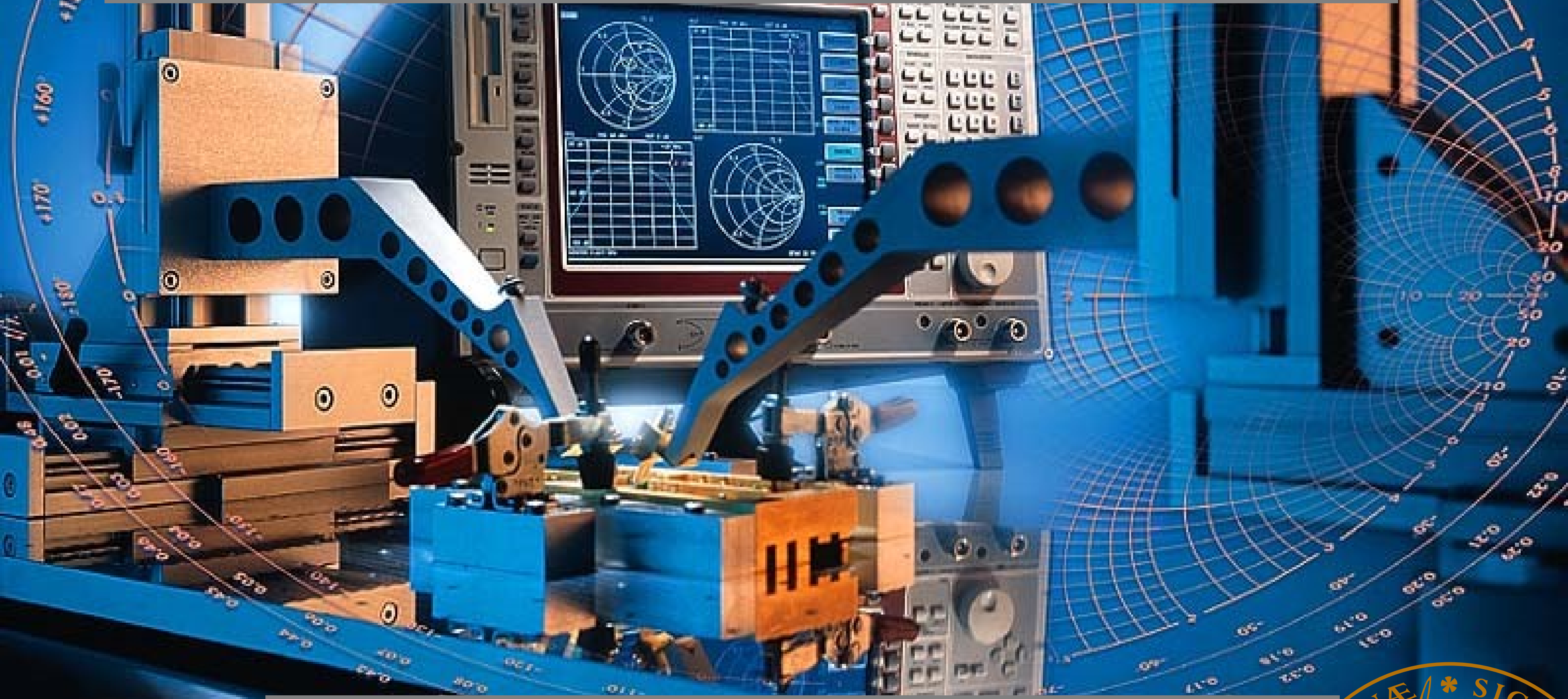
2019-11-27

Lecture 7

RF Amplifier Design



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Schedule Reminder

- Lab 2: Thursday, Nov 28, 8:15-12:00
  - S-parameter measurement using a calibrated VNA
- Hand-in 2: Friday, Dec 6, 23:59
  - Matching and bias design for an LNA
  - …to be validated in laboratory session 3
- Hand-in 1: with Hand-in 2
  - Revise according to feedback from supervisor
  - Re-submit corrected version

Lecture 7

- Amplifier Design
  - Stability Analysis by S-parameters
  - Design Cases
    - unilateral two-port, maximum gain (Case 1)
    - unilateral two-port, specific gain (Case 2)
    - bilateral two-port, maximum gain (Case 3)
      - "simultaneous conjugate match"
    - bilateral two-port, specific gain (Case 4)
      - conjugate match at one port, mismatch the other port
      - design method using "operating gain"
      - design method using "available gain"
  - Noise in a Two-Port
  - Design of Low Noise Amplifiers (LNA)

Stability Analysis by S-Parameters

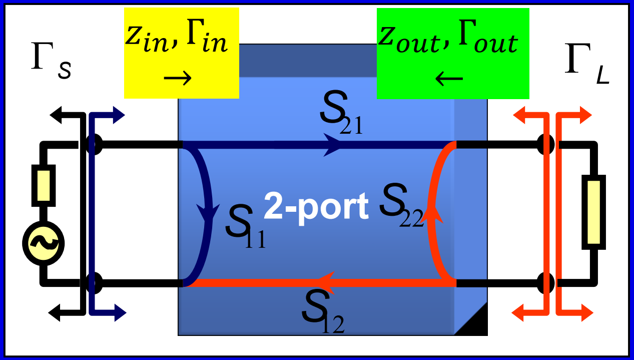
$\mathrm{Re}[z_{in}] > 0$ for any value of z_L $(\mathrm{Re}[z_L] > 0)$

$|\Gamma_{in}| < 1$ for all $|\Gamma_L| < 1$

$\mathrm{Re}[z_{out}] > 0$ for any value of z_S $(\mathrm{Re}[z_S] > 0)$

$|\Gamma_{out}| < 1$ for all $|\Gamma_S| < 1$

$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L}$$



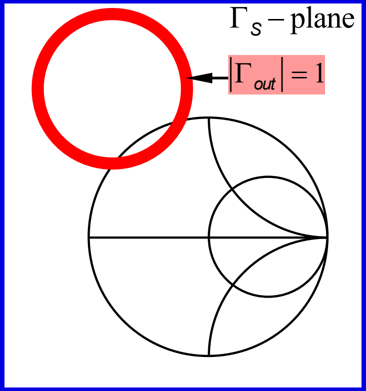
$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S}$$

A unilateral two-port ($S_{12} = 0$) is unconditionally stable if $|\Gamma_{in}| = |S_{11}| < 1$ and $|\Gamma_{out}| = |S_{22}| < 1$

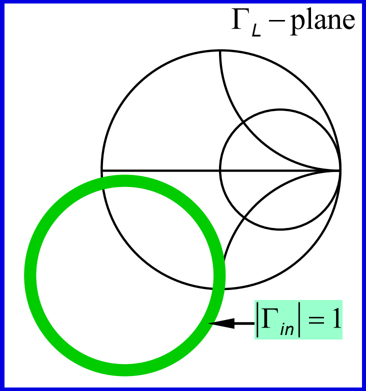
A bilateral two-port is

1) unconditionally stable if $|\Delta| < 1$ and $K > 1$ where $\Delta = S_{11}S_{22} - S_{12}S_{21}$ and $K = \dfrac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$

2) conditionally stable if some Γ_S gives $|\Gamma_{out}| < 1$

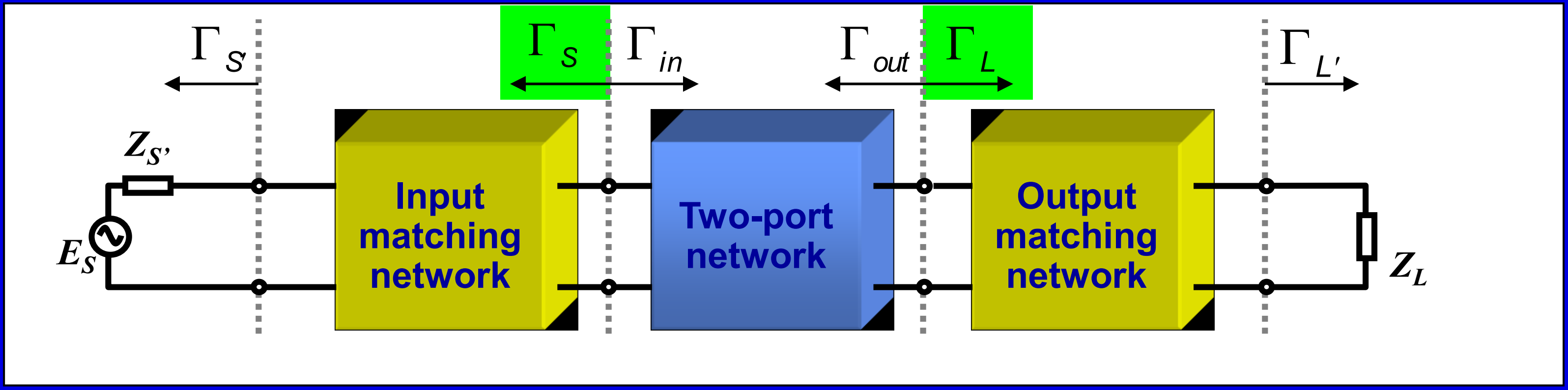


and some Γ_L gives $|\Gamma_{in}| < 1$



Amplifier Design

General design case

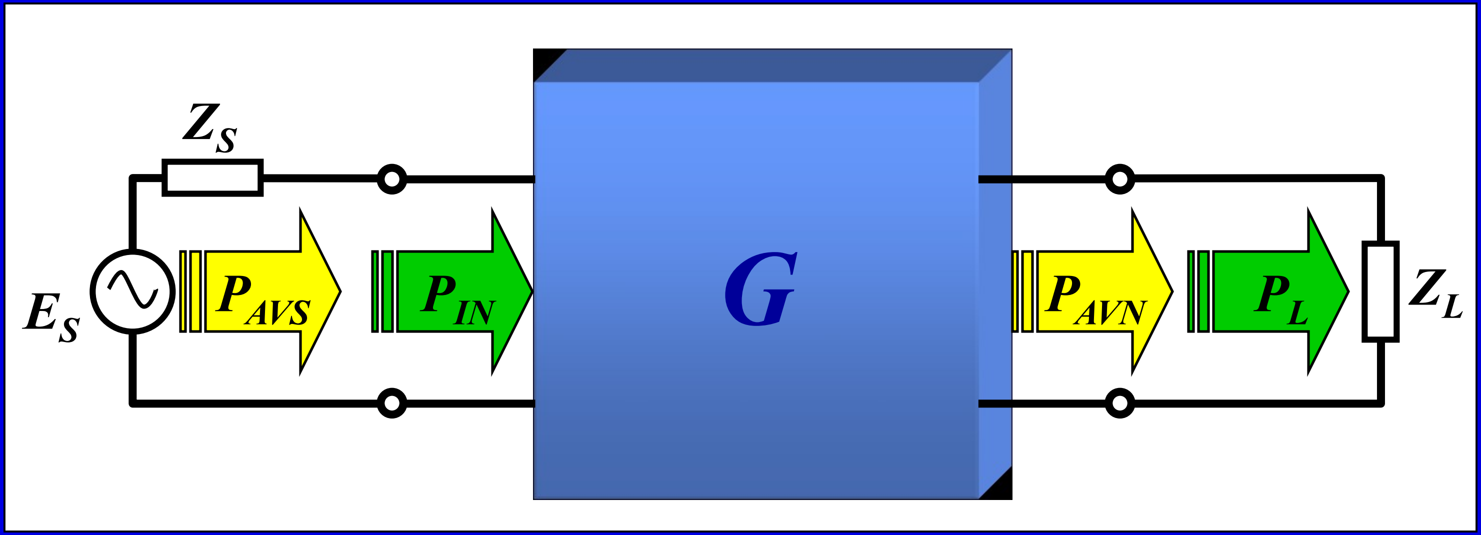


- Given this:
  - two-port (S-parameters) and
  - source $\Gamma_{S'}$ and load $\Gamma_{L'}$

- The stability analysis gives allowed values of Γ_S and Γ_L

- After a proper choice of Γ_S and Γ_L
  the matching networks may be designed

Power Gain Definitions

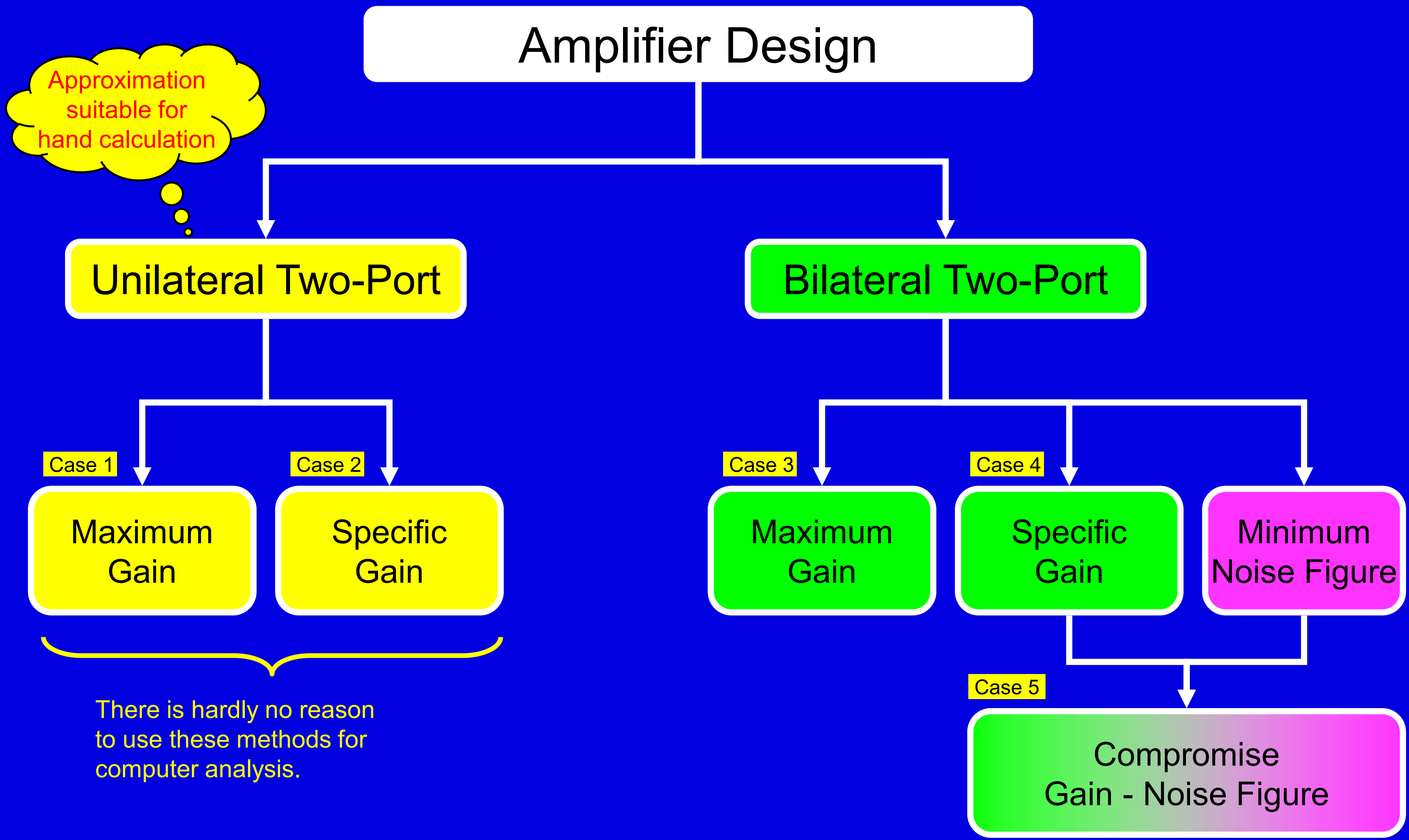


available gain $G_A = \dfrac{P_{AVN}}{P_{AVS}}$

operating gain $G_P = \dfrac{P_L}{P_{IN}}$

transducer gain $G_T = \dfrac{P_L}{P_{AVS}}$

Design Cases - Gain and Noise Figure



Case 1

Unilateral Two-Port, Maximum Gain

- Choose $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$

  i.e. apply conjugate match to both input and output

- The gain is then

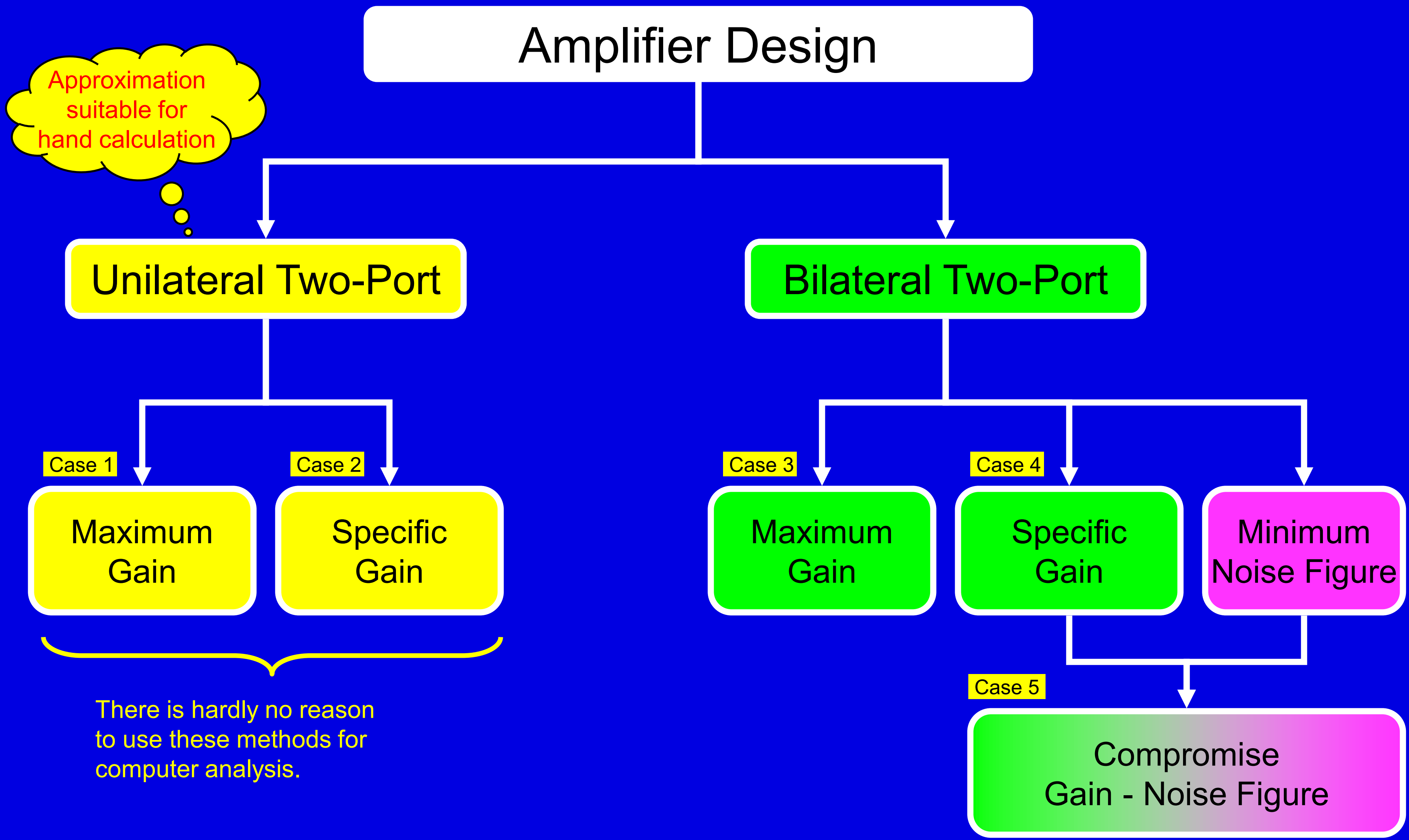
$$G_T = \frac{P_L}{P_{AVS}} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \\ = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

Maximum Unilateral Transducer Gain:

$$G_{TUM} = \frac{P_L}{P_{AVS}} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}$$

Assumption: $|\Gamma_{in}| = |S_{11}| < 1$ and $|\Gamma_{out}| = |S_{22}| < 1$

Design Cases - Gain and Noise Figure



Case 2

Unilateral Two-Port, Specific Gain

- The gain is expressed by $G_{TU} = \frac{P_L}{P_{AVS}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$

  (Unilateral Transducer Gain)

- Split up $G_{TU} = \alpha_S|S_{21}|^2\alpha_L$ note that $\max(\alpha_S) = \frac{1}{1-|S_{11}|^2}$, $\max(\alpha_L) = \frac{1}{1-|S_{22}|^2}$

- Result: $G_{TU} = \frac{\alpha_S}{\max(\alpha_S)}\max(\alpha_S)\,|S_{21}|^2\max(\alpha_L)\frac{\alpha_L}{\max(\alpha_L)}$

  $$G_{TU} = \frac{\alpha_S}{\max(\alpha_S)}G_{TUM}\frac{\alpha_L}{\max(\alpha_L)} = g_S G_{TUM} g_L$$

  where $0 \le g_S \le 1$, $0 \le g_L \le 1$

- When $g_S = g_L = 1$ only one solution exists: $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$

- If $g_S < 1$ and $g_L < 1$ there are lot of solutions. Described by circles in the Γ_S-plane and the Γ_L-plane

Case 2

Unilateral Two-Port, Specific Gain

$$G_{TU} = g_S \cdot G_{TUM} \cdot g_L$$

For $g_S < 1$ and $g_L < 1$ there are lot of solutions. Described by circles in the Γ_S-plane and the Γ_L-plane

Input:

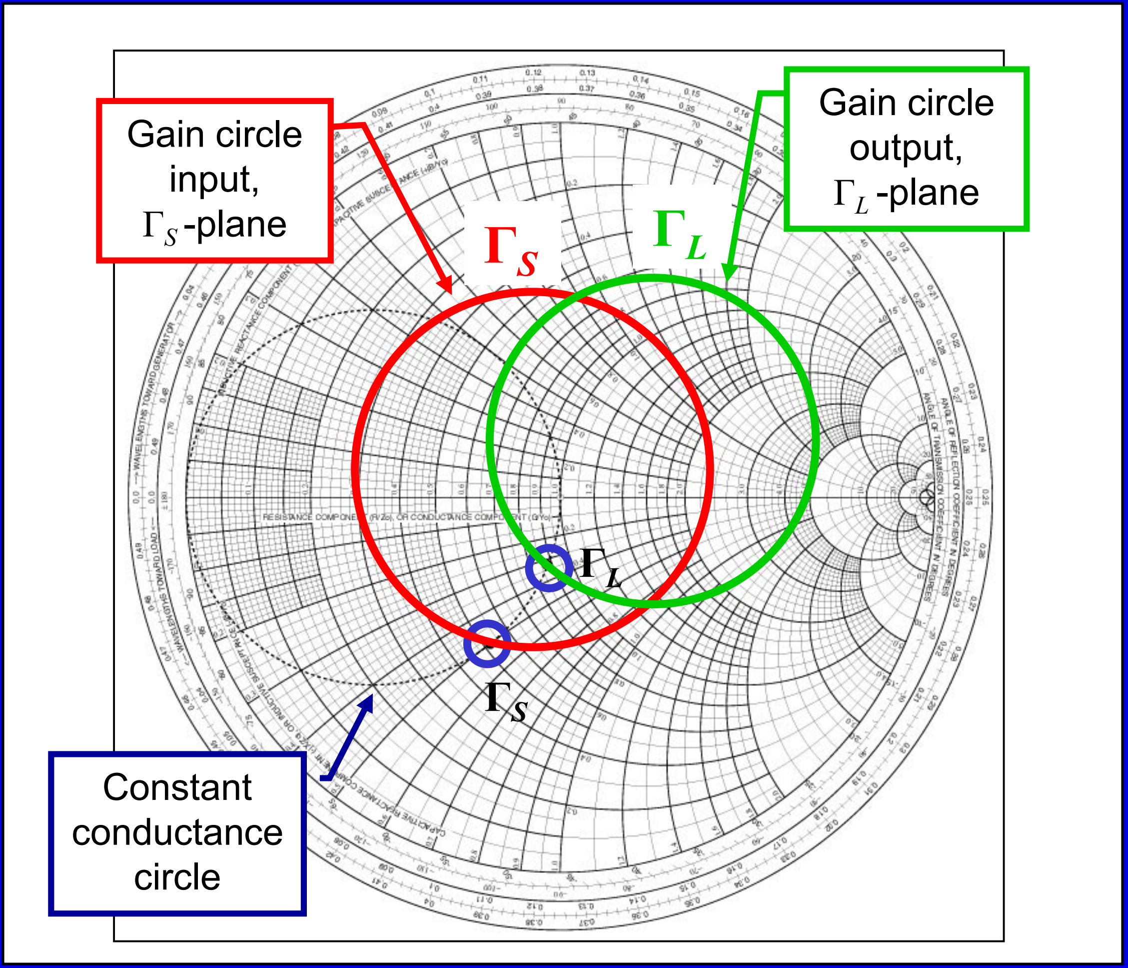
$$\text{radius: } r_S = \frac{\sqrt{1-g_S}\left(1-|S_{11}|^2\right)}{1-|S_{11}|^2\left(1-g_S\right)}$$

$$\text{centre: } \Gamma_{SO} = \frac{g_S|S_{11}|}{1-|S_{11}|^2\left(1-g_S\right)} \cdot e^{j\arg\left(S_{11}^*\right)}$$

Output:

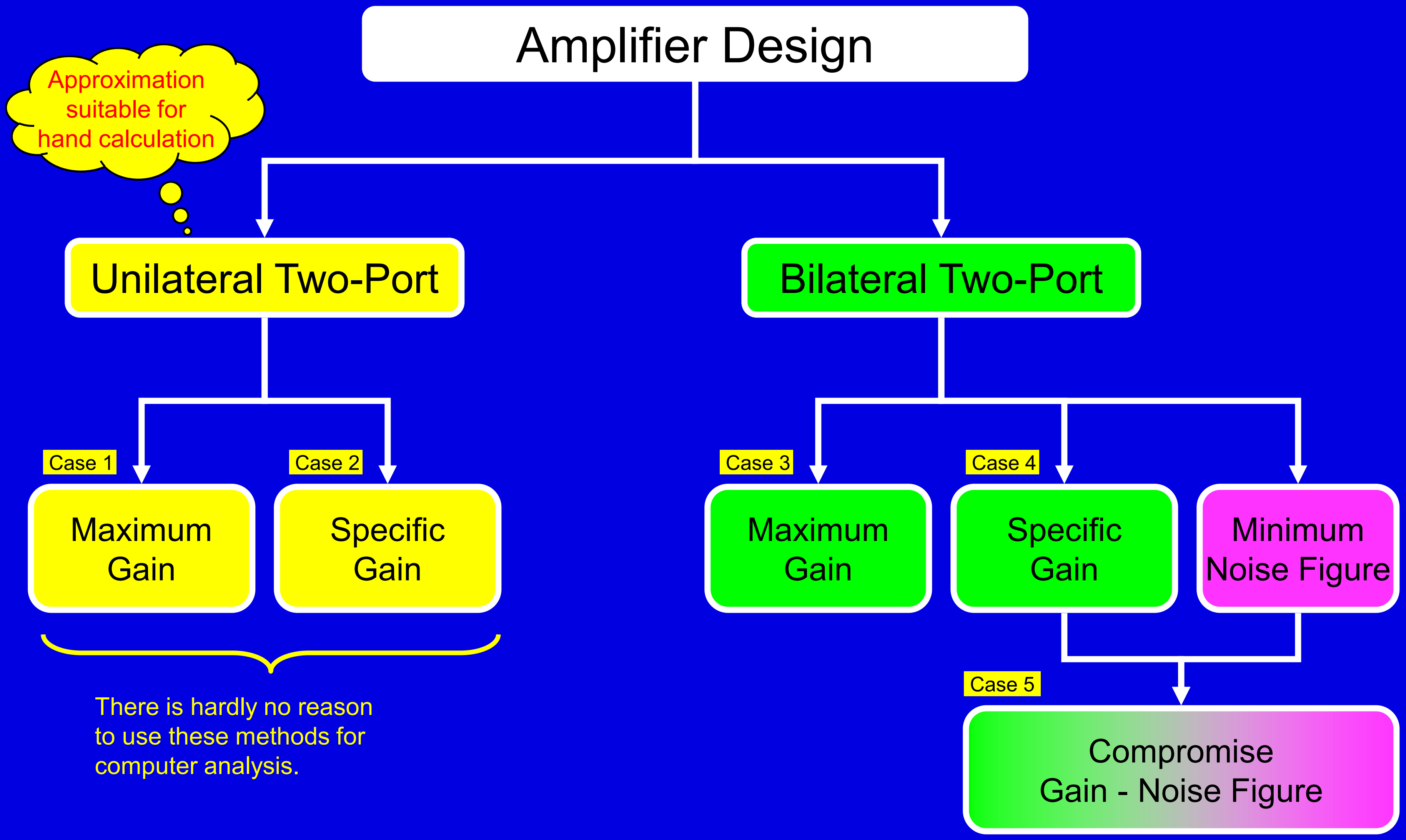
$$\text{radius: } r_L = \frac{\sqrt{1-g_L}\left(1-|S_{22}|^2\right)}{1-|S_{22}|^2\left(1-g_L\right)}$$

$$\text{centre: } \Gamma_{LO} = \frac{g_L|S_{22}|}{1-|S_{22}|^2\left(1-g_L\right)} \cdot e^{j\arg\left(S_{22}^*\right)}$$



How do you select "smart" Γ_S and Γ_L?

Design Cases - Gain and Noise Figure



Case 3

Bilateral Two-Port, Maximum Gain

- Maximum gain is achieved when $\Gamma_S = \Gamma_{IN}^*$ and $\Gamma_L = \Gamma_{OUT}^*$

  i.e. apply conjugate match to both input and output

$$\Gamma_S = \left(S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L}\right)^* \quad \text{and} \quad \Gamma_L = \left(S_{22} + S_{21}S_{12}\frac{\Gamma_S}{1 - S_{11}\Gamma_S}\right)^*$$

- These equations need to be solved simultaneously, that's why it's called "simultaneous conjugate match"

- Explicit solution:

$$\Gamma_{SM} = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \quad \text{and} \quad \Gamma_{LM} = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \quad \text{where} \quad \begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta S_{22}^* \\ C_2 = S_{22} - \Delta S_{11}^* \end{cases}$$

- NOTE! The solution only exists when the two-port is unconditionally stable, i.e. $|\Delta| < 1$ and $K > 1$!

Bilateral Two-Port

- At "simultaneous conjugate match" the maximum transducer gain is:

$$G_{TM} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right)$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \text{ and } K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

- With K set to 1 the quantity Maximum Stable Gain is derived:

$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

- For conditionally stable two-port, the stability factor K may be altered by resistive loading of the input or output without changing the ratio $|S_{21}|/|S_{12}|$.
- But for a conditionally stable two-port
  - it doesn't make any sense to the quantity "maximum transducer gain" and
  - the simultaneous conjugate match doesn't have any solution.
- Therefore, the method changes to case 4 when there is conditional stability.

Bilateral Two-Port

- The procedure will be like this:

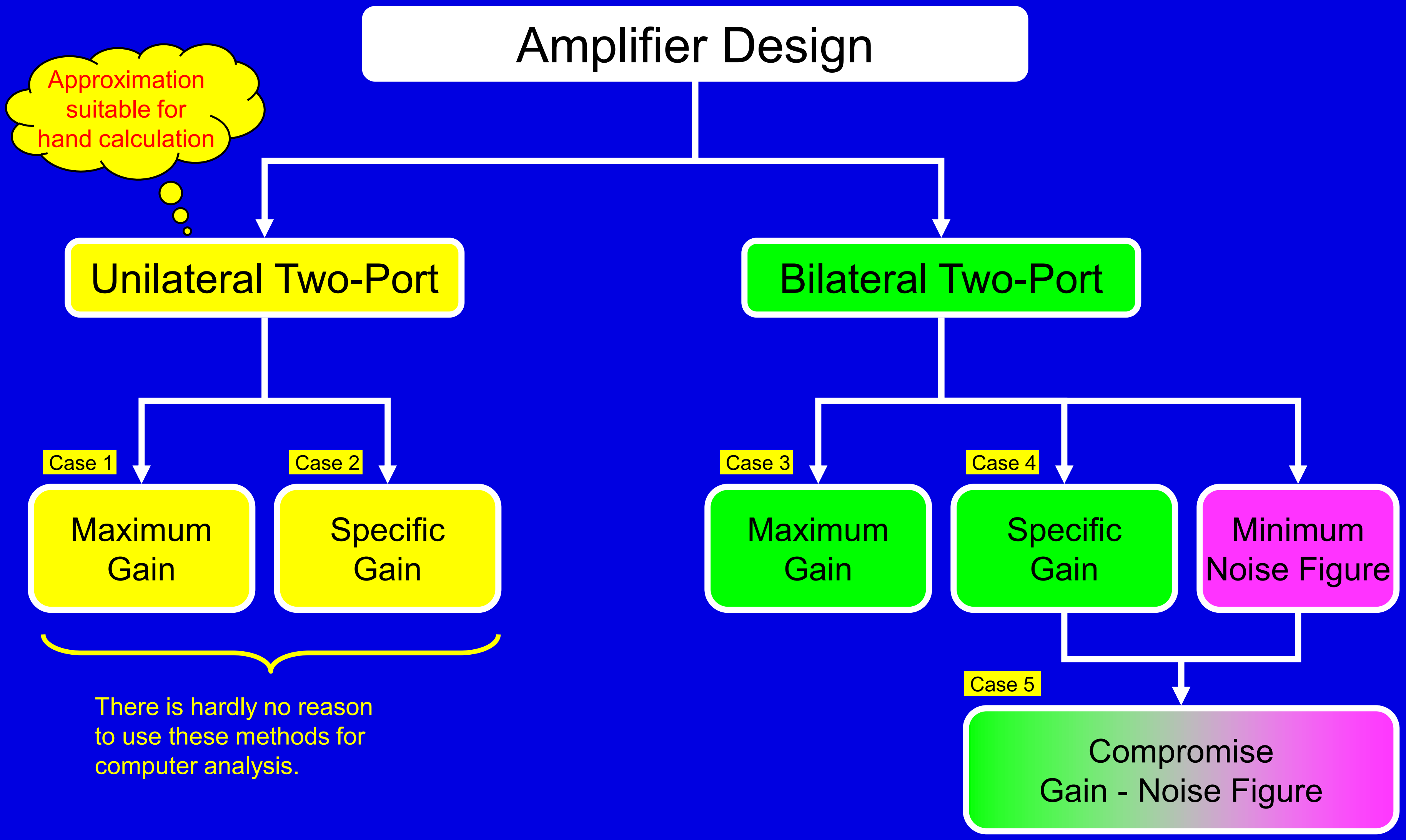
1. Calculate the "maximum stable gain": $$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

2. Back off a few dB or so to set a safety margin.

3. Use the reduced gain as specific gain and design according to case 4.

- For conditionally stable two-port
  - may strictly any arbitrary gain be selected
  - but as the gain increases towards G_{MSG}, the risk for self-oscillation escalates
  - G_{MSG} is in this sense the absolute maximum level.

Design Cases - Gain and Noise Figure



Case 4

Bilateral Two-Port, Specific Gain

- In the unilateral case it was possible to handle the input and output ports separately.
- BUT at the bilateral case the conditions at the input port depends on the load and vice versa.

$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L} \qquad \Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S}$$

Is it possible to disengage the ports from each other?

Yes it is:
Assume conjugate match at one of the ports and the other is mismatched to obtain the specified gain (G_T)!

- Then

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{out}\Gamma_L|^2} = \{\Gamma_L = \Gamma_{out}^*\} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1}{1 - |\Gamma_{out}|^2} = G_A$$

Case 4

(BREAK)

Bilateral Two-Port, Specific Gain

Assume conjugate match at one port and mismatch is applied to the other!

If a mismatch is wanted at the output:

1. use "operating gain"

$$G_P = \frac{P_L}{P_{IN}} = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

2. apply mismatch at the output so that $G_P = G_T$ and solve Γ_L
3. conjugate match the input (Γ_{in} known) then $G_P = P_L/P_{IN} = P_L/P_{AVS} = G_T$

$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1-S_{22}\Gamma_L}$$

If a mismatch is wanted at the input:

1. use "available gain"

$$G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1}{1-|\Gamma_{out}|^2}$$

2. apply mismatch at the input so that $G_A = G_T$ and solve Γ_S
3. conjugate match the output (Γ_{out} known) then $G_A = P_{AVN}/P_{AVS} = P_L/P_{AVS} = G_T$

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1-S_{11}\Gamma_S}$$

Case 4

Bilateral Two-Port, Specific Gain

Design by “operating gain”

$$G_P = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

can be written as

$$G_P = |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{\left(1-\left|\frac{S_{11}-\Delta\Gamma_L}{1-S_{22}\Gamma_L}\right|^2\right)|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 g_P$$

Can we affect S_{21}?

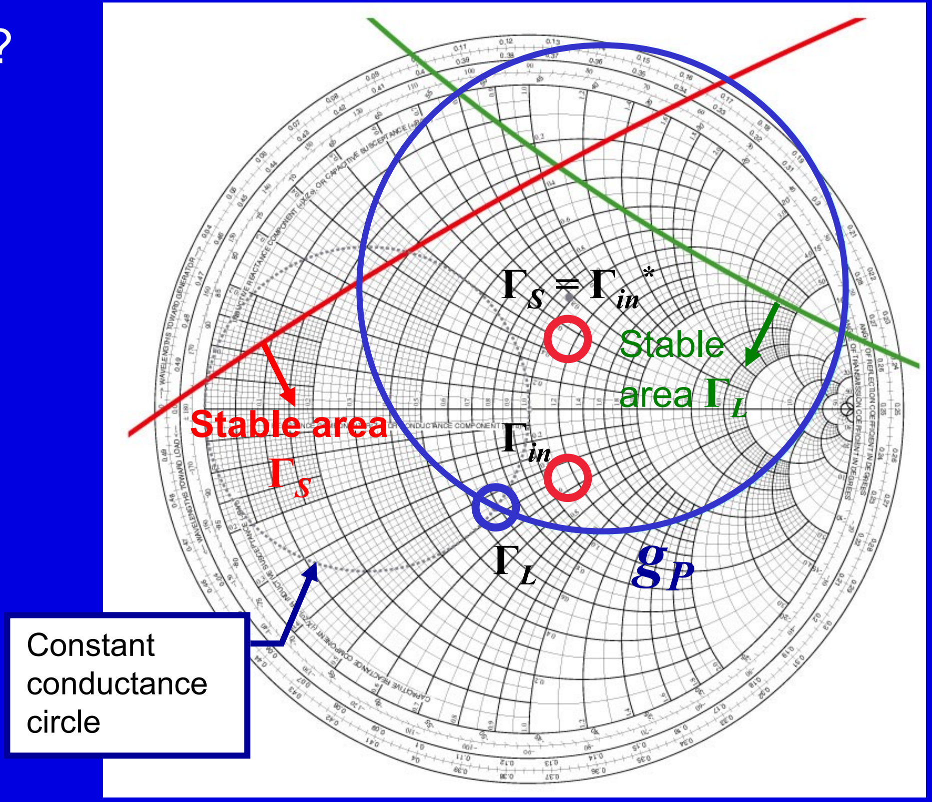
1. S_{21} is known, determine g_P to obtain the wanted gain
2. what Γ_L complies with the selected g_P?
   - there are a number of solutions at a circle in the Γ_L-plane

$$\text{radius: } r_L = \frac{\sqrt{1-2K|S_{12}S_{21}|g_P+|S_{12}S_{21}|^2 g_P^2}}{\left|1-g_P\left(|S_{22}|^2-|\Delta^2|\right)\right|}$$

$$\text{centre: } \Gamma_{LO} = \frac{g_P C_L^*}{1-g_P\left(|S_{22}|^2-|\Delta^2|\right)}$$

$$\text{where } C_L = S_{22}-\Delta S_{11}^*$$

3. select a “smart” Γ_L!
4. calculate Γ_{in} and conjugate match the input



Case 4

Bilateral Two-Port, Specific Gain

Design by “available gain”

$$G_A = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1}{1-|\Gamma_{out}|^2}$$

can be written as

$$G_A = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{\left(1-\left|\frac{S_{22}-\Delta\Gamma_S}{1-S_{11}\Gamma_1}\right|\right)|1-S_{11}\Gamma_S|^2} = |S_{21}|^2 g_A$$

1. S_{21} is known, determine g_A to obtain the wanted gain
2. what Γ_S complies with the selected g_A?

- there are a number of solutions at a circle in the Γ_S-plane

$$\text{radius: } r_S = \frac{\sqrt{1-2K|S_{12}S_{21}|g_A+|S_{12}S_{21}|^2 g_A^2}}{\left|1-g_A\left(|S_{11}|^2-|\Delta^2|\right)\right|}$$

$$\text{centre: } \Gamma_{SO} = \frac{g_A C_S^*}{1-g_A\left(|S_{11}|^2-|\Delta^2|\right)}$$

where $C_S = S_{11} - \Delta S_{22}^*$

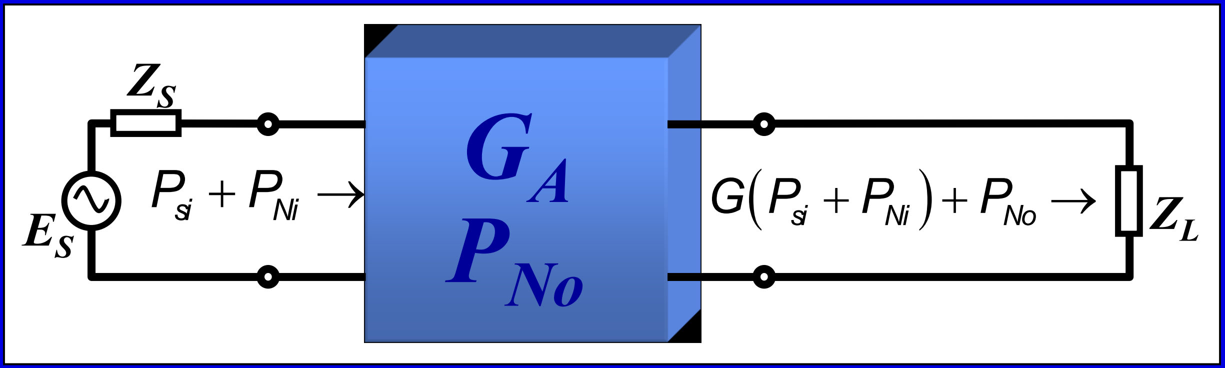
Case 4

Bilateral Two-Port, Specific Gain

summary

- If a two-port is conditionally stable:
  1. Calculate stability circles
  2. Calculate a gain circle to obtain the wanted G_P, (G_A)
  3. Select Γ_L, (Γ_S) at the gain circle in the stable area
  4. Calculate Γ_{IN}, (Γ_{OUT})
  5. Check if conjugate match is possible
     - i.e. if $\Gamma_S = \Gamma^*_{IN}$ ($\Gamma_L = \Gamma^*_{OUT}$) is located in the stable area
     - if not, return to step 3 and make a new choice
     - alternatively lower the demand for gain

Noise in a Two-Port



- The signal-to-noise ratio $SNR = \dfrac{P_s}{P_N}$ is deteriorated due to noise power added by the two-port

- The noise factor (F), linear, denotes the increase of noise by the two-port, assumed a source noise temperature of T_0 = 290 K

$$F = \frac{P_{No} + P_{Ni}G_A}{P_{Ni}G_A}$$ but $$G_A = \frac{P_{So}}{P_{Si}}$$ leads to $$F = \frac{\dfrac{P_{Si}}{P_{Ni}}}{\dfrac{P_{So}}{P_{No} + P_{Ni}G_A}} = \frac{SNR_i}{SNR_o}$$

- Alternatively noise figure (NF), decibel scale, $NF = 10\log_{10}(F)$.

Noise in Cascaded Two-Ports

- The total noise factor (remember: linear scale) is

$$F_{TOT} = \frac{\text{total available noise power at the output}}{\text{available noise at the output originating from the source}}$$

$$F_{TOT} = 1 + \frac{P_{No1}}{P_{Ni} G_{A1}} + \frac{P_{No2}}{P_{Ni} G_{A1} G_{A2}} + \cdots$$

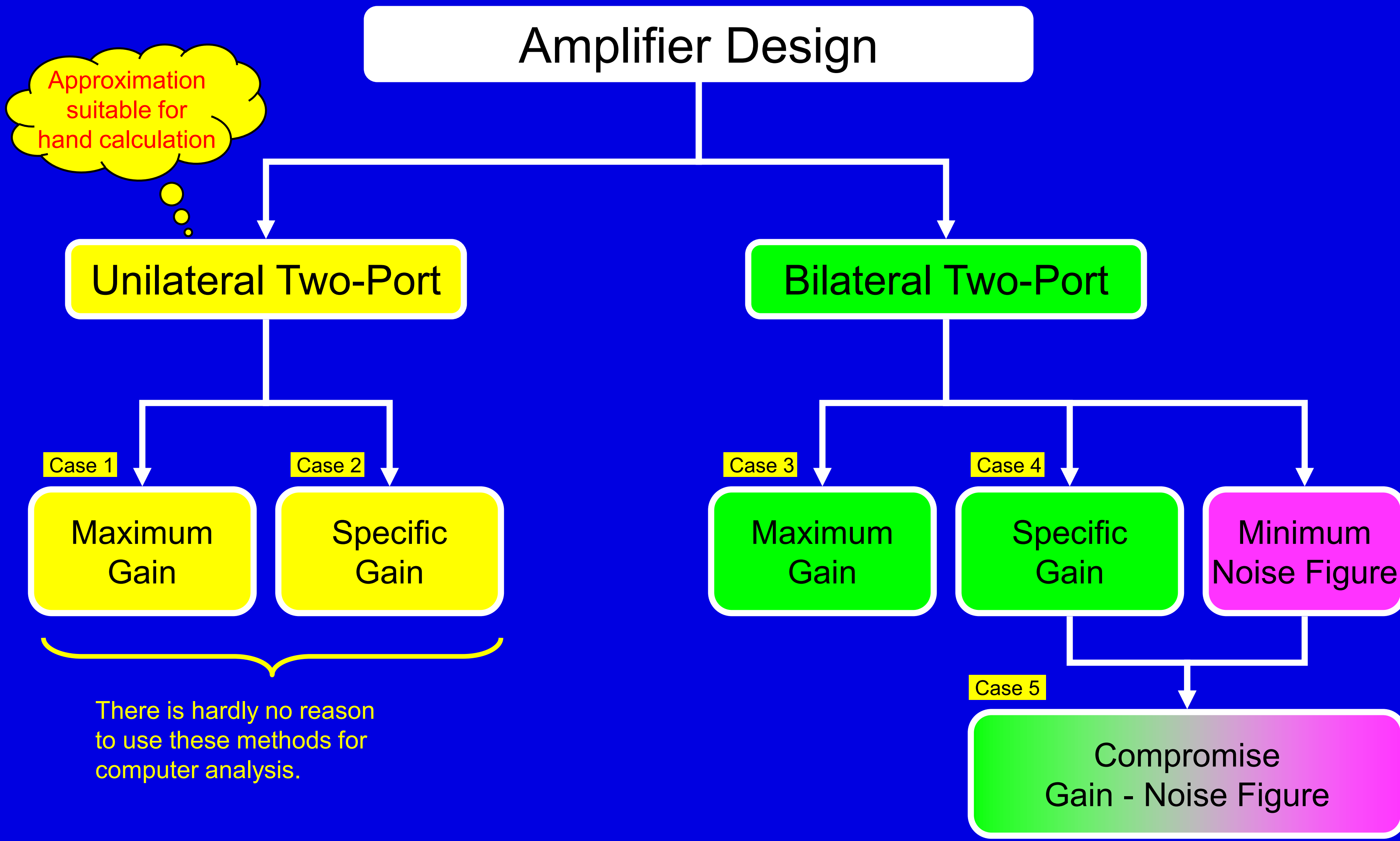
- Friis' formula:

$$F_{TOT} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1} G_{A2}} + \cdots$$

No decibels here!

- NOTE: all variables must be denoted in linear quantities!

Design Cases - Gain and Noise Figure



Case 5

Design of Low Noise Amplifiers (LNA)

- The noise power from a transistor depends on
  - the source impedance
  - the quiescent point (I_C , V_{CE})
- There is an optimum source impedance that gives the minimum noise factor (or figure) for a specified quiescent point
- The source impedance for minimum noise factor does unfortunately NOT coincide with the source impedance for maximum gain

$\rightarrow$ noise match $\Leftrightarrow$ power mismatch at the input

Case 5

Design of Low Noise Amplifiers (LNA)

- The noise factor: $$F = F_{min} + \frac{R_N}{G_S}\left|Y_S - Y_{opt}\right|^2$$ where $$G_S = \mathrm{Re}\left[Y_S\right]$$

F_{min} - the minimum noise factor

R_N - determines how much F increases when Y_S deviates from Y_{opt}

Y_{opt} - the source admittance providing F_{min}

- The noise factor denoted by normalised parameters:

$$F = F_{min} + \frac{r_N}{g_S}\left|y_S - y_{opt}\right|^2$$ where $$g_S = \mathrm{Re}\left[y_S\right]$$

- The noise factor denoted by reflection coefficients:

$$F = F_{min} + \frac{4r_N\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1 - \left|\Gamma_S\right|^2\right)\left|1 + \Gamma_{opt}\right|^2}$$

Case 5

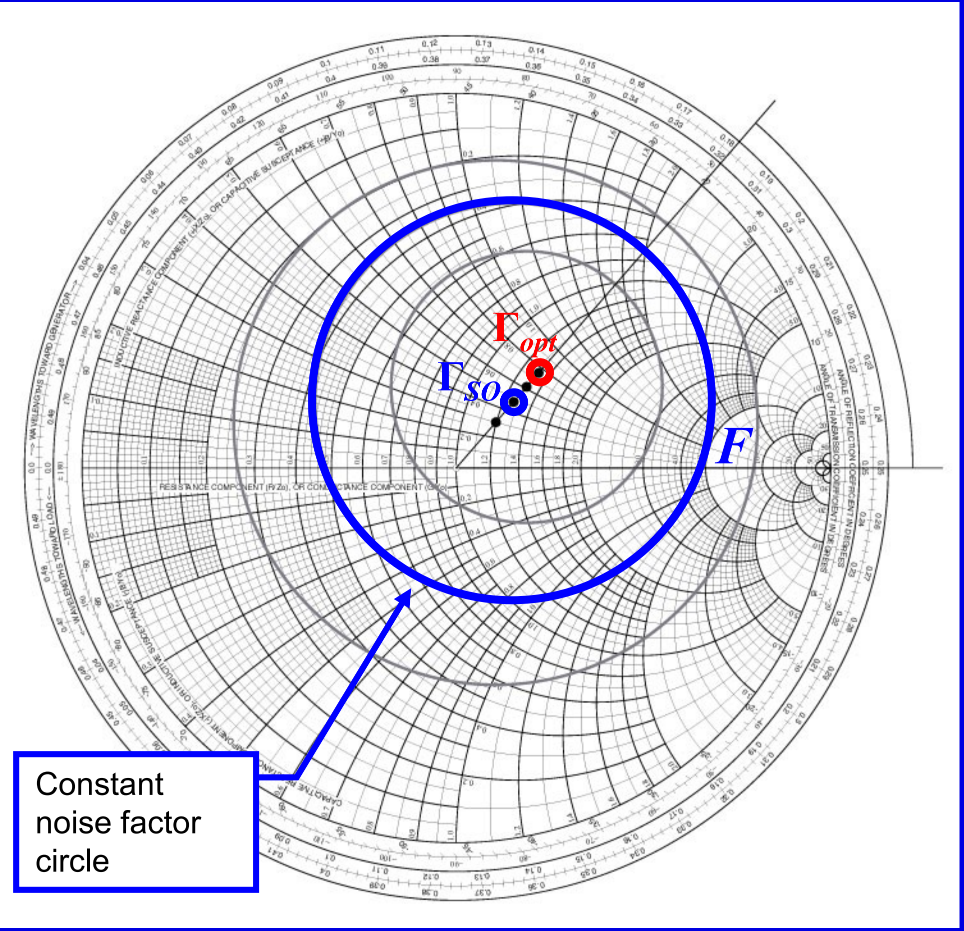
Design of Low Noise Amplifiers (LNA)

- There are a number of Γ_S that provides a specified noise factor
- These are found at circles in the Γ_S-plane

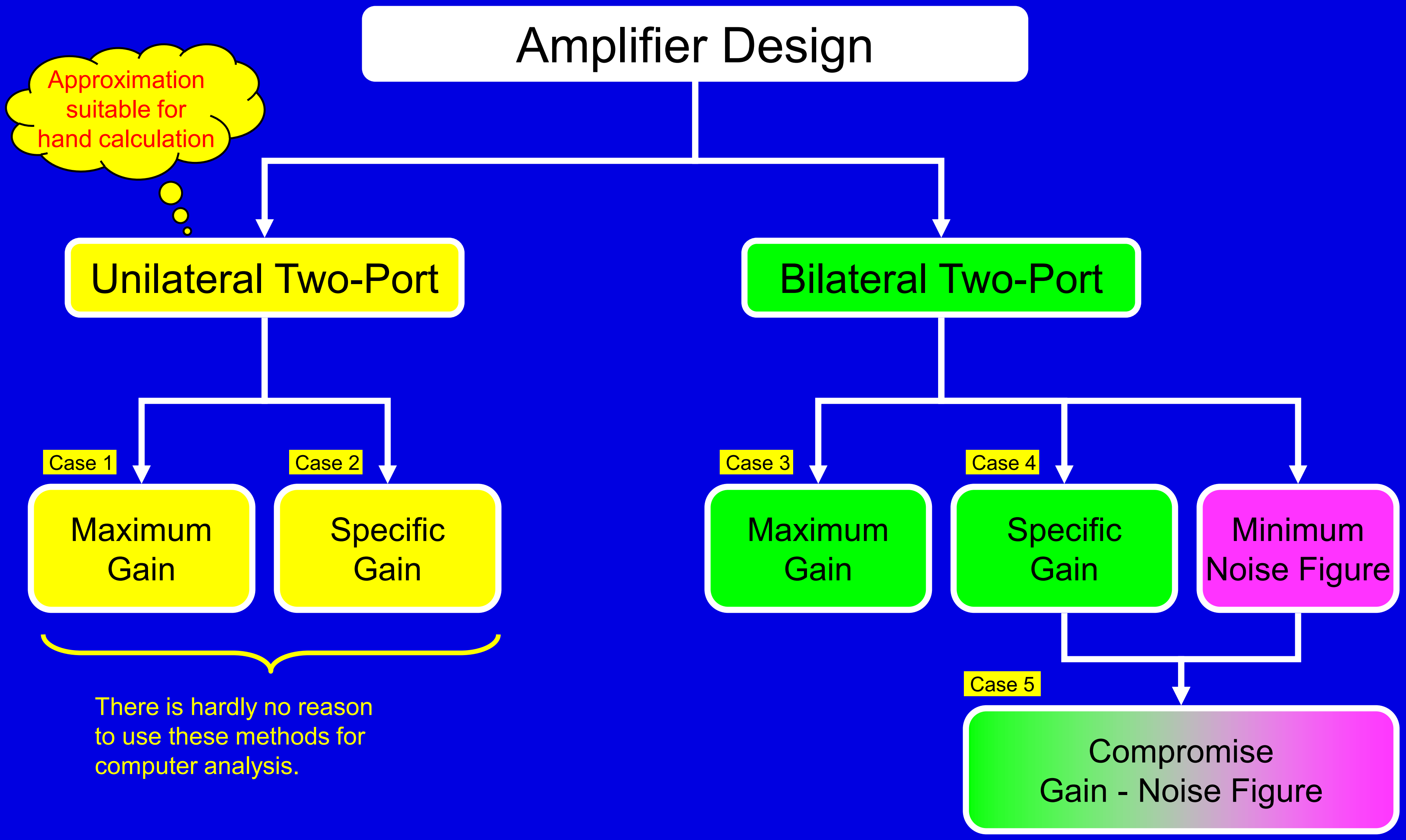
$$\text{radius: } r_S = \frac{\sqrt{N_i^2 + N_i\left(1 - \left|\Gamma_{opt}\right|^2\right)}}{1 - N_i}$$

$$\text{centre: } \Gamma_{SO} = \frac{\Gamma_{opt}}{1 + N_i}$$

$$\text{where } N_i = \left(F - F_{min}\right)\frac{\left|1 + \Gamma_{opt}\right|^2}{4 r_N}$$



Design Cases - Gain and Noise Figure



Summary of Amplifier Design

1. Decide if the transistor is unconditionally stable
2. Calculate stability circles if necessary

G_T

3. Choose a method for specific or maximum gain
4. Assume conjugate match at the input (or the output)
5. Calculate a gain circle to obtain the wanted G_P (or G_A)
6. Select Γ_L (or Γ_S) at the gain circle in the stable area
7. Calculate Γ_{IN} (or Γ_{OUT})

G_T, F

3. Choose the method for specific gain using available gain
4. Assume conjugate match at the output
5. Draw noise and gain circles
6. Select Γ_S in the stable area that provides a suitable compromise of noise and gain
7. Calculate Γ_{OUT}

8. Check if conjugate match is possible
   - i.e. if $\Gamma_S = \Gamma^*_{IN}$ ($\Gamma_L = \Gamma^*_{OUT}$) is located in the stable area
   - if not, return to step 6 and make a new choice
   - alternatively lower the demand for gain and return to step 5
9. Design the matching networks and verify stability at all frequencies of interest