depletion region vanishes !

ex: charge / electric field / Potential



- Voltage drops over low-doped part
- depletion region is mainly in p-doped part.

Small signal model

Want to calculate $i_D(t) = I_D + i_d(t)$ Simplified diode: $I = I_s e^{qV_D/kT}$

when applying $v_D(t) = V_D + v_d(t)$ large = DC, small = AC

(includes area)

$$i_D(V_D+v_d) = I_s e^{q(V_D+v_d(t))/kT} = \underbrace{I_s e^{qV_D/kT}}_{I_D = \text{const.}} \cdot e^{q v_d(t)/kT} = I_D e^{q v_d(t)/kT}$$

Taylor expansion around $x = x_0$

$$f(x_0+dx) \approx f(x_0) + \frac{df(x)}{dx}\Big|_{x=x_0} dx + \frac{d^2 f(x)}{dx^2}\Big|_{x=x_0} \frac{dx^2}{2!} \overset{\{x_0=0\}}{\Longrightarrow} e^x = 1 + x + \frac{x^2}{2!} + \dots$$

$$\left\{\text{If } v_d << kT\right\} \approx I_D\left(1 + \frac{qV_d}{kT} + \cancel{\left(\frac{qV_d}{kT}\right)^2 \frac{1}{2}} \dots\right) = \underbrace{I_D\left(1 + \frac{q v_d(t)}{kT}\right)}_{\text{small signal approximation}} = \underbrace{I_D}_{\text{steady state}} + \underbrace{\frac{1}{r_d} v_d(t)}_{\text{time varying}}$$

linear variation of current (i_D) with voltage (v_d)

Have actually transformed

$$r_d = \frac{1}{\text{slope}} = \cancel{\frac{1}{\frac{dI}{dV_D}\Big|_{V_D}}} = \begin{cases} \infty & \text{for } V_D << 0 \\ \frac{kT}{qI_D} & \leftarrow \text{decreasing resistance with increasing current} \end{cases}$$

small signal resistance

Depletion-region (junction) capacitance (on power-point)

Consider a p^+n junction $(N_A >> N_D)$ remember $W_1 N_D = W_2 N_A$

$$W_1 = \sqrt{\frac{2\varepsilon_0\varepsilon_r(\psi_0 + V_R)}{qN_D\left(1+\frac{N_D}{N_A}\right)}} = \qquad W_2 \approx 0$$

Only W_1 change with V_R

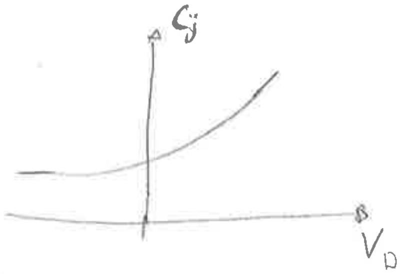
only depletion width on n-side

$$C_j = \frac{dQ}{dV_R} = \frac{dQ}{dW_1}\cdot\frac{dW_1}{dV_R}$$

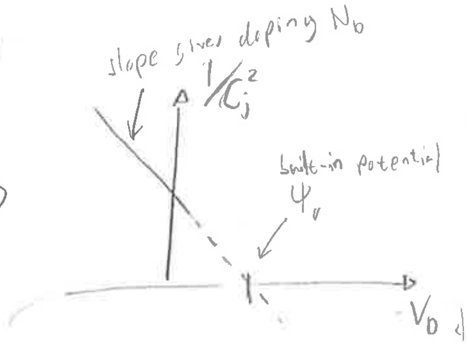
since $Q = A q N_D \cdot W_1 \rightarrow \frac{dQ}{dW_1} = A q N_D$

$$\frac{dW_1}{dV_R} = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{qN_D\left(1+\frac{N_D}{N_A}\right)}}\cdot\frac{1}{2}(\psi_0 + V_R)^{-0,5}$$

$$C_j = A q N_D \cdot \sqrt{\frac{\varepsilon_0 \varepsilon_r}{2qN_D\left(1+\frac{N_D}{N_A}\right)}}\cdot\frac{1}{\sqrt{\psi_0 + \underset{\text{negative}}{V_R}}} \Rightarrow$$



rewrite as
$$\frac{1}{C_j^2} = \frac{2\cdot\left(1+\frac{N_D}{N_A}\right)}{A^2 q N_D \varepsilon_0 \varepsilon_r}\cdot(\psi_0 + \underset{(-V_D)}{V_R}) \Rightarrow$$



Diffusion capacitance (n^+p-junction)

forward bias → excess minority carriers (electrons) in p-region

$$Q = \frac{qAW_p\left(n_p(W_2) - n_p(W_p)\right)}{2} = \frac{q\, n_{p0} \cdot A \cdot W_p}{2}\left(e^{qV_D/kT} - 1\right)$$

$$C_{diff} = \frac{dQ}{dV_D} = \frac{q\, n_{p0} A W_p}{2} \cdot \frac{1}{kT}\, e^{qV_D/kT}$$