Advanced AD/DA converters

Continuous-Time ΔΣ Modulators

Pietro Andreani
Dept. of Electrical and Information Technology
Lund University, Sweden

Overview

- Example: continuous-time 2^{nd}-order modulator
- General approach
- Implementation issues

Continuous-time design

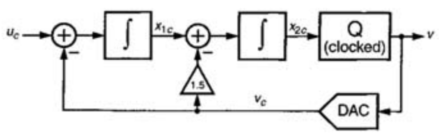
1) CT modulators possess inherent anti-aliasing! Degree of alias suppression = degree of q-noise suppression → an explicit anti-alias filter is nor needed!
2) Sampling occurs at the output of the loop filter, and not at the input → imperfection of the sampling process (e.g. clock jitter, etc.) and folding of white noise take place at a much less sensitive node in the loop!
3) Highest clock rate determined by regeneration time in quantizer and update rate of the feedback DAC, while in DT DSM this is set by the integrators settling requirements → highest rate is approx. 20%-50% of the unity-gain frequency of the CT opamp → CT allows 2-4 higher clock rate than equivalent DT (although with question marks about linearity and accuracy) → this difference is due to the fact that the settling error in a SC integrator depends exponentially on the ratio of clock frequency to unity-gain frequency, while in CT the error increases much more gradually with this ratio.
4) The time constants in a CT modulator must be calibrated, while they are locked to the sampling frequency in DT (→ SC filter)

Example of CT modulator

2^{nd}-order modulator as example

Time-domain equations – we assume that the quantizer is delay-free, and that the DAC is a delay-free sample-and-hold stage

We derive a sampled-data model for the CT loop filter, which provides the correct input to the clocked quantizer



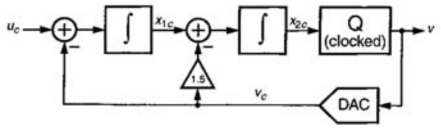
Output of 1^{st} integrator:

$$x_{1c}(t) = x_{1c}(n) + \int_n^t \left[u_c(\tau) - v(n)\right] d\tau, \qquad n \le t \le n+1$$

$$x_{1c}(n+1) = x_{1c}(n) - v(n) + u_1(n), \qquad u_1(n) = \int_n^{n+1} u_c(\tau)\, d\tau$$

Integrations

Output of 1st integrator:



$$x_{1c}(n+1) = x_{1c}(n) - v(n) + u_1(n), \qquad u_1(n) = \int_n^{n+1} u_c(\tau)\,d\tau$$

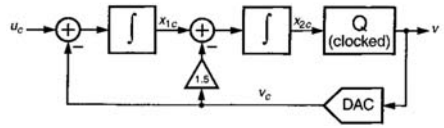
Change of variable: $\tau \to n-\tau$

$$u_1(n) = -\int_0^{-1} u_c(n-\tau)\,d\tau = \int_{-1}^{0} u_c(n-\tau)\,d\tau = \int_{-1}^{0} 1\cdot u_c(n-\tau)\,d\tau = (g_{p1c} * u_c)(n)$$

where we have defined the (non-causal) pre-filter $g_{p1c}(t) = \begin{cases} 1, & -1 \le t \le 0 \\ 0, & \text{otherwise} \end{cases}$

Integrations – II

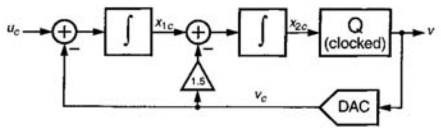
Output of 2nd integrator:



$$\begin{aligned} x_{2c}(n+1) &= x_{2c}(n) + \int_n^{n+1} \left[x_{1c}(\tau) - 1.5v(n) \right] dt \\ &= x_{2c}(n) + \int_n^{n+1} \left\{ x_{1c}(n) + \int_n^{t} \left[u_c(\tau) - v(n) \right] d\tau \right\} dt - 1.5v(n) \\ &= x_{2c}(n) + x_{1c}(n) + \int_n^{n+1} \left\{ \int_n^{t} u_c(\tau)\,d\tau \right\} dt - \int_n^{n+1} (t-n)v(n)\,dt - 1.5v(n) \\ &= x_{2c}(n) + x_{1c}(n) + \int_n^{n+1} \left\{ \int_n^{t} u_c(\tau)\,d\tau \right\} dt - 2v(n) \end{aligned}$$

Integrations – III

Output of 2nd integrator:



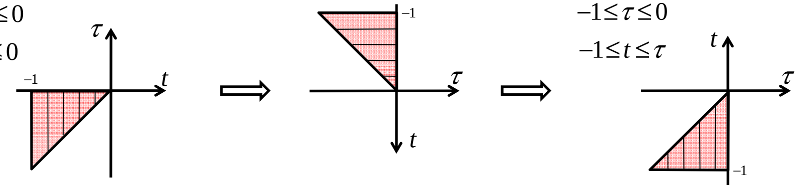
$$x_{2c}(n+1) = x_{2c}(n) + x_{1c}(n) - 2v(n) + \int_n^{n+1} \left\{ \int_n^{t} u_c(\tau)\,d\tau \right\} dt$$

$$\int_n^{n+1} \left\{ \int_n^{t} u_c(\tau)\,d\tau \right\} dt \xrightarrow{t \to n-t} -\int_0^{-1} \left\{ \int_n^{n-t} u_c(\tau)\,d\tau \right\} dt = \int_{-1}^{0} \left\{ \int_n^{n-t} u_c(\tau)\,d\tau \right\} dt$$

$$\int_{-1}^{0} \left\{ \int_n^{n-t} u_c(\tau)\,d\tau \right\} dt \xrightarrow{\tau \to n-\tau} -\int_{-1}^{0} \left\{ \int_0^{t} u_c(n-\tau)\,d\tau \right\} dt = \int_{-1}^{0} \left\{ \int_t^{0} u_c(n-\tau)\,d\tau \right\} dt$$

Change the integration order ($t \leftrightarrow \tau$)

$\begin{cases} -1 \le t \le 0 \\ t \le \tau \le 0 \end{cases}$



$-1 \le \tau \le 0$

$-1 \le t \le \tau$

$$\int_{-1}^{0} \left\{ \int_t^{0} u_c(n-\tau)\,d\tau \right\} dt \rightarrow \int_{-1}^{0} \left\{ \int_{-1}^{\tau} u_c(n-\tau)\,dt \right\} d\tau = \int_{-1}^{0} (\tau+1)\,u_c(n-\tau)\,d\tau$$

$$x_{2c}(n+1) = x_{2c}(n) + x_{1c}(n) - 2v(n) + \int_{-1}^{0} (\tau+1)\,u_c(n-\tau)\,d\tau$$

Yet another convolution

$$x_{2c}(n+1) = x_{2c}(n) + x_{1c}(n) - 2v(n) + \int_{-1}^{0} (\tau+1) u_c(n-\tau)\, d\tau$$

Thus, the samples values are given by

$$x_2(n+1) = x_2(n) + x_1(n) - 2v(n) + u_2(n)$$

where

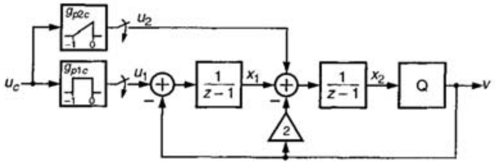
$$u_2(n) = (g_{p2c} * u_c)(n), \qquad g_{p2c}(t) = \begin{cases} t+1, & -1 \le t \le 0 \\ 0, & \text{otherwise} \end{cases}$$

(also the filter g_{p2c} is non-causal, as g_{p1c}, but we will see that the final result is causal, as it should)

DT/CT equivalence

Equivalent DT circuit – notice that the feedback coefficient of the 2nd integrator is 2, while it was 1.5 in CT → more complicated than just replacing DT integrators with CT!

Furthermore, the equivalence CT = DT requires that the CT input $u_c(t)$ is pre-filtered by g_{p1c} and g_{p2c}, whose outputs are sampled and injected into the loop

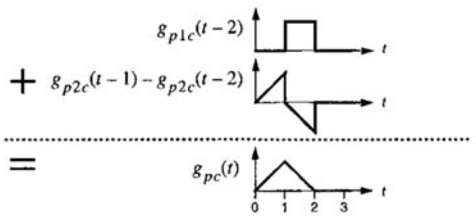


We obtain: $V(z) = z^{-2}U_1(z) + z^{-1}(1-z^{-1})U_2(z) + (1-z^{-1})^2 E(z)$

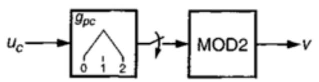
NTF → same as for the DT MOD2

STF → the impulse response of the SFT is: twice delayed g_{p1c}, plus once delayed g_{p2c}, minus twice delayed g_{p2c}

DT/CT equivalence



This is equivalent to (notice that now g_{pc} is indeed causal):



$$g_{pc}(t) = \begin{cases} t, & 0 \le t < 1 \\ 2-t, & 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$$

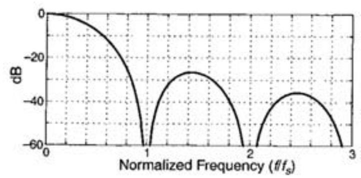
g_{pc} is the convolution of a unit pulse with itself, and its Laplace transform is therefore:

$$L[g_{pc}(t)] = \{L[u(t) - u(t-1)]\}^2 = \left(\frac{1-e^{-s}}{s}\right)^2$$

Anti-alias pre-filtering

Thus, the amplitude response of the STF is

$$|STF(f)| = \left|\frac{1-e^{-s}}{s}\right|^2_{s=j2\pi f} = \left|\frac{1-e^{-j2\pi f}}{j2\pi f}\right|^2 = \left|\frac{e^{j\pi f} - e^{-j\pi f}}{e^{j\pi f} \cdot j2\pi f}\right|^2 = \left|\frac{\sin(\pi f)}{\pi f}\right|^2$$

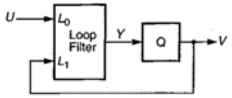


2nd-order zeros at all multiples of the sampling frequencies → anti-alias! (of same order as modulator order)

In general, degree of alias protection is the same as degree of q-noise suppression – this is because both q-noise and sampling occur at the same point in the loop

General case

General case: the samples entering the quantizer are given by



$$y(n) = [l_{0c} * u_c](n) + [l_1 * v](n)$$

l_{0c} is the CT impulse response from $u_c(t)$ input to quantizer input

l_1 is the DT impulse response from quantizer output through DAC (or DACs), through loop filter, and back to quantizer input; we obtain

$$NTF(z) = \frac{1}{1 - L_1(z)}$$

$$STF = \frac{L_{0c}(s)}{1 - L_1(z)} = L_{0c}(s)\, NTF(z)$$

STF → mixture of CT and DT!

$$STF(f) = L_{0c}(j2\pi f)\, NTF\left(e^{j2\pi f}\right)$$

CT STF

At a pole frequency of L_{0c}, the NTF is zero (remember: L_0 and L_1 have the same poles, which are also the zeros of the NTF) → pole-zero cancellation in the STF → finite signal transmission; e.g. in 2[nd]-order CT:

$$L_{0c}(s) = \frac{1}{s^2}$$

$$NTF(z) = \left(1 - z^{-1}\right)^2$$

$$STF = \frac{\left(1 - z^{-1}\right)^2}{s^2}$$

$$\left|STF(f)\right| = \left|\frac{1 - e^{-j2\pi f}}{2\pi f}\right|^2 = \left|\frac{\sin(\pi f)}{\pi f}\right|^2$$

Double pole – double zero cancellation → STF(0)=1

Consider instead a frequency that aliases to an NTF zero → L_{0c} finite (we assume all poles lie in the passband) → L_{0c} x NTF = STF = 0

This also shows that the amount of alias suppression is intimately linked to the NTF! – for out-of-band frequencies L_{0c} is usually less than unity → alias attenuation usually higher than q-noise attenuation

Key result: CT = DT + anti-aliasing

From DT to CT – general procedure

Select a DT prototype, and transform DT into CT with same NTF → this is equivalent to ensure that the path from quantizer output back to quantizer input has the same impulse response in CT as in DT

In DT: impulse response is $l_1(n)$

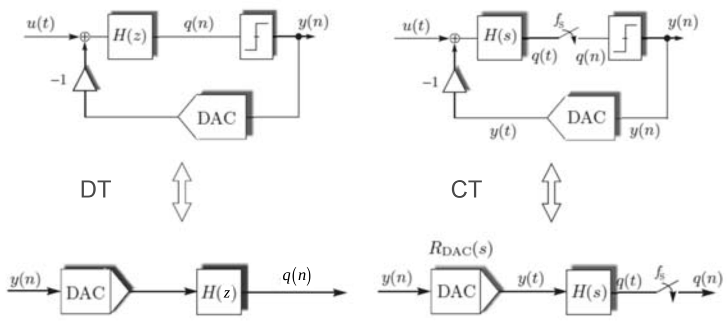
In CT: impulse response must be calculated, assuming a given DT quantizer output, DT-to-CT transformation in the DACs, CT loop filter, and finally sampling at the quantizer input, which yields a DT response

Thus:

The sampled impulse response of the CT loop filter must match the impulse response of the DT loop filter

DT to CT – I

The same statement with figures:



DT to CT – II

And with equations: we want $q(n)=q(t)\big|_{t=nT_s}$, i.e.

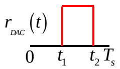
$$Z^{-1}\left[H(z)\right]=L^{-1}\left[R_{DAC}(s)H(s)\right]\Big|_{t=nT_s}$$

$$h(n)=\left[r_{DAC}(t)*h(t)\right]\Big|_{t=nT_s}=\int_{-\infty}^{+\infty} r_{DAC}(\tau)h(t-\tau)d\tau\Big|_{t=nT_s}$$

For instance, a single DT pole in the DT loop yields

$$H(z)=\frac{1}{z-z_k}\quad\rightarrow\quad h(n)=z_k^{n-1}$$

What should the CT loop contain? Let us start assuming that the CT DAC delivers a general rectangular pulse contained within a sampling period:



$$r_{DAC}(t)=\begin{cases}1, & t_1=\alpha T_s\le t\le t_2=\beta T_s\\ 0, & \text{otherwise}\end{cases}\quad\rightarrow\quad R_{DAC}(s)=\frac{1}{s}\left(e^{-s\alpha T_s}-e^{-s\beta T_s}\right)$$

DT to CT – III

A leaky CT integrator yields $H(s)=\dfrac{C}{s-s_k}\quad\rightarrow\quad h(t)=Ce^{s_k t}$

Therefore, the impulse response of the CT loop is

$$\int_{-\infty}^{+\infty} r_{DAC}(\tau)h(t-\tau)d\tau=\int_{\alpha T_s}^{\beta T_s} Ce^{s_k(t-\tau)}d\tau=\frac{C}{s_k}\left(e^{s_k(t-\alpha T_s)}-e^{s_k(t-\beta T_s)}\right)$$

Sampling at nT_s:

$$\frac{C}{s_k}\left(e^{s_k(nT_s-\alpha T_s)}-e^{s_k(nT_s-\beta T_s)}\right)=\frac{C}{s_k}e^{s_k nT}\left(e^{-\alpha T_s s_k}-e^{-\beta T_s s_k}\right)=\frac{C}{s_k}e^{s_k(n-1)T}e^{s_k T}\left(e^{-\alpha T_s s_k}-e^{-\beta T_s s_k}\right)$$

The equality of DT and CT impulse responses at nT_s yields

$$z_k^{n-1}=\frac{C}{s_k}e^{s_k(n-1)T_s}e^{s_k T_s}\left(e^{-\alpha T_s s_k}-e^{-\beta T_s s_k}\right)\quad\rightarrow\quad\begin{cases}C=\dfrac{s_k}{e^{s_k T_s}\left(e^{-\alpha T_s s_k}-e^{-\beta T_s s_k}\right)}\\ z_k=e^{s_k T_s}\quad\rightarrow\quad s_k=\dfrac{1}{T_s}\ln(z_k)\end{cases}$$

DT to CT – IV

In particular, for ideal integrators we obtain $z_k=1\quad\rightarrow\quad s_k=0$, and

$$C=\lim_{s_k\to 0}\frac{s_k}{e^{s_k T_s}\left(e^{-\alpha T_s s_k}-e^{-\beta T_s s_k}\right)}=\frac{1}{T_s(\beta-\alpha)}$$

Finally, we have obtained the expression of the CT loop transfer function we were looking for:

$$H(s)=\frac{1}{s}\frac{1}{T_s(\beta-\alpha)}=\frac{1}{sT_s}\frac{1}{\beta-\alpha}$$

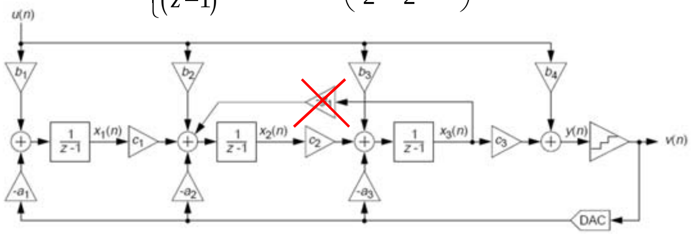
Thus, assuming in general a coefficient a_{DT} in front of the DT integrator, and normalizing T_s to unity as usual, the coefficient a_{CT} in front of the CT integrator is given by

$$a_{CT}=\frac{a_{DT}}{\beta-\alpha}$$

Extension to a 3rd-order DT/CT modulator

From the basic theory of the Z-transform we find the impulse response for 1st, 2nd and 3rd order DT integration (simulations show that the local feedback via g_1 can be neglected)

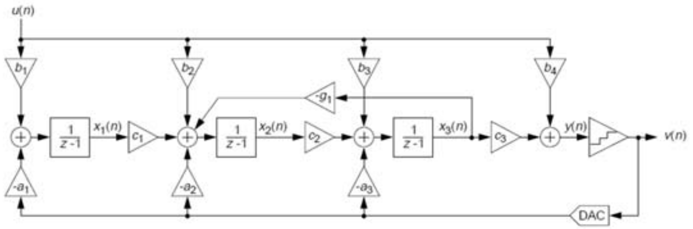
$$\begin{cases}\dfrac{a_{3,DT}}{z-1}\quad\rightarrow\quad a_{3,DT}\\[2mm] \dfrac{a_{2,DT}}{(z-1)^2}\quad\rightarrow\quad a_{2,DT}(n-1)\\[2mm] \dfrac{a_{1,DT}}{(z-1)^3}\quad\rightarrow\quad a_{1,DT}\left(\dfrac{n^2}{2}-\dfrac{3}{2}n+1\right)\end{cases}$$



Extension to a 3rd-order DT/CT modulator

With e.g. Maple we find the sampled impulse response for 1st, 2nd and 3rd order CT integration, still with the same DAC feedback pulse (and T=1):

$$\begin{cases} \left[\frac{1}{s}\left(e^{-s\alpha_3 T}-e^{-s\beta_3 T}\right)\right]\cdot\frac{a_{3,CT}}{s} & \rightarrow \quad a_{3,CT}\left(\beta_3-\alpha_3\right) \\ \left[\frac{1}{s}\left(e^{-s\alpha_2 T}-e^{-s\beta_2 T}\right)\right]\cdot\frac{a_{2,CT}}{s^2} & \rightarrow \quad a_{2,CT}\left[\left(\beta_2-\alpha_2\right)n+\frac{\alpha_2^2-\beta_2^2}{2}\right] \\ \left[\frac{1}{s}\left(e^{-s\alpha_1 T}-e^{-s\beta_1 T}\right)\right]\cdot\frac{a_{1,CT}}{s^3} & \rightarrow \quad \frac{a_{1,CT}}{2}\left[\left(\beta_1-\alpha_1\right)n^2+\left(\alpha_1^2-\beta_1^2\right)n+\frac{\beta_1^3-\alpha_1^3}{3}\right] \end{cases}$$



Extension to a 3rd-order DT/CT modulator

Identity of DT impulse response and sampled CT impulse response demands:

$$a_{3,DT}+a_{2,DT}\left(n-1\right)+a_{1,DT}\left(\frac{n^2}{2}-\frac{3}{2}n+1\right)=$$

$$a_{3,CT}\left(\beta_3-\alpha_3\right)+a_{2,CT}\left[\left(\beta_2-\alpha_2\right)n+\frac{\alpha_2^2-\beta_2^2}{2}\right]+\frac{a_{1,CT}}{2}\left[\left(\beta_1-\alpha_1\right)n^2+\left(\alpha_1^2-\beta_1^2\right)n+\frac{\beta_1^3-\alpha_1^3}{3}\right]$$

From this, we obtain all CT coefficients as functions of the DT ones:

$$a_{1,CT}=\frac{a_{1,DT}}{\beta_1-\alpha_1}$$

$$a_{2,CT}=\frac{a_{2,DT}+\frac{a_{1,DT}}{2}\left(\alpha_1+\beta_1-3\right)}{\beta_2-\alpha_2}$$

$$a_{3,CT}=\frac{a_{3,DT}+a_{2,DT}\left[\frac{1}{2}\left(\alpha_2+\beta_2\right)-1\right]+a_{1,DT}\left[1-\frac{1}{6}\left(\alpha_1^2+\alpha_1\beta_1+\beta_1^2\right)+\frac{1}{4}\left(\alpha_2+\beta_2\right)\left(\alpha_1+\beta_1-3\right)\right]}{\beta_3-\alpha_3}$$

Non-rectangular CT DAC waveforms

If the CT feedback DAC waveforms are not rectangular pulses, we can find the values of the CT feedback coefficients with the following procedure:

1) Set up the circuit for simulating the impulse response (IR) of the open-loop feedback path in a CT modulator with NRZ feedbacks (which acts as the reference design) and in the CT modulator with the desired feedback waveforms
2) Let us e.g. assume that we are targeting a 3rd-order CIFB, where the NRZ DACs have known coefficients $a_{1,NRZ}$, $a_{2,NRZ}$, $a_{3,NRZ}$, and that we must find the corresponding coefficients a_1, a_2, a_3; then it can be shown that we can adopt the following procedure:

Non-rectangular CT DAC waveforms

a) Simulate the impulse response (IR) of the NRZ modulator with $a_{2,NRZ} = a_{3,NRZ} = 0$
b) Simulate the IR of the desired modulator with $a_2 = a_3 = 0$, and adjust the value of a_1 until the IR after the first integrator is the same in the two modulators (this can be done quickly enough in Cadence); this is the sought value for a_1
c) Set $a_{3,NRZ} = 0$ and $a_3 = 0$; simulate the IR of the NRZ modulator; simulate the IR of the desired modulator, adjusting the value of a_2 until the IR after the second integrator is the same in the two modulators; this is the sought value for a_2
d) Now use all coefficients, and adjust the value of a_3 until the IRs of the two modulators after the third integrator are the same; this is the sought value for a_3

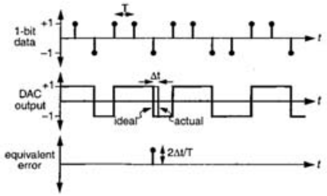
Non-idealities

A quantizer does not provide feedback immediately after sampling its input

Assume single-bit modulator, 100MHz clock, and that every 100 cycles the comparator output is delayed from its nominal timing by 0.1ns

The equivalent relative error signal from the feedback DAC has an average power of

$$\frac{\left(2\frac{\Delta t}{T}\right)^2}{100} = \frac{\left(2\frac{0.1ns}{100ns}\right)^2}{100} = 4\cdot 10^{-6}$$



This is only 51dB below a full-scale sine wave

If noise is white → in-band power reduced by OSR; however, the error is likely to be correlated to the input signal → distortion, 9-bit linearity

Non-idealities

Solution to input-dependent quantizer delay → allocate a fixed amount of time for the quantizer to resolve its output → delayed feedback

Delayed feedback → loop delay → must be taken into account in the DT-to-CT transformation

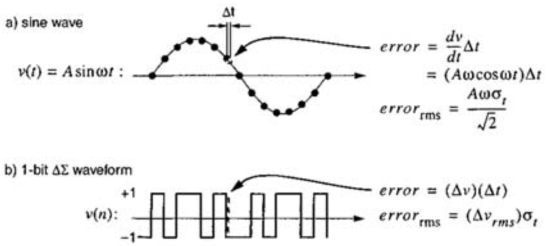
Excess loop delay may severely deteriorate SNR or even lead to instability → an extra feedback DAC may be needed (as well as an adjustment of the loop coefficients), or equivalent circuit techniques

Clock jitter – I

Clock jitter, problem as usual! With a sine wave of amplitude A, the error introduced by jitter in the sampling stage of a DT modulator is

$$\varepsilon_{j,rms} = \frac{\omega A \sigma_t}{\sqrt{2}} = \sqrt{2}\pi A f \sigma_t$$

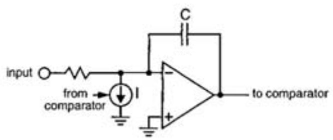
The error caused by jitter in the feedback of a 1-bit CT modulator is $(\Delta v)_{rms}\,\sigma_t$ where $(\Delta v)_{rms}$ is the *rms* change in the 1-bit feedback. Assuming $(\Delta v)_{rms} \approx 1$ this error is the same as that associated with sampling a full-scale sine wave having a frequency of $0.2f_s$ → very high frequency → CT modulator has a jitter disadvantage compared to DT!

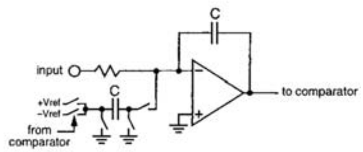


Clock jitter – II

Straightforward solution: increase the # of bits in the feedback DAC → with sufficient number of levels, the feedback waveform approximates the input signal → *rms* sample-to-sample feedback error is approx. the same as the sample-to-sample error on the input signal → DT and CT have then comparable sensitivities to jitter

More elegant solution: change the feedback waveform to a timing-insensitive one (compared to rectangular feedback current pulses)





Replace current generator with switched capacitor → the charge delivered by the pre-charged capacitor is $\pm V_{ref}$, basically irrespective of the length of the clock period → insensitive to jitter (however, the large amplitude of the discharge current is now an issue in itself!)