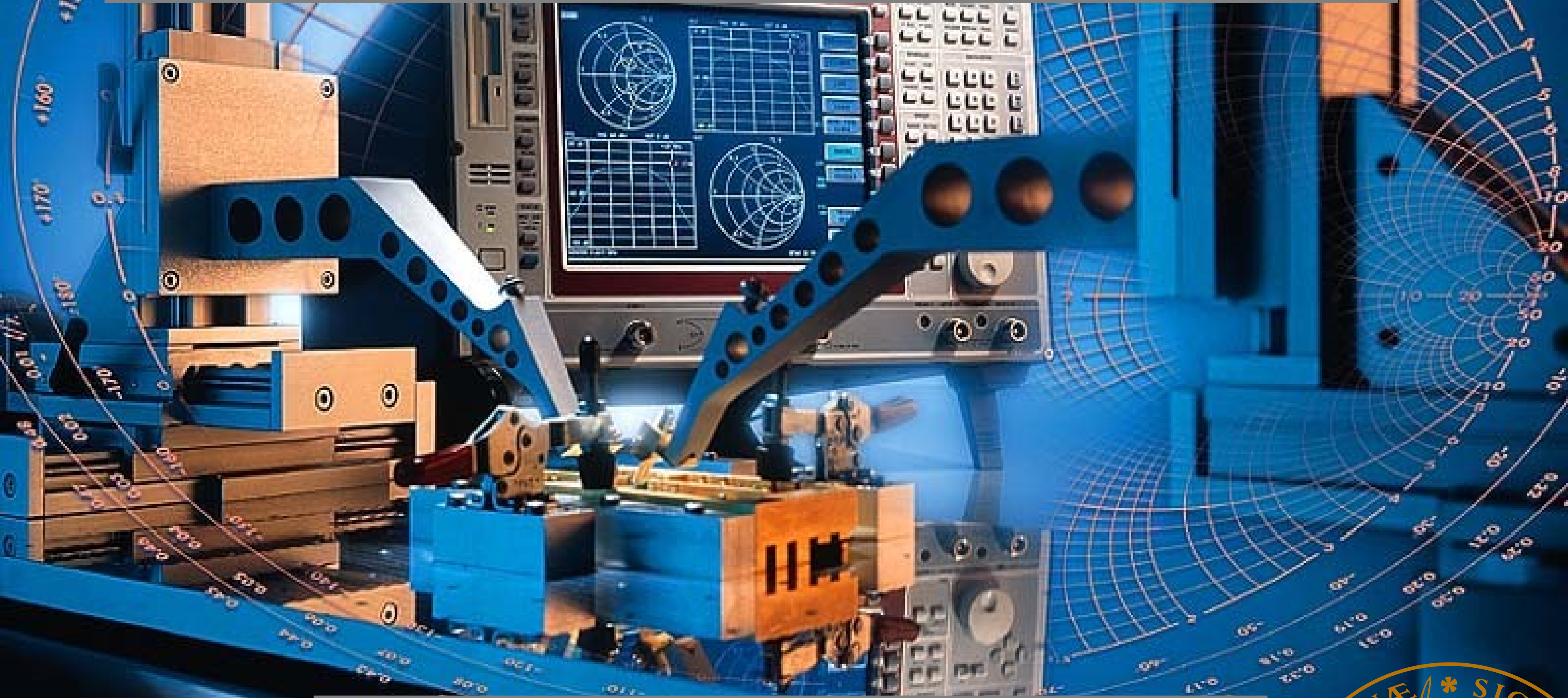
2019-11-25

Lecture 6

RF Amplifier Design



Lars Ohlsson Fhager

Electrical and Information Technology

Application: Millimetre Wave Radar

- Courses at LU
  - ETIN90 – Radar and Remote Sensing
- Work in Lund
  - Acconeer, www.aconeer.com
- Work abroad
  - Google SOLI, www.atap.google.com/soli

Lecture 6

- Amplifier Design
  - S-Parameters
    - Definitions
    - Power Waves
    - Applications
    - Parameter Conversion
    - Signal Flow Graphs
  - Stability Analysis
  - Power Gain Definitions
  - Design Methods
    - Maximum Gain
    - Minimum Noise Figure
  - The Vector Network Analyser (refresher from Introduction to…)

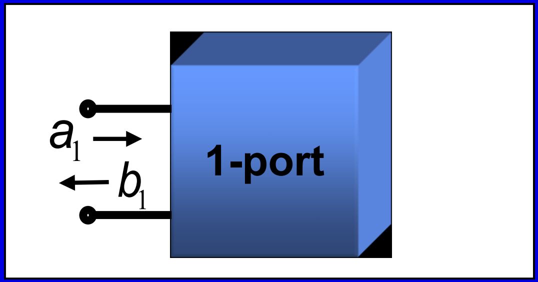
Toughest week
in the course,
hang in there

S-Parameters

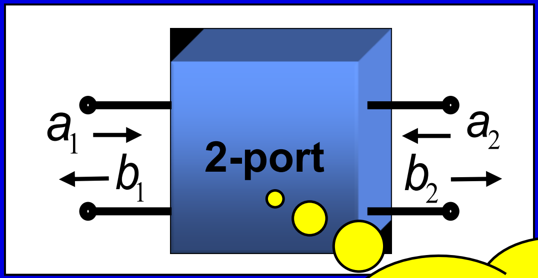
- The circuit is characterized by wave quantities

- 1-port network
  - reflection coefficient

a_x = incident wave
b_x = reflected wave



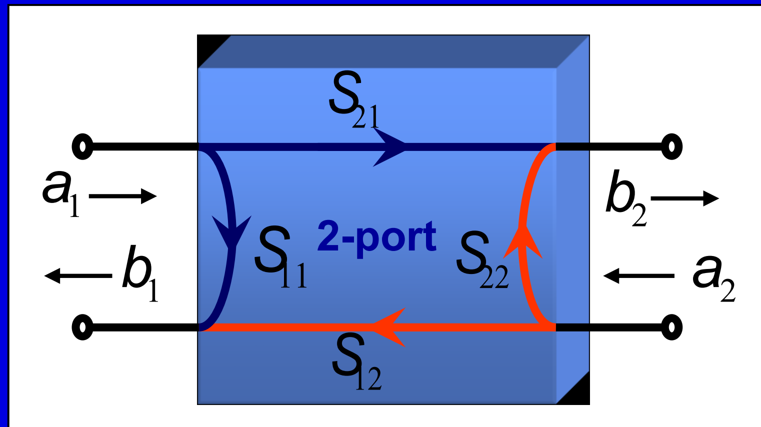
- N-port network
  (the course only deals with 2-ports)
  - S-parameters
    (scattering parameters)
  - T, transmission parameters
  - X, large signal scattering parameters



NOTE!
The definition typically utilizes $Z_0 = 50\ \Omega$ as reference impedance

S-Parameters Definition

- Model:



- Definition:

$$\begin{cases} b_1 = s_{11} \cdot a_1 + s_{12} \cdot a_2 \\ b_2 = s_{21} \cdot a_1 + s_{22} \cdot a_2 \end{cases}$$

or in matrix format:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

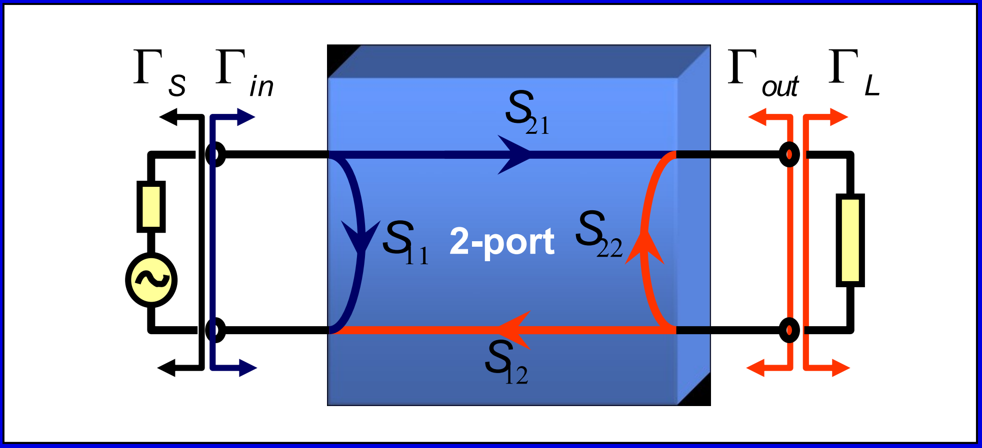
- Boundary conditions:

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

NOTE!
The definition typically utilizes $Z_0 = 50\ \Omega$ as reference impedance

Measurement of S-Parameters



$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11}\Big|_{\Gamma_L = 0}$$

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S} = S_{22}\Big|_{\Gamma_S = 0}$$

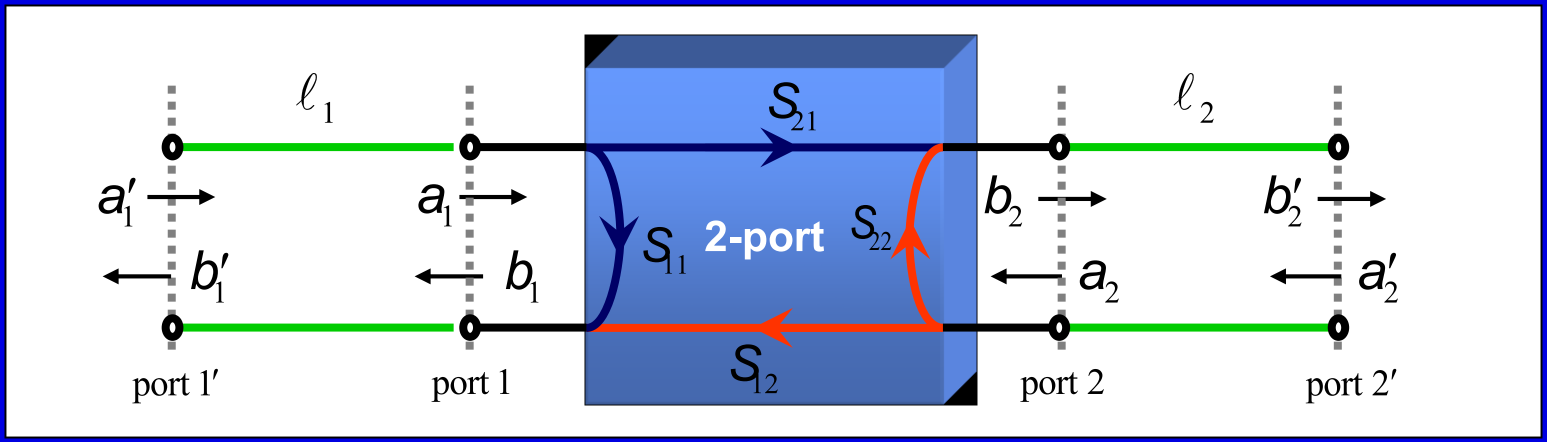
S-parameters are easily measured if the ports are terminated by the reference impedance $Z_0 = 50\ \Omega$ ($\Gamma_L = 0$ respectively $\Gamma_S = 0$)

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2 = 0} \qquad S_{12} = \frac{b_1}{a_2}\Big|_{a_1 = 0}$$

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2 = 0} \qquad S_{22} = \frac{b_2}{a_2}\Big|_{a_1 = 0}$$

Transfer of the Reference Plane

A simple example when you need to compensate for the effect of the test cables:



S-parameters including test cables:

$$\begin{bmatrix} b_1' \\ b_2' \end{bmatrix} = \begin{bmatrix} s_{11}' & s_{12}' \\ s_{21}' & s_{22}' \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

The change of the waves between the reference planes:

$$b_1 = b_1' \exp(\gamma l_1) \qquad a_1 = a_1' \exp(-\gamma l_1)$$

$$b_2 = b_2' \exp(\gamma l_2) \qquad a_2 = a_2' \exp(-\gamma l_2)$$

Substitute:

$$\begin{bmatrix} b_1 \cdot e^{-\gamma \ell_1} \\ b_2 \cdot e^{-\gamma \ell_2} \end{bmatrix} = \begin{bmatrix} s_{11}' & s_{12}' \\ s_{21}' & s_{22}' \end{bmatrix} \begin{bmatrix} a_1 \cdot e^{\gamma \ell_1} \\ a_2 \cdot e^{\gamma \ell_2} \end{bmatrix}$$

Result:

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} = \begin{bmatrix} s_{11}' \cdot e^{2\gamma \ell_1} & s_{12}' \cdot e^{\gamma \ell_1 + \gamma \ell_2} \\ s_{21}' \cdot e^{\gamma \ell_1 + \gamma \ell_2} & s_{22}' \cdot e^{2\gamma \ell_2} \end{bmatrix}$$

Power Waves

- At any port:

$$V = V^+ + V^-, \qquad I = I^+ - I^- = \frac{V^+}{Z_0} - \frac{V^-}{Z_0}, \qquad \Gamma = \frac{V^-}{V^+} = -\frac{I^-}{I^+}$$

- Apply normalized quantities:

$$v = \frac{V}{\sqrt{2Z_0}}, \qquad i = \sqrt{\frac{Z_0}{2}}\, I, \qquad a = \frac{V^+}{\sqrt{2Z_0}}, \qquad b = \frac{V^-}{\sqrt{2Z_0}}$$

where a and b are referred as power waves

- The first equations may now be written:

$$v = a + b, \quad i = a - b \quad \text{and} \quad b = \Gamma a$$

- The power in the waves are:

$$P^+ = \frac{\left|V^+\right|^2}{2Z_0} = |a|^2 \quad \text{and} \quad P^- = \frac{\left|V^-\right|^2}{2Z_0} = |b|^2$$

- The power delivered to the two-port is then:

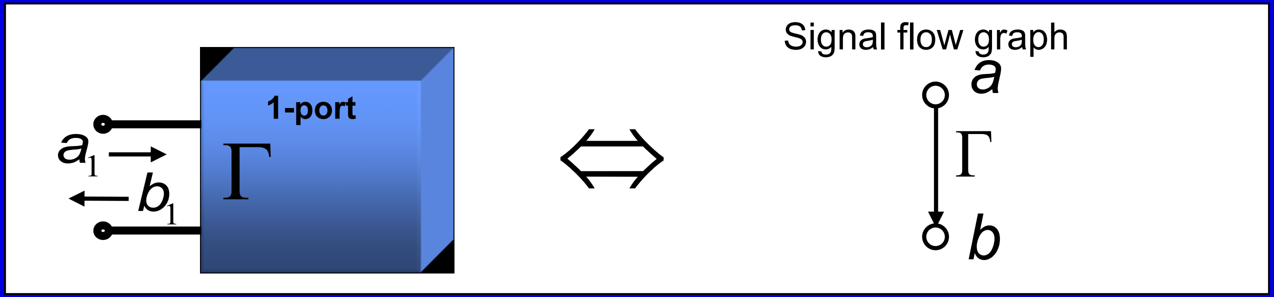
$$P_{IN} = P^+ - P^- = |a|^2 - |b|^2$$

NOTE!
V and I are denoted as *peak values*

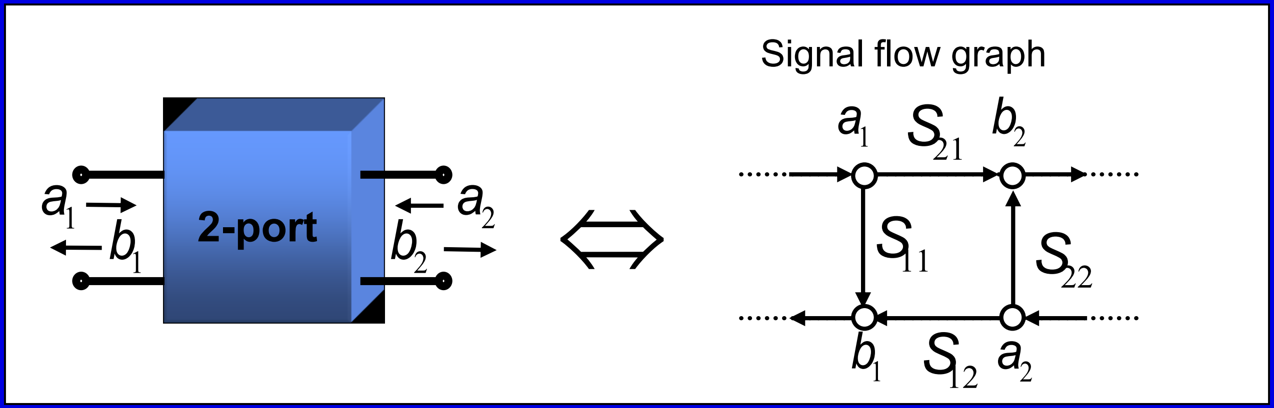
Graphic Representation of Waves in Circuits

- Ordinary circuit diagrams are not effective as they only represents nodes (voltages) and branches (currents).
- Signal flow graphs are useful tools as waves in different directions are separated and represented by nodes.

example:
1-port



2-port



- Methods for systematic analysis by signal flow graphs are depicted in chapter 8.4.3

S-Parameters, Parameter Conversion

- The S-parameters may be converted into y, z or ABCD parameters and vice versa without any loss of information
- Conversion of S-parameters results in normalised y, z and ABCD parameters
- Example: 1) Convert S-parameters to z parameters
  2) Denormalise: $Z = z \cdot 50\Omega$

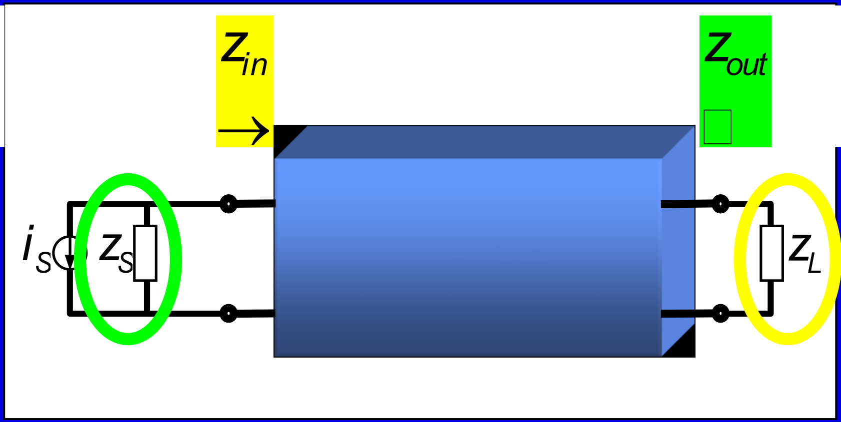
Note!
The definition utilizes
50Ω as
reference impedance

Stability Analysis by S-Parameters

- Definition of unconditional stability:

$$y_{in} = y_{11} - \frac{y_{21}y_{12}}{y_{22} + y_L}$$

$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L}$$



$$y_{out} = y_{22} - \frac{y_{12}y_{21}}{y_{11} + y_S}$$

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S}$$

$$\mathrm{Re}\left[z_{in}\right] > 0 \text{ for any value of } z_L \left(\text{assume } \mathrm{Re}\left[z_L\right] > 0\right)$$

$$\mathrm{Re}\left[z_{out}\right] > 0 \text{ for any value of } z_S \left(\text{assume } \mathrm{Re}\left[z_S\right] > 0\right)$$

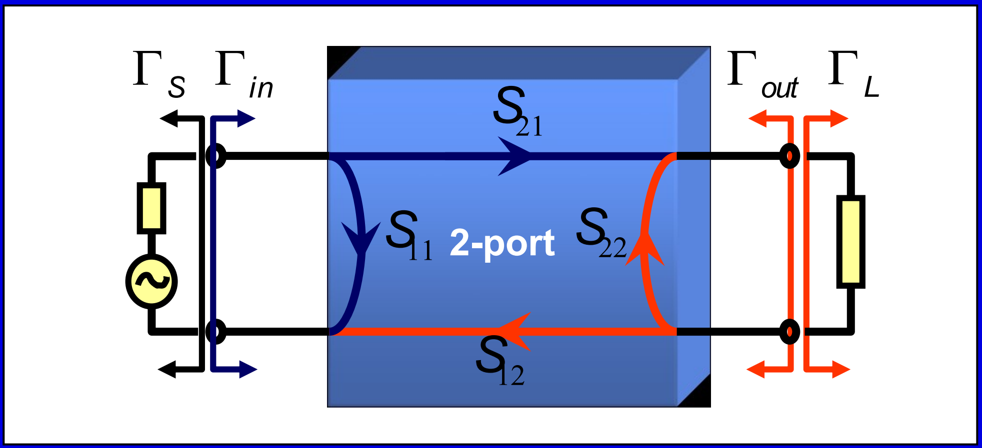
or

$$\left|\Gamma_{in}\right| < 1 \text{ for all } \left|\Gamma_L\right| < 1$$

$$\left|\Gamma_{out}\right| < 1 \text{ for all } \left|\Gamma_S\right| < 1$$

Bilateral and Unilateral Two-Port

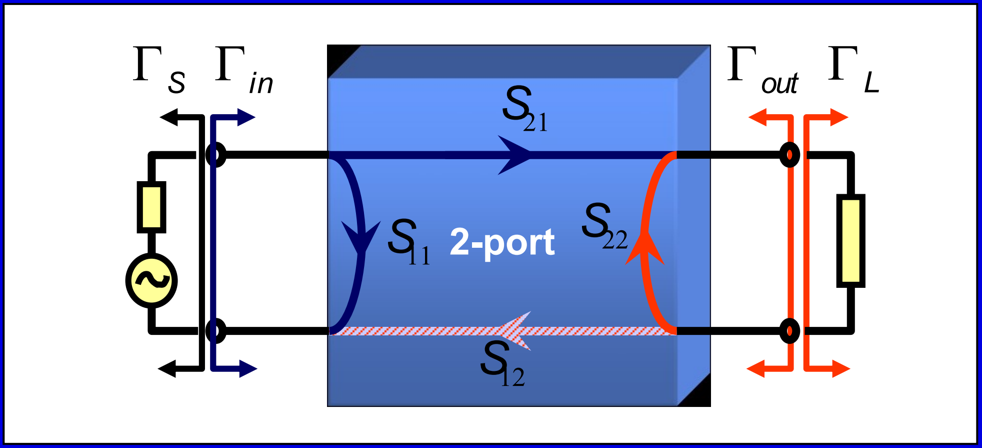
Bilateral two-port



$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S}$$

Unilateral two-port: $S_{12} = 0$ (does these really exist?)



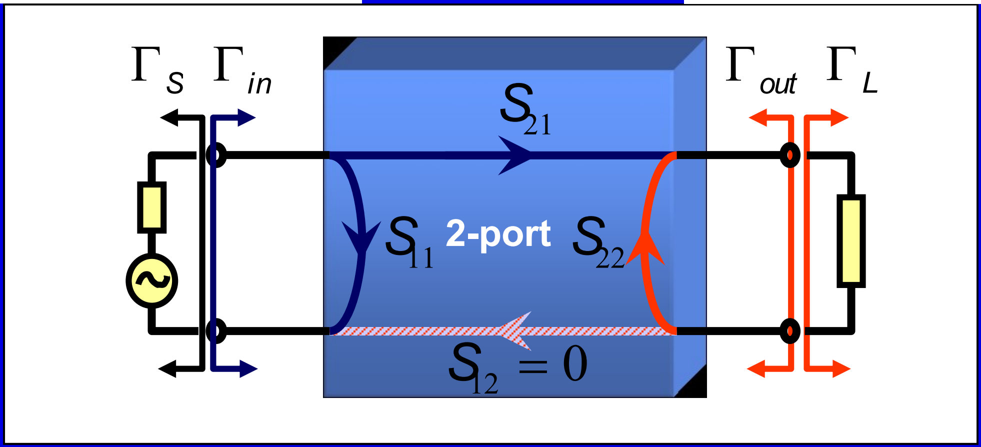
$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11}\Big|_{S_{12}=0}$$

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S} = S_{22}\Big|_{S_{12}=0}$$

Stability Analysis, Unilateral Two-Port

$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11}\Big|_{S_{12}=0}$$

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S} = S_{22}\Big|_{S_{12}=0}$$

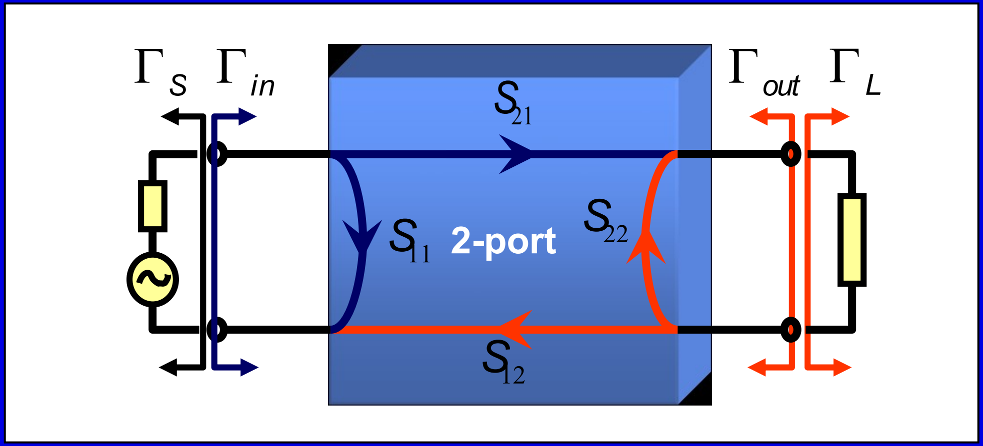


The unilateral two-port ($S_{12} = 0$) is unconditionally stable if

$$\begin{cases} |\Gamma_{in}| < 1 \text{ for all } |\Gamma_L| < 1 \\ |\Gamma_{out}| < 1 \text{ for all } |\Gamma_S| < 1 \end{cases} \quad \text{i.e.} \quad \begin{cases} |\Gamma_{in}| = |S_{11}| < 1 \text{ and} \\ |\Gamma_{out}| = |S_{22}| < 1 \end{cases}$$

Stability Analysis, Bilateral Two-Port

How can we show if a bilateral two-port is unconditionally stable?



$$\begin{cases} |\Gamma_{in}| = \left| S_{11} + S_{12}S_{21}\dfrac{\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1 \\ |\Gamma_{out}| = \left| S_{22} + S_{12}S_{21}\dfrac{\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1 \end{cases} \quad \text{for all } |\Gamma_S| < 1 \text{ and } |\Gamma_L| < 1$$

After an extensive analysis the stability criteria can be reformulated as

$$|\Delta| < 1 \text{ and } K > 1 \quad \text{where} \quad \Delta = S_{11}S_{22} - S_{12}S_{21} \text{ and } K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

Stability Analysis, Bilateral Two-Port (cont.)

(BREAK)

But if $|\Delta| < 1$ and $K > 1$ not are fulfilled, what then?

- There may be some
  - Γ_L that provides $|\Gamma_{in}| < 1$ and
  - Γ_S that provides $|\Gamma_{out}| < 1$
- The two-port is then considered to be conditionally stable
- Where is the boundary for stability?
- Find
  - all Γ_L that gives $|\Gamma_{in}| = 1$ and
  - all Γ_S that gives $|\Gamma_{out}| = 1$

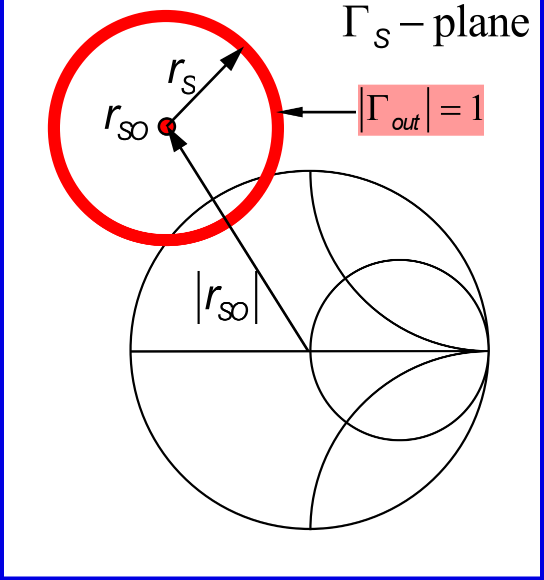
Stability Analysis, Bilateral Two-Port (cont.)

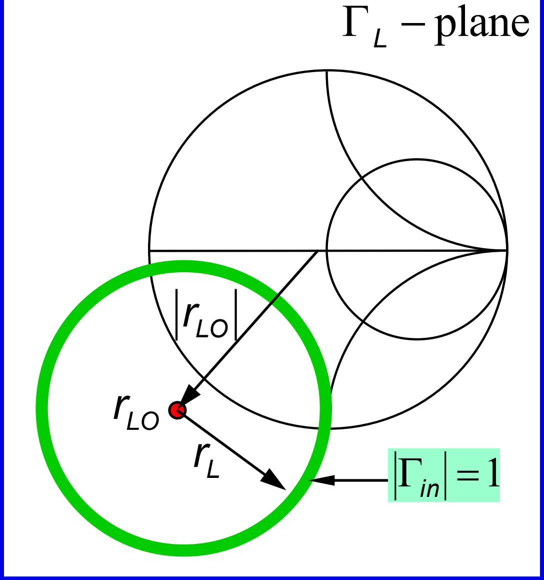
Solve $|\Gamma_{out}| = 1$ with respect to Γ_S and $|\Gamma_{in}| = 1$ with respect to Γ_L

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S}$$

$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1 - S_{22}\Gamma_L}$$

The result is stability circles:





Γ_S -plane:

radius $$r_S = \frac{|S_{12}S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|}$$

centre $$\Gamma_{SO} = \frac{S_{11}^* - \Delta^* S_{22}}{|S_{11}|^2 - |\Delta|^2}$$

Γ_L -plane:

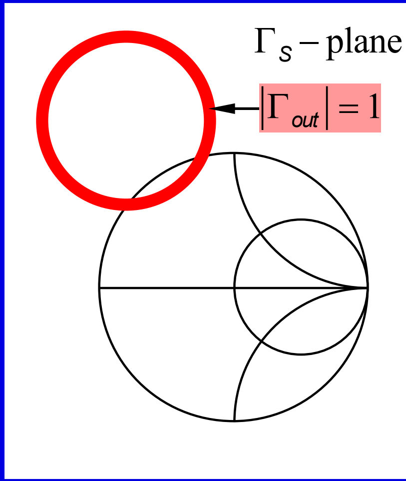
radius $$r_L = \frac{|S_{12}S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|}$$

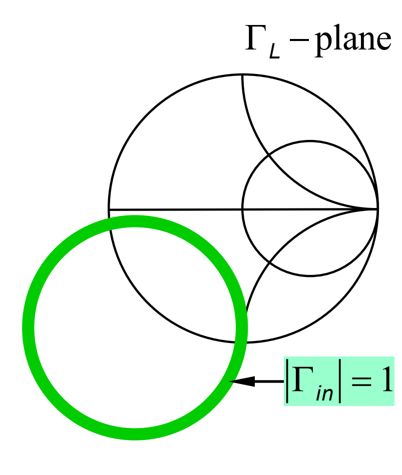
centre $$\Gamma_{LO} = \frac{S_{22}^* - \Delta^* S_{11}}{|S_{22}|^2 - |\Delta|^2}$$

Stability Circles

- The stability circles denotes
  - all Γ_S that equals $|\Gamma_{out}| = 1$ and
  - all Γ_L that equals $|\Gamma_{in}| = 1$

    the circles accordingly shows the boundary for instability...





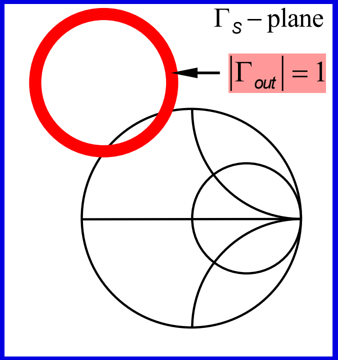
- But which Γ_S and Γ_L implies stability?

  i.e. are the Γ that provides stability found inside or outside respectively stability circle?
- Test at one location!
- It is understood that if one point is part of the stable set, e.g. in the Γ_S-plane, then this holds for all other points belonging to the same set.
- Tip: test at the centre of the Smith chart! Why is this smart?

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1 - S_{11}\Gamma_S} = S_{22}\Big|_{\Gamma_S = 0}$$

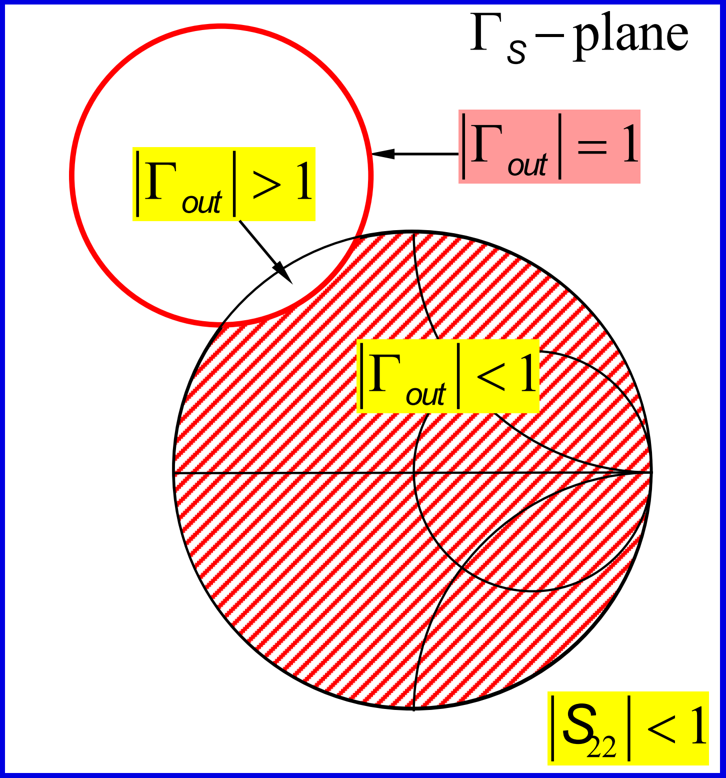
- Perform the same test in the Γ_L-plane.

Stability Circles (cont.)

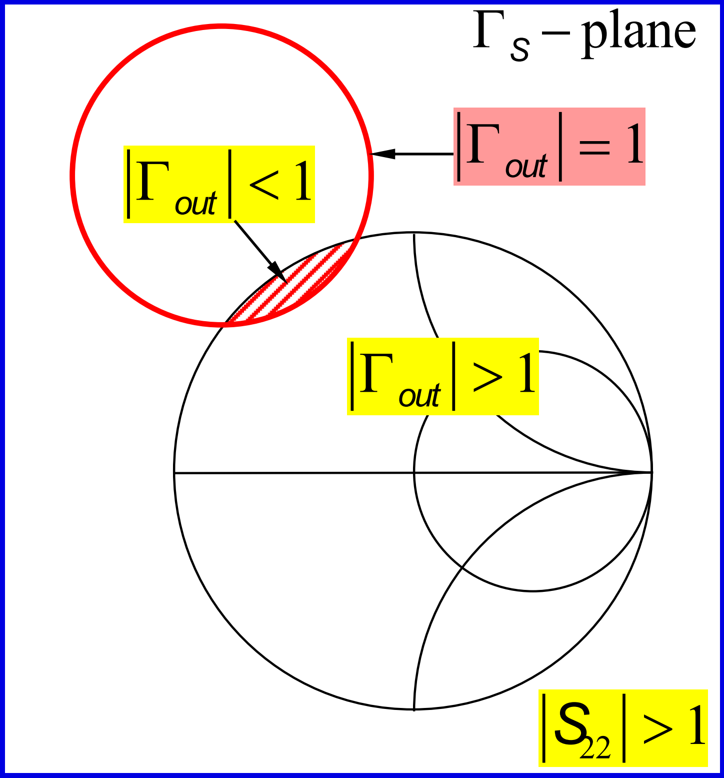


- In which area is the output stable?
- Test at the centre in the Γ_S-plane: $\Gamma_S = 0 \rightarrow \Gamma_{out} = S_{22}$
  - is $|S_{22}|$ greater or less than one?

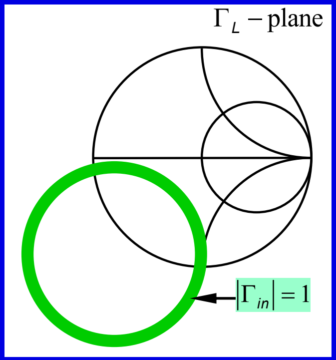
$|S_{22}| < 1 \rightarrow$ stable outside the circle



$|S_{22}| > 1 \rightarrow$ stable inside the circle

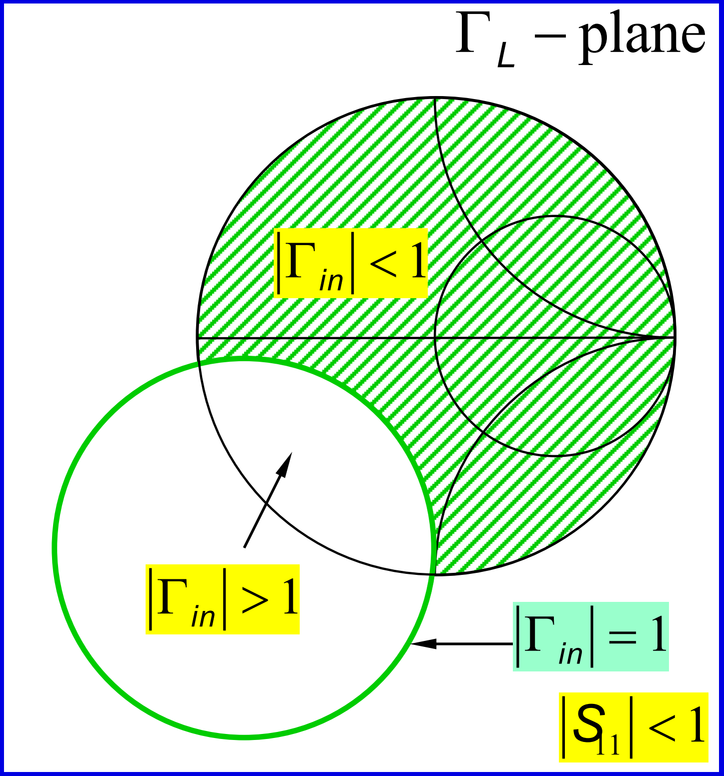


Stability Circles (cont.)

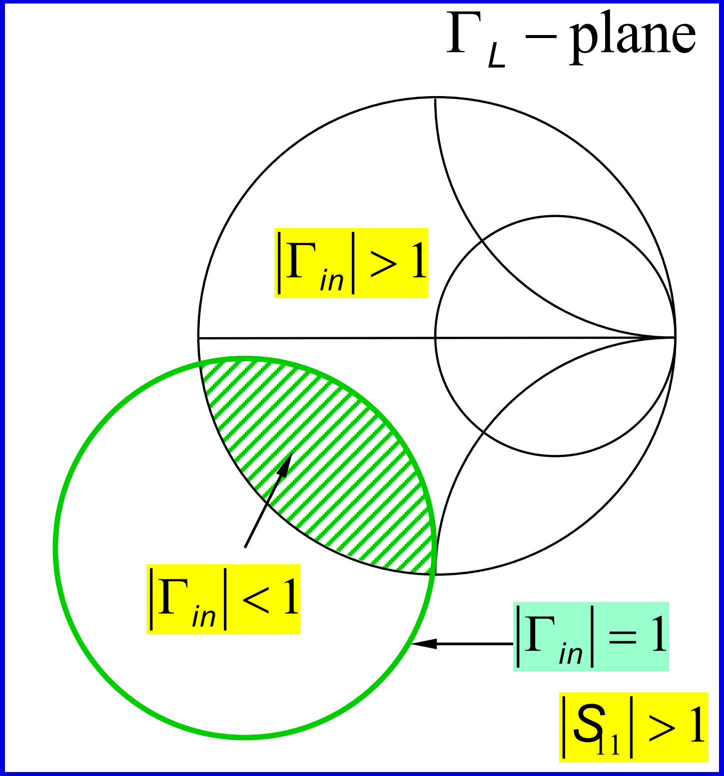


- In which area is the input stable?
- Test at the centre in the Γ_L-plane: $\Gamma_L = 0 \rightarrow \Gamma_{in} = S_{11}$
  - is $|S_{11}|$ greater or less than one?

$|S_{11}| < 1 \rightarrow$ stable outside the circle

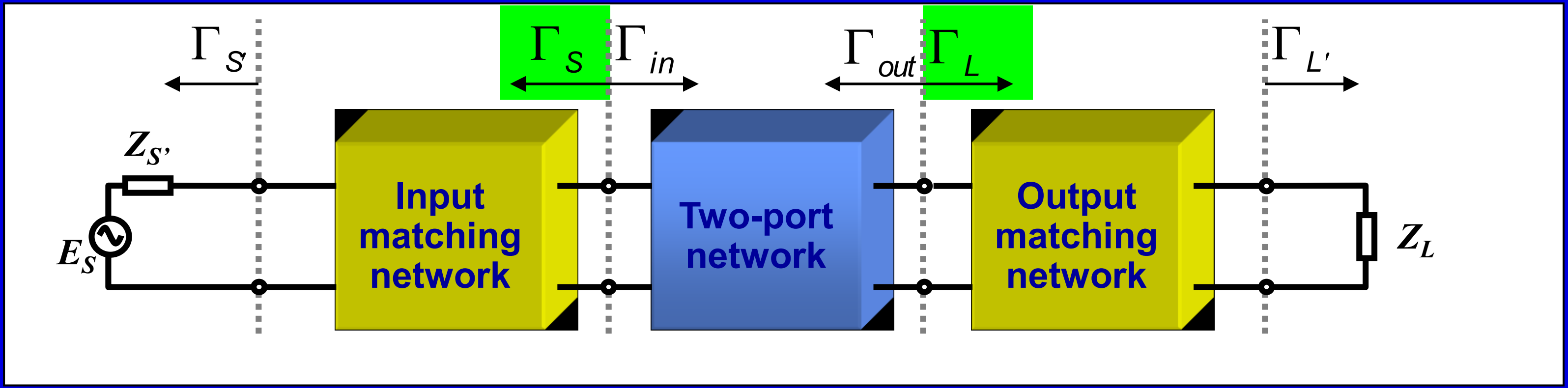


$|S_{11}| > 1 \rightarrow$ stable inside the circle



Amplifier Design

General design case

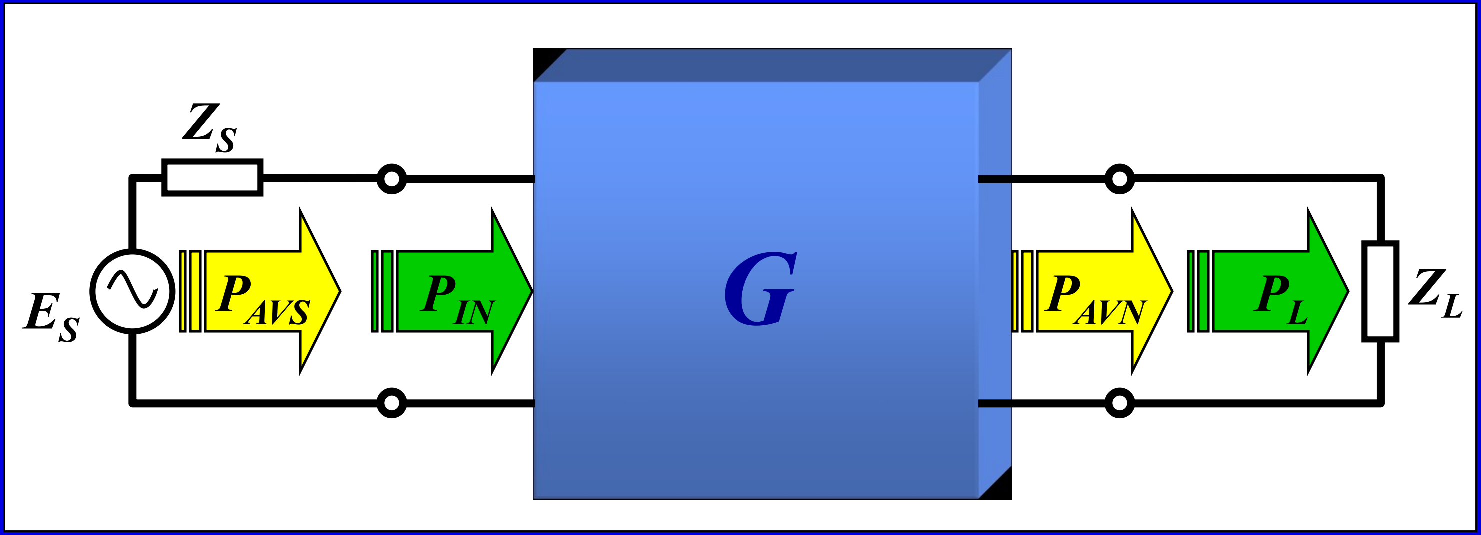


- Given this:
  - two-port (S-parameters) and
  - source $\Gamma_{S'}$ and load $\Gamma_{L'}$

- The stability analysis gives allowed values of Γ_S and Γ_L

- After a proper choice of Γ_S and Γ_L
  the matching networks may be designed

Power Gain Definitions

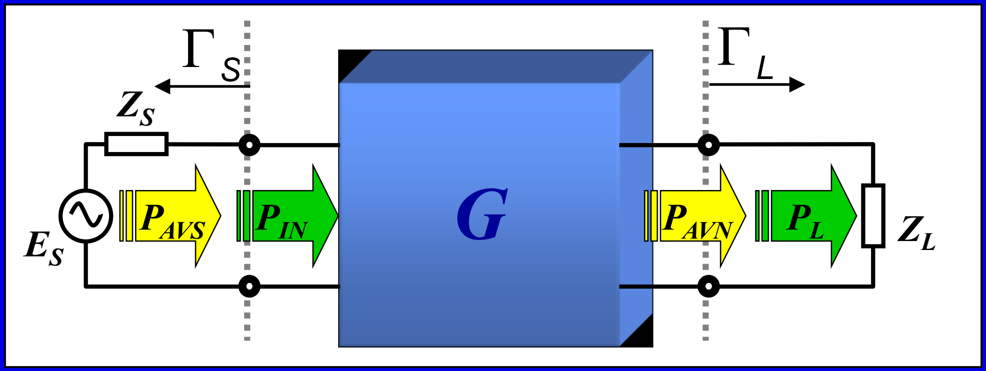


available gain $G_A = \dfrac{P_{AVN}}{P_{AVS}}$

operating gain $G_P = \dfrac{P_L}{P_{IN}}$

transducer gain $G_T = \dfrac{P_L}{P_{AVS}}$

The Gain Expressed by S-Parameters



available gain

$$G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1}{1-|\Gamma_{out}|^2}$$

operating gain

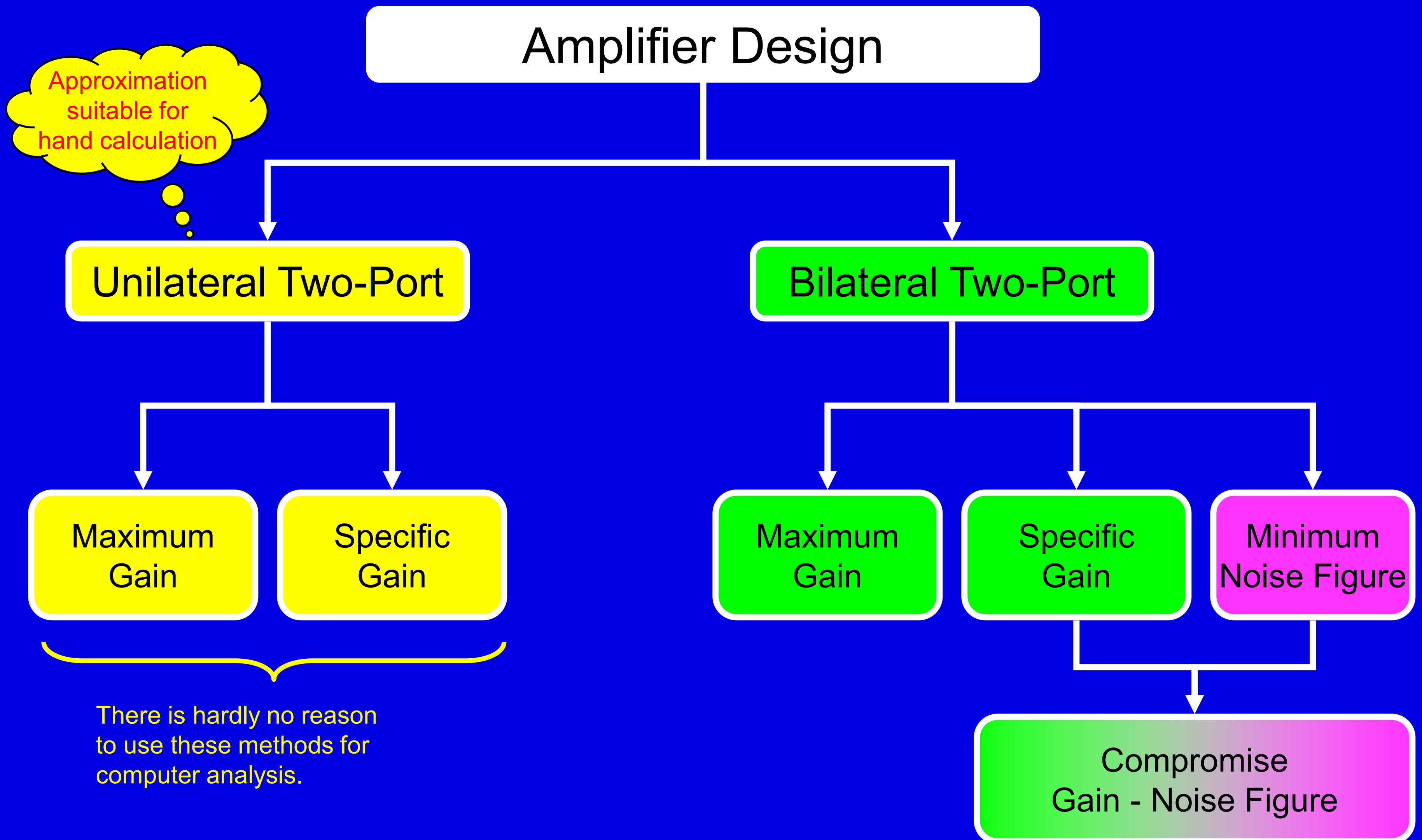
$$G_P = \frac{P_L}{P_{IN}} = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

transducer gain

$$G_T = \frac{P_L}{P_{AVS}} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = $$

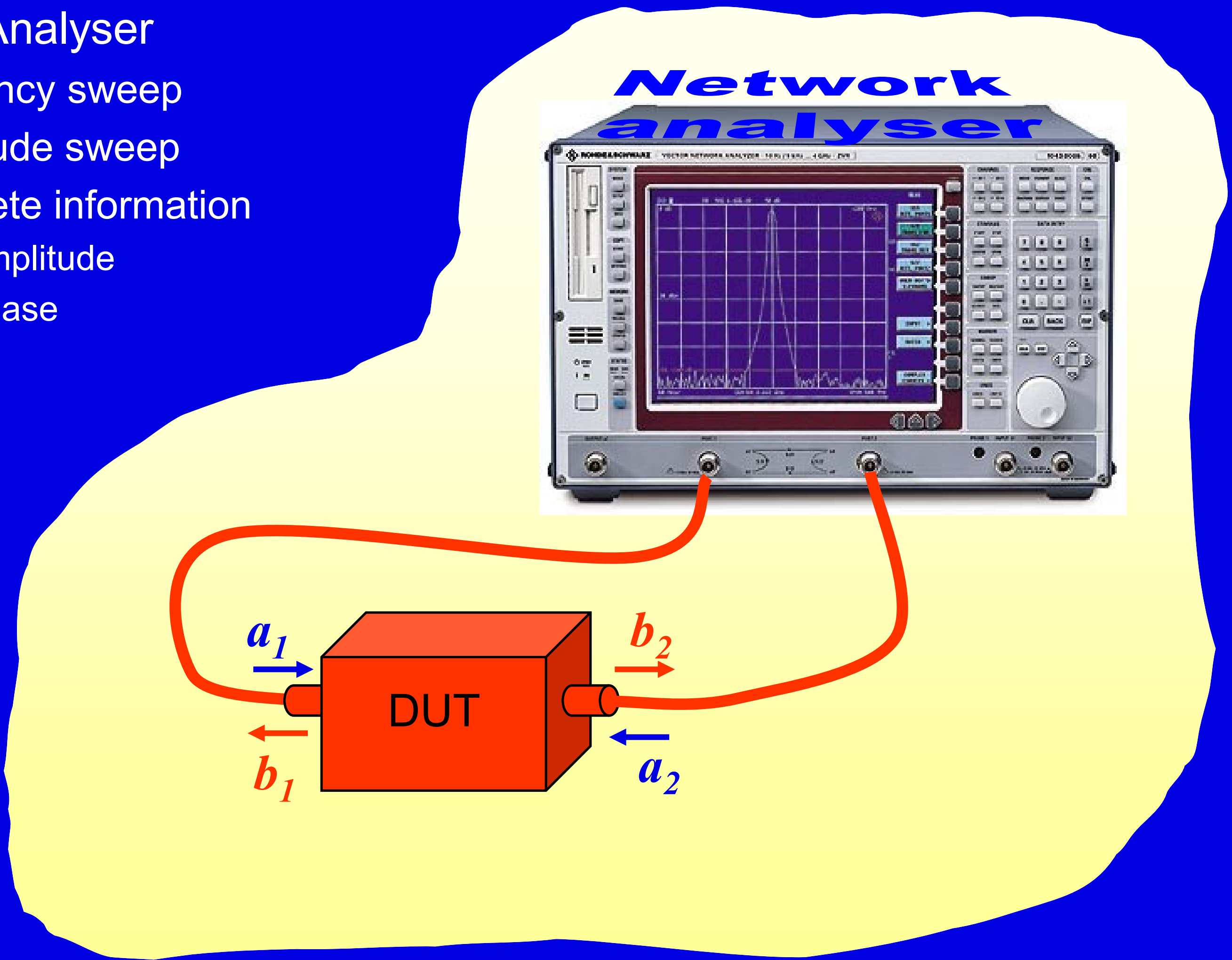
$$= \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

Design Cases - Gain and Noise Figure



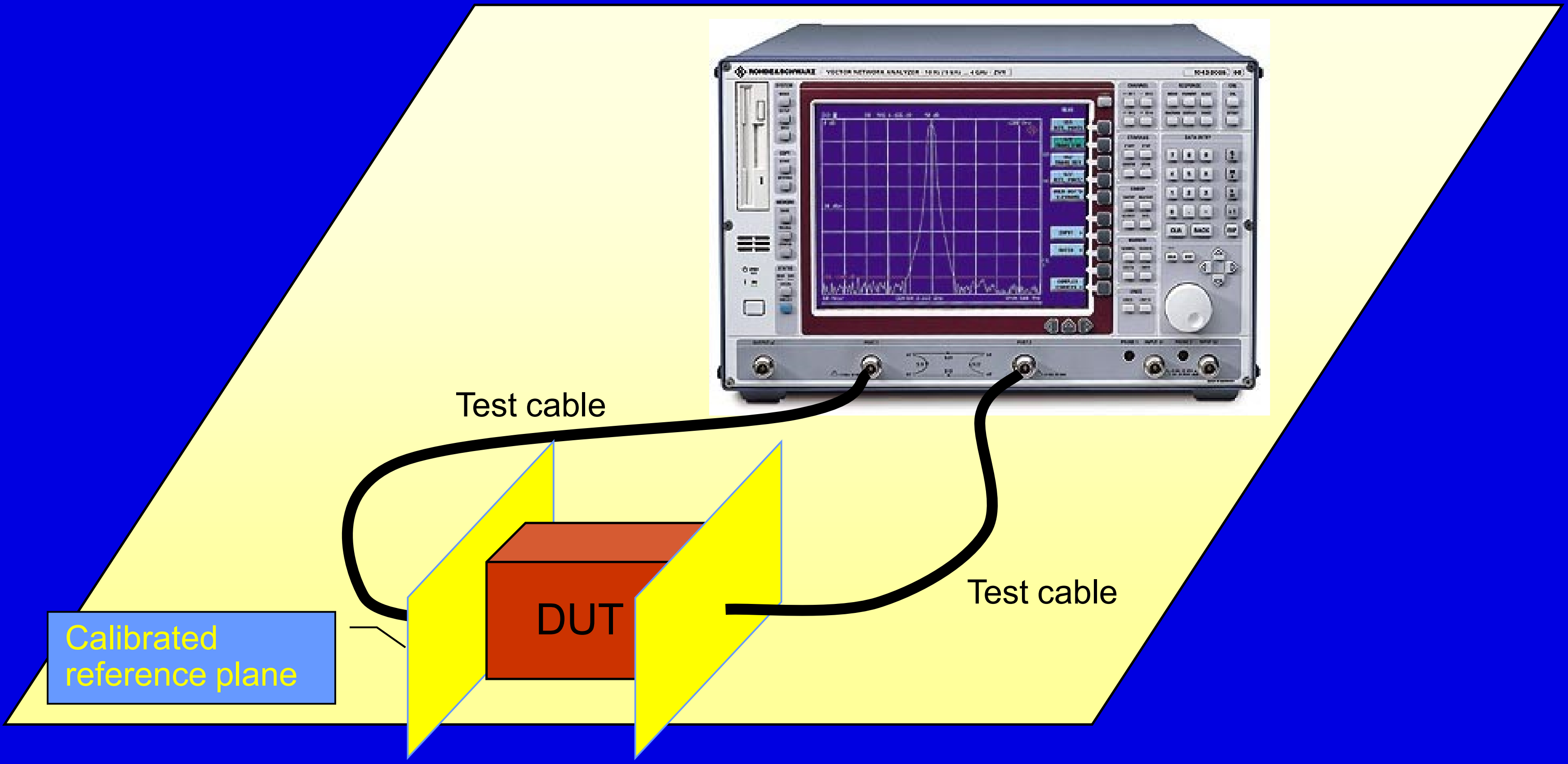
Vector Network Analysis

- Characterising the Device Under Test properties
- Network Analyser
  - frequency sweep
  - amplitude sweep
  - complete information
    - amplitude
    - phase



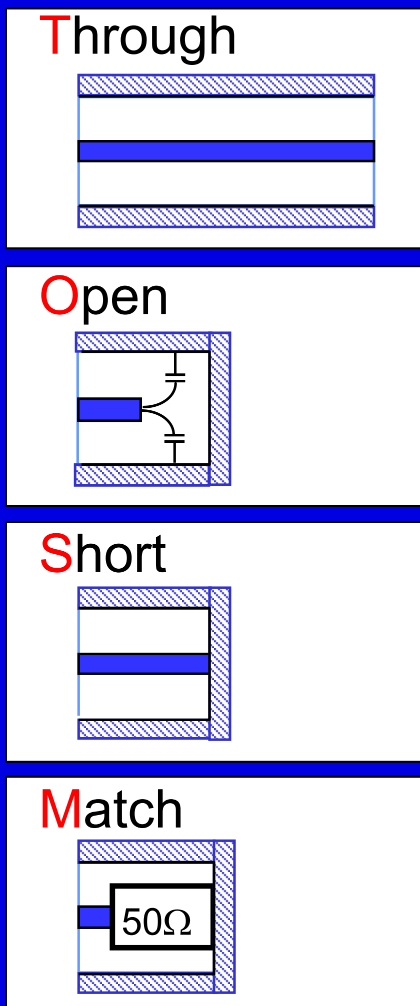
Calibration

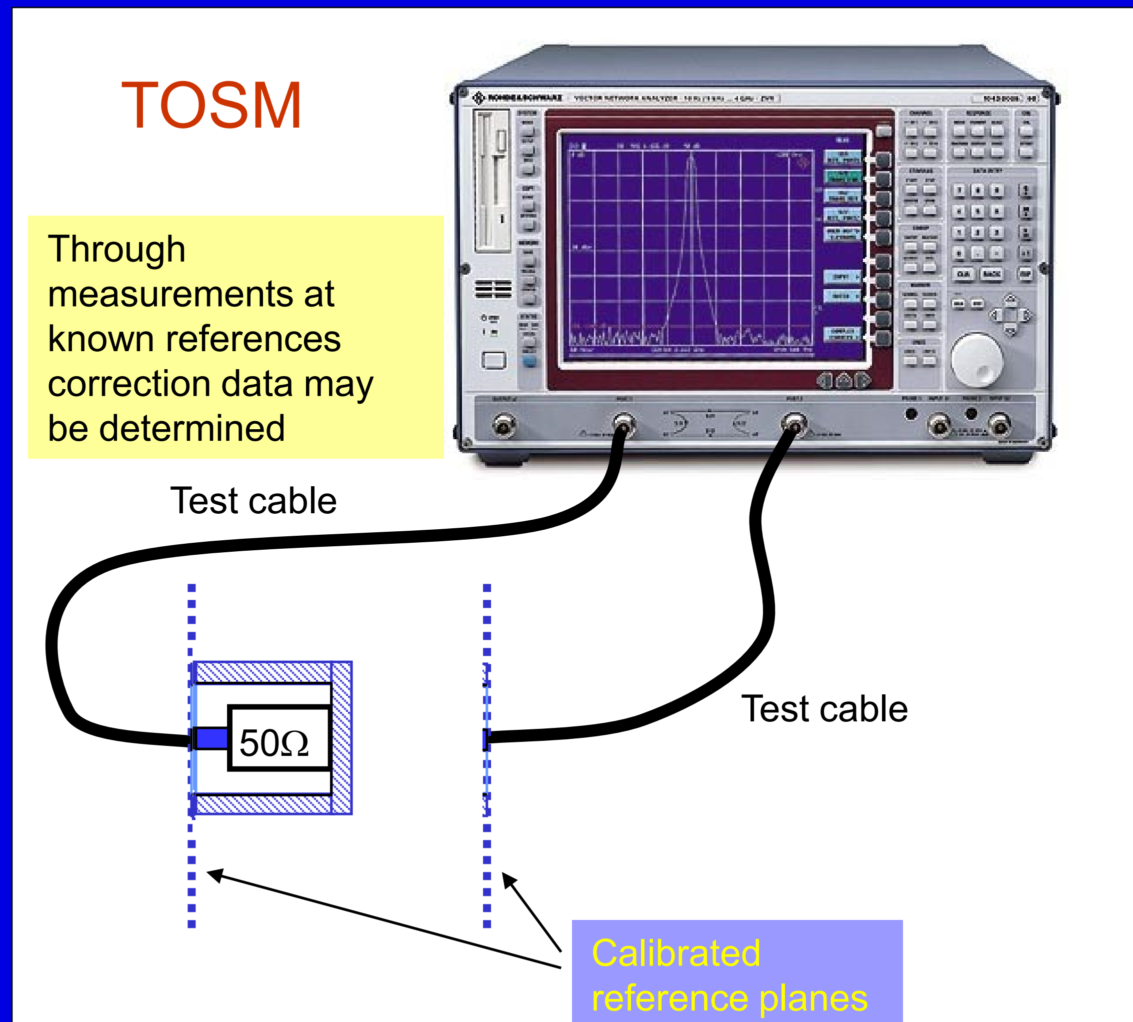
- Attenuation and phase shift in the test cables must be compensated
- Calibrated reference planes are therefore created where the device under test is connected



Calibration

- Calibrated reference planes will be created where the DUT is to be connected

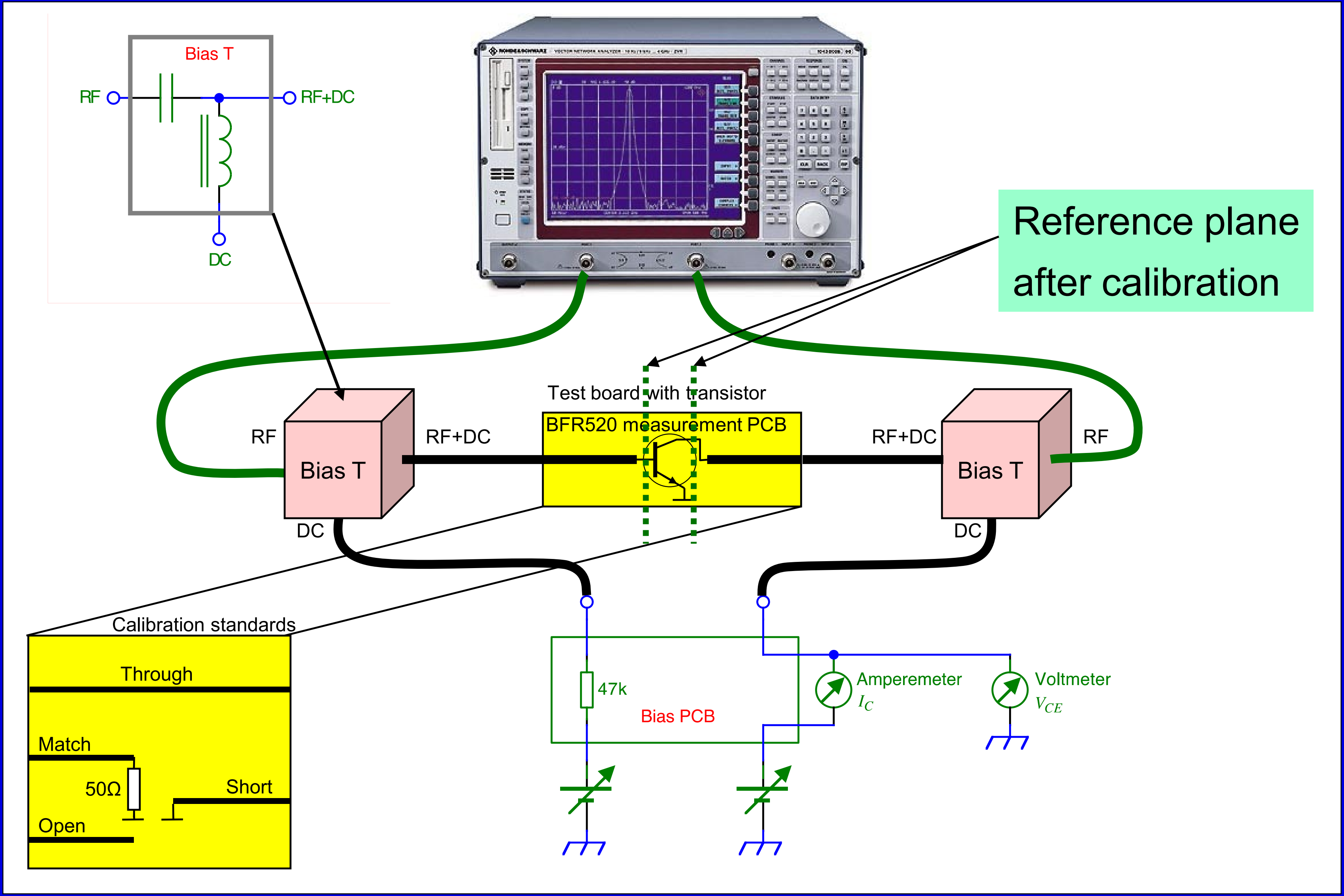




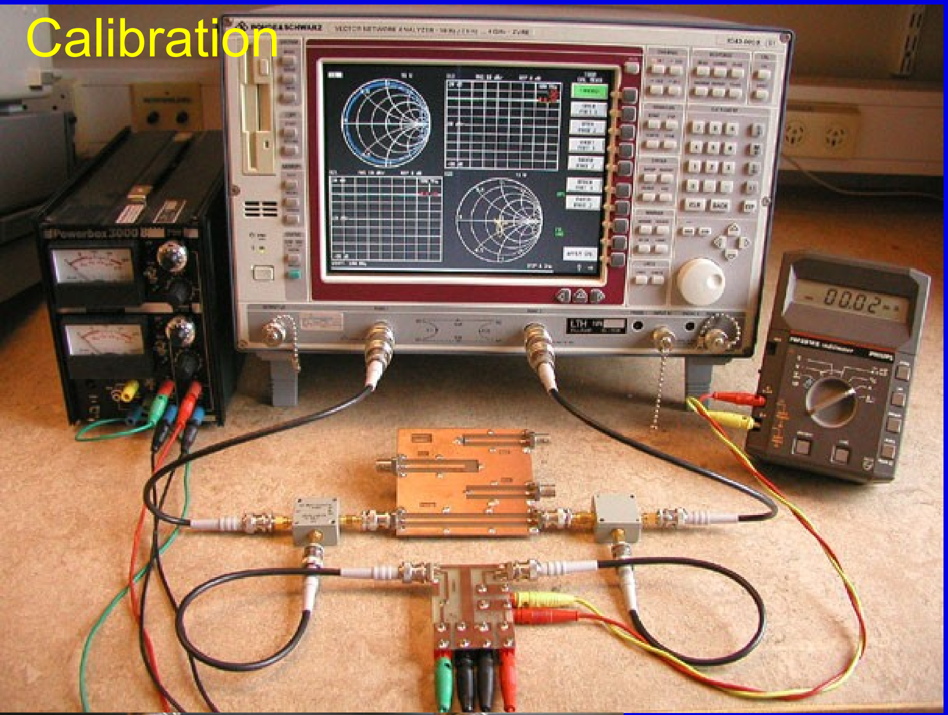
Be careful about torn connectors!

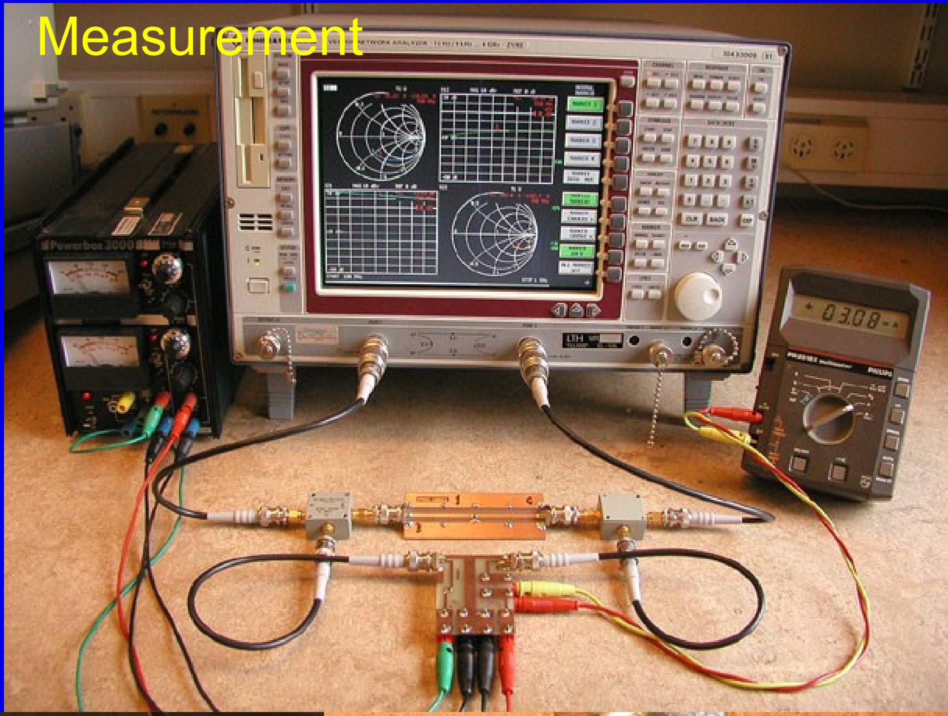
- The wear and tear when connectors are connected and disconnected may result in measurement errors.
  - always check that the connectors are clean
  - only turn the socket or the nut
    - the contact pin may **never** spin round
  - always use a torque wrench or **fingers**
    - the connector may never be fastened by other tools if you tighten up to hard the thread is harmed
- Test cables and connectors for professional use are only used for a limited period until they will be exchanged or reconditioned.

S-Parameter Measurements on a Transistor

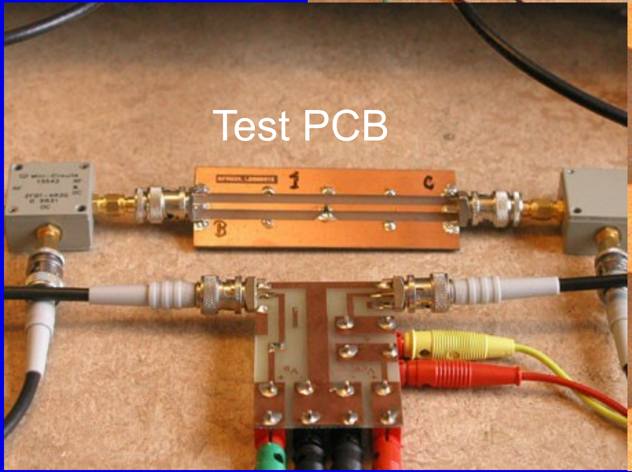


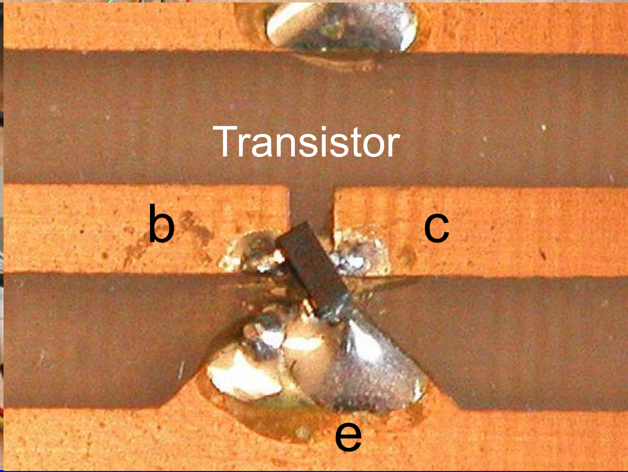
Lab 2


Calibration


Measurement


Calibration standards


Test PCB


Transistor
b
c
e