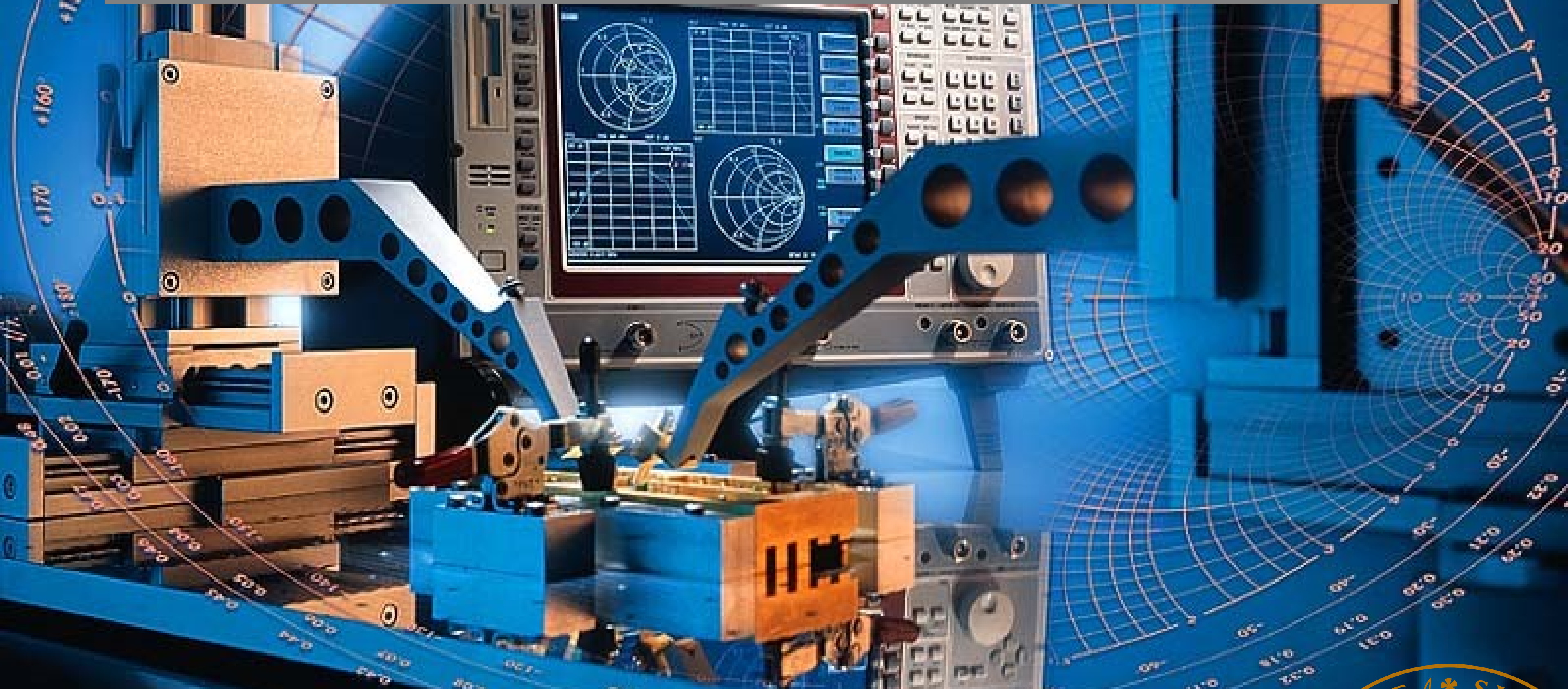
Lecture 5

Radio Electronics



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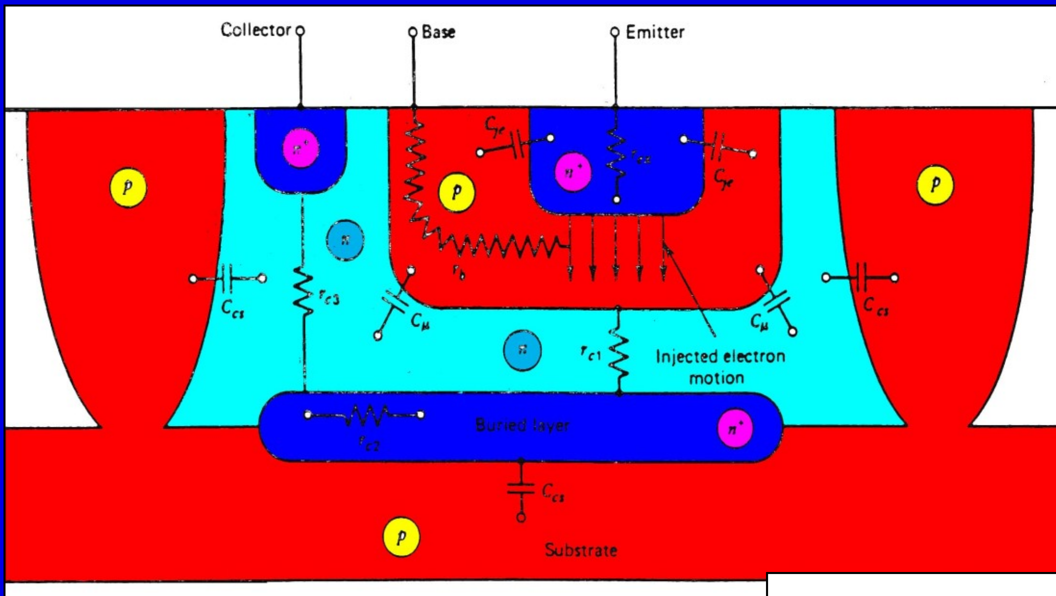
Electrical and Information Technology

Lecture 5

- RF Amplifier Analysis and Design
  - The hybrid-π-model
    - transistor configurations
    - amplifier classes
    - example of a tuned amplifier
  - Two-Port Network Representation
    - y, z, and ABCD parameters, definition and applications
    - measurement of parameters
    - power gain definitions
    - stability

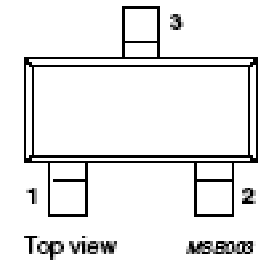
Transistor Models

Physical model

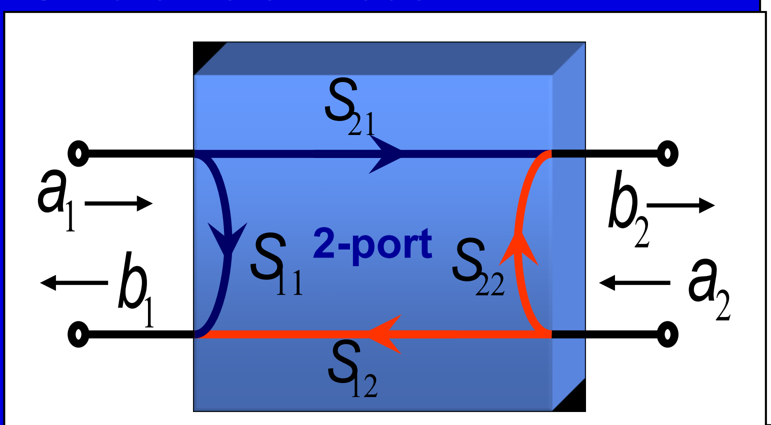


PINNING

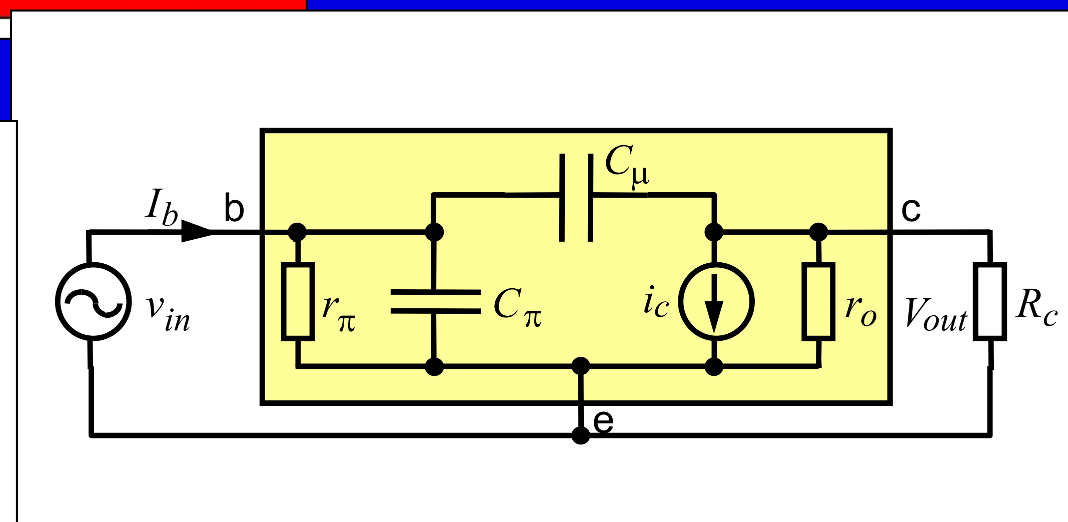
PIN	DESCRIPTION
Code: N28	
1	base
2	emitter
3	collector



S-Parameter model



Simplified hybrid-π-model



Transistors

$r_{b'e} = r_\pi$

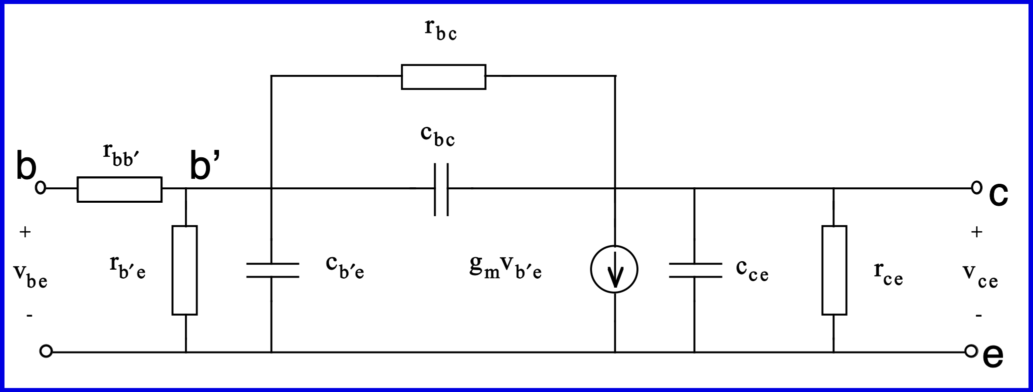
$C_{b'e} = C_\pi$

$r_{bc} = r_\mu$

$C_{bc} = C_\mu$

$r_{ce} = r_0$

The hybrid-π model for bipolar transistors



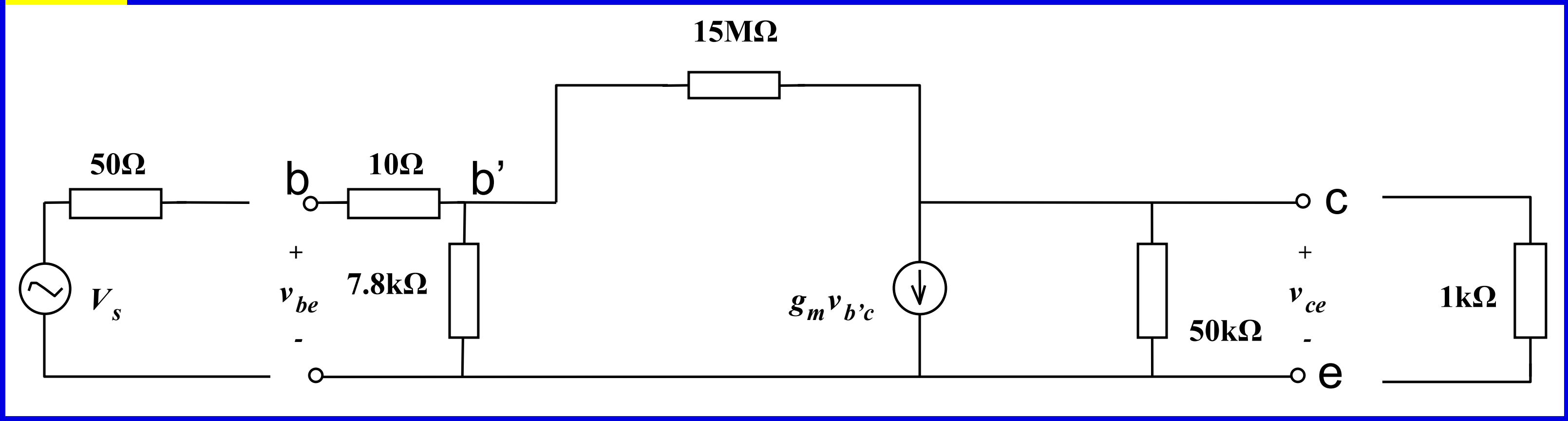
- Example:

$$c_{b'c} = 0.5\text{pF} \quad c_{b'e} = 9\text{pF} \quad c_{ce} = 1\text{pF}$$

$$r_{b'b} = 10\Omega \quad \beta = 300 \quad V_A = 50\text{V} \quad I_C = 1\text{mA}$$

What happens when the frequency increases from 0 to 1 GHz?

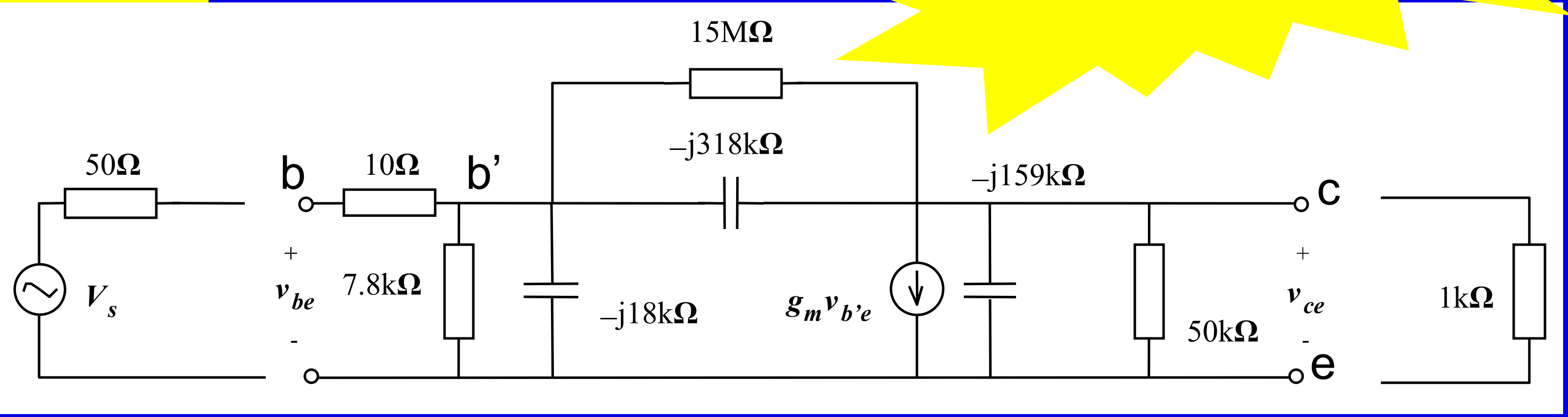
DC:



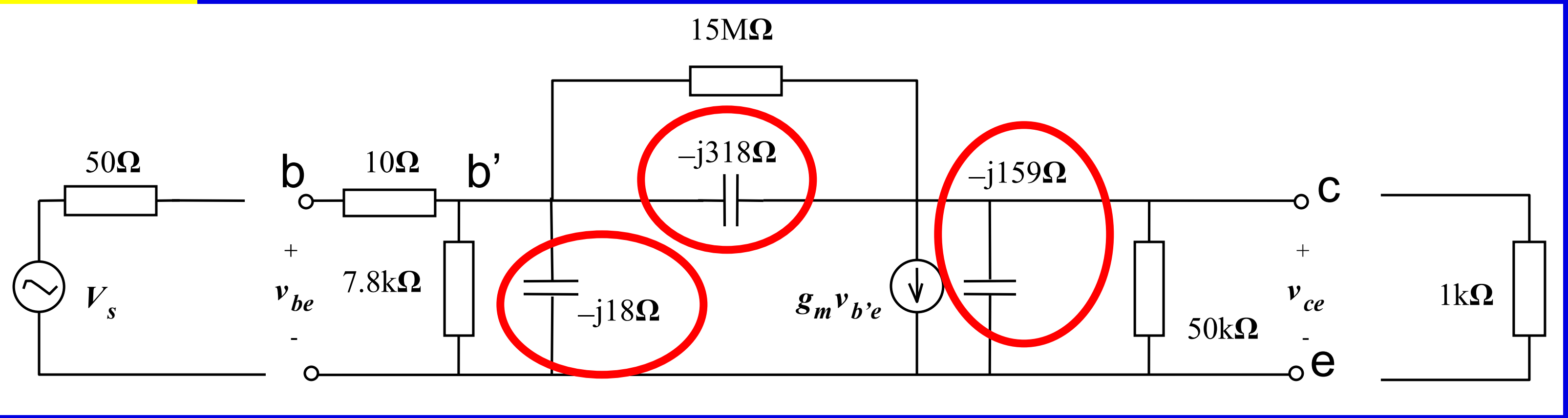
Transistors (con…

You will use these models in the labs/exam/exercise

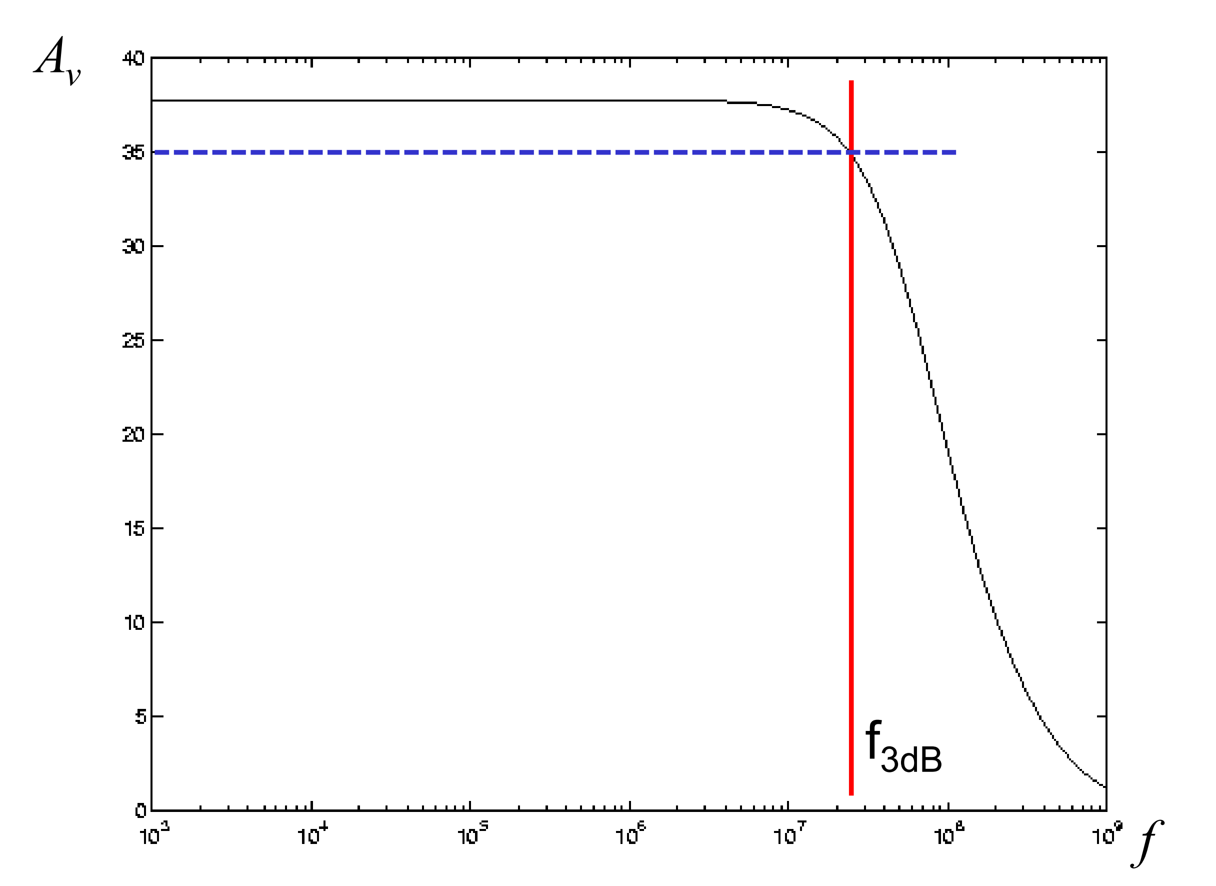
1 MHz:



1 GHz:



Voltage Gain



Tuned Amplifiers

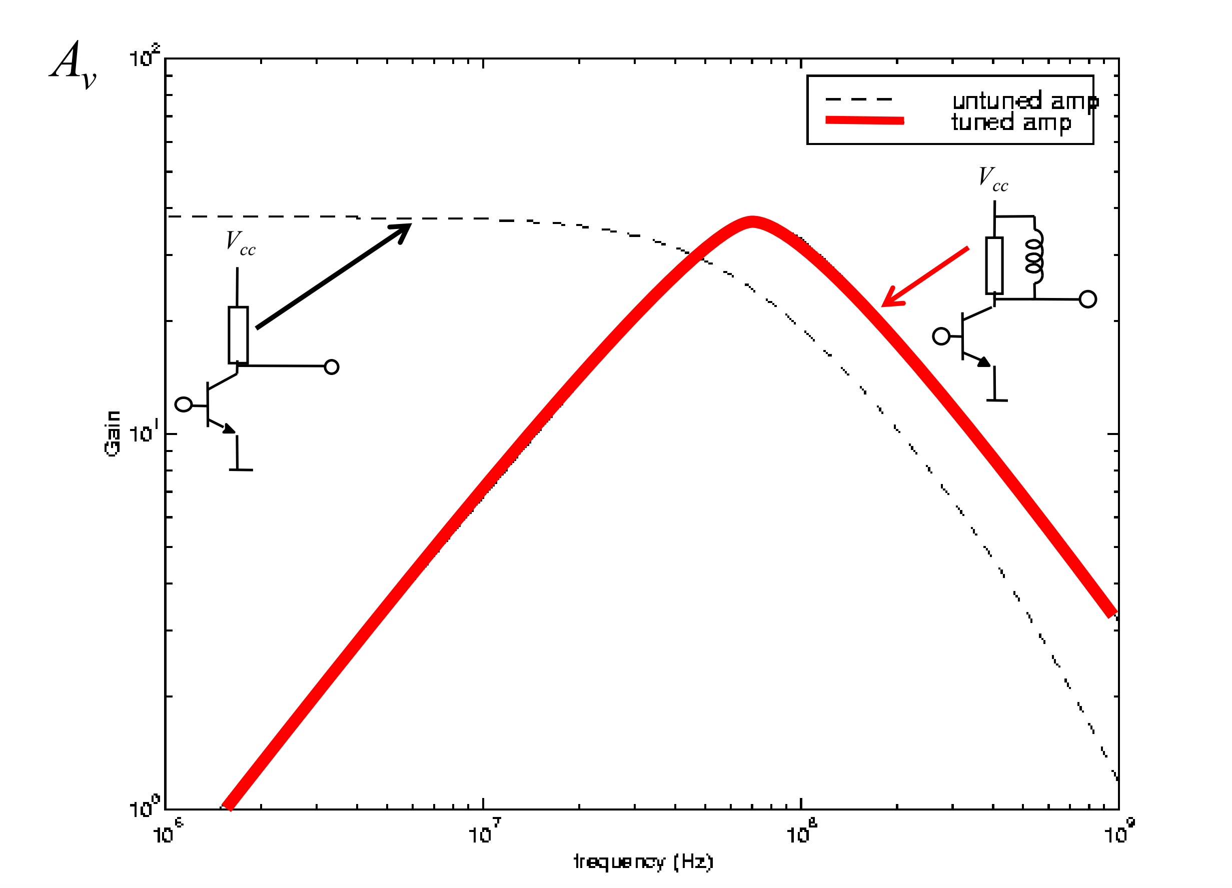
- Conventional design of amplifiers results in low-pass behaviour due to intrinsic device capacitances and this causes problems at higher frequencies

- By neutralizing the problematic capacitances the high-frequency behaviour can be improved
  - with preserved bandwidth!

- How to? Connect an inductor in parallel to the capacitance to form a resonant circuit tuned to the operation frequency.

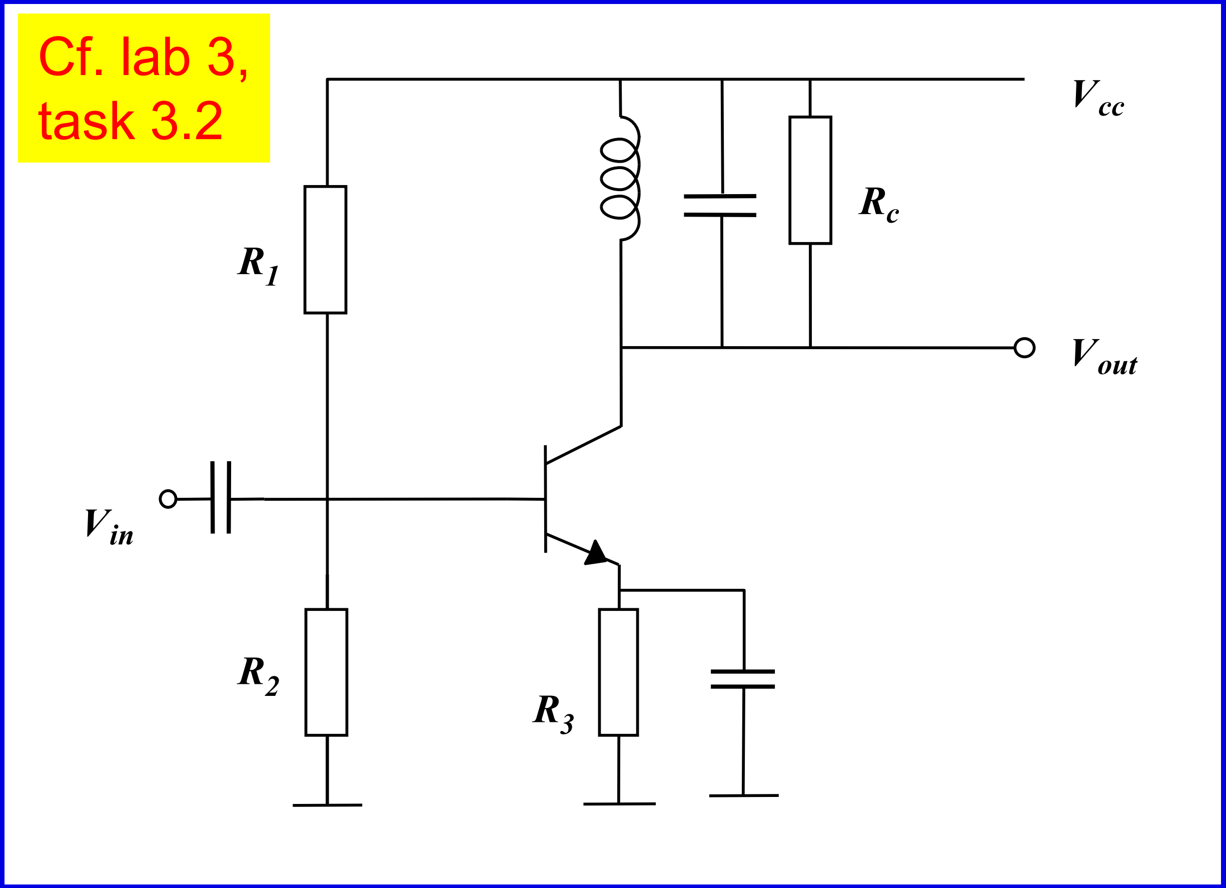
  The rest of the amplifier is designed by conventional low-frequency methods

Result of Neutralization (Inductive Peaking)

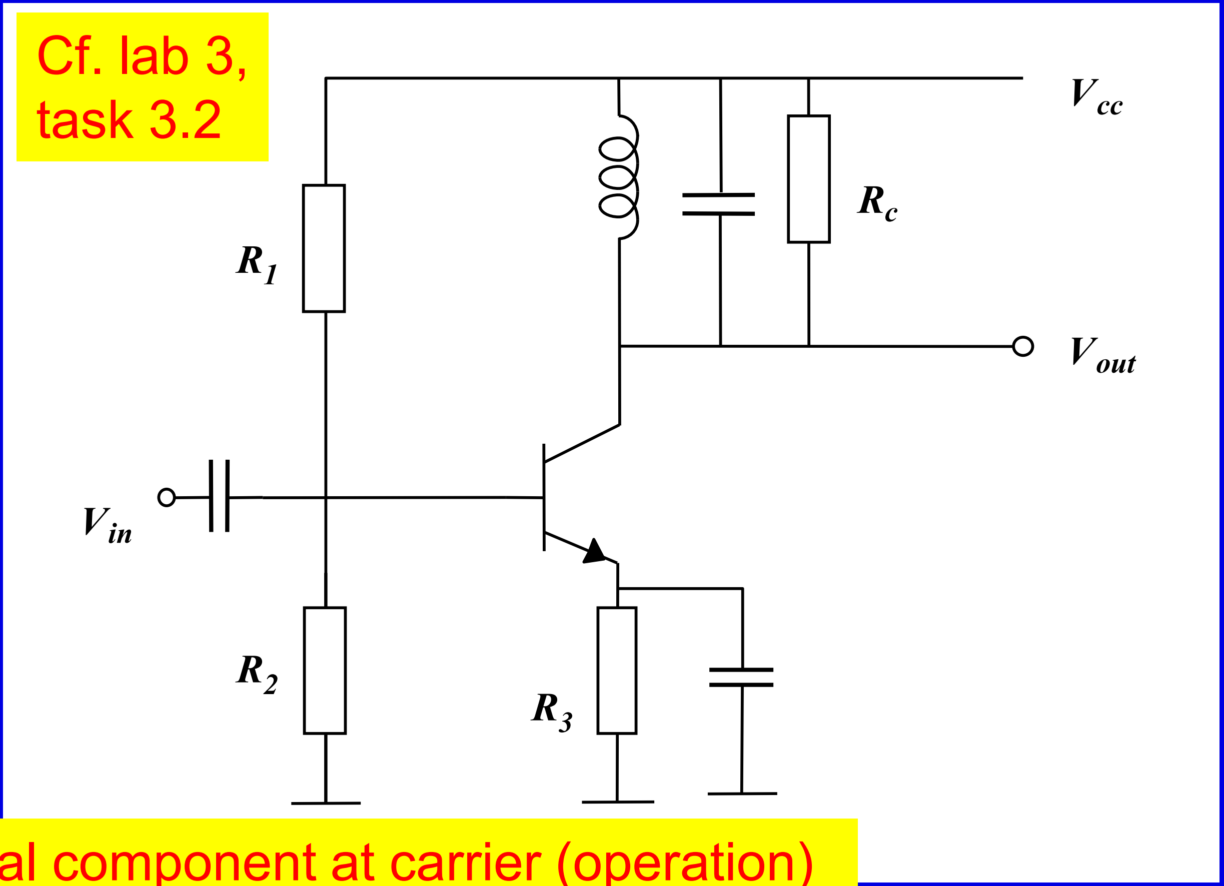


Both amplifiers have the same bandwidth, don't let the log-scale fool you.
$Q = f_0/B_{3dB} = R/X_L = R/X_C = R\omega_0 C \Rightarrow B_{3dB} = 1/2\pi RC$

Example of a Tuned Amplifier



Note the Signal Notation

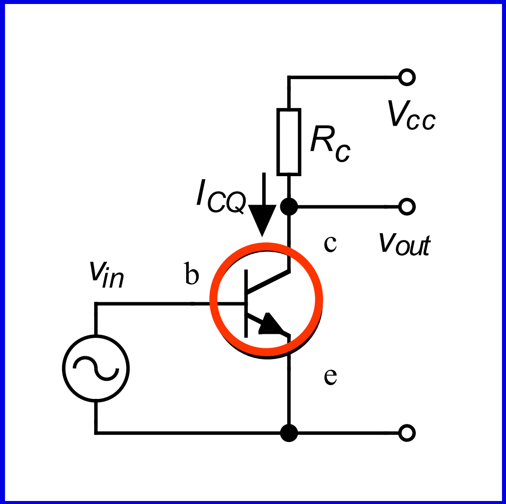


Small-signal component at carrier (operation) frequency often implied:

$$V_{in}(f_0) = \mathcal{F}\{v_{IN}(t)\}(f_0), \qquad v_{IN}(t) = V_{IN} + v_{in}(t)$$
$$V_{out}(f_0) = \mathcal{F}\{v_{OUT}(t)\}(f_0), \qquad v_{OUT}(t) = V_{OUT} + v_{out}(t)$$

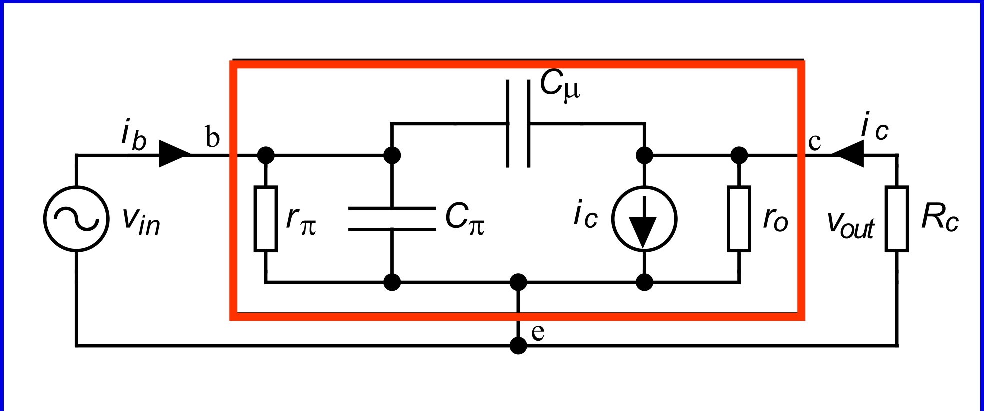
Small Signal Model of Bipolar Junction Transistor (BJT)

Circuit diagram



$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$$

Simplified hybrid-π-model



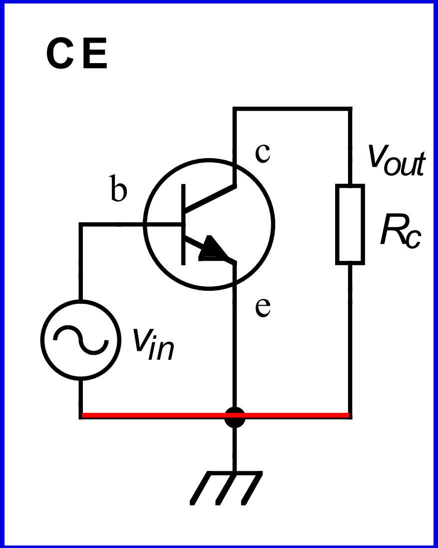
$$i_c = \beta i_b = g_m v_{in} = g_m r_\pi i_b$$

$$g_m = \frac{I_{CQ}}{V_T} \qquad r_o = \frac{V_A}{I_{CQ}}$$

The BJT model is fitted at a quiescent point and only valid at small-signal input:

$$V_{in} \ll V_T = \frac{kT}{q} \approx 25.0\ \mathrm{mV\ at}\ T = T_0 = 290\ \mathrm{K}$$

Common Emitter (CE) Transistor Configuration



- High voltage gain

$$\boxed{A_v = -g_m R_c}$$

- High input resistance and capacitance

$$\boxed{R_{in} = \beta / g_m = (\beta + 1) r_e}$$

$$\boxed{C_{in} = C_\pi + (1 - A_v) C_\mu}$$

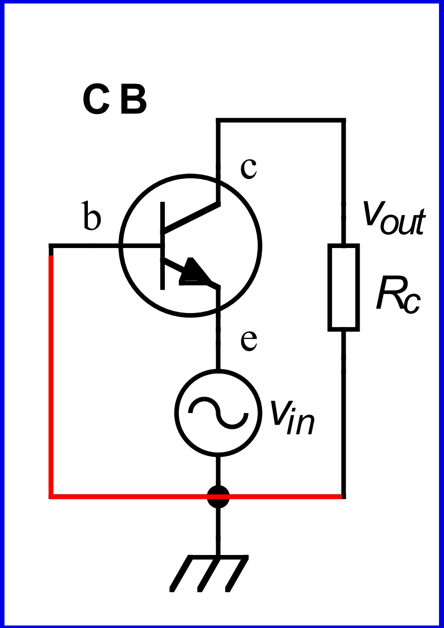
Result of Miller's theorem
(Miller approximation)

- High output resistance and low output capacitance

$$\boxed{R_{out} = r_o || R_c}$$

$$\boxed{C_{out} \approx C_\mu}$$

Common Base (CB) Transistor Configuration



- High voltage gain

$$A_v = g_m R_c$$

- Low input resistance and capacitance

$$R_{in} = r_e \approx \frac{1}{g_m}$$

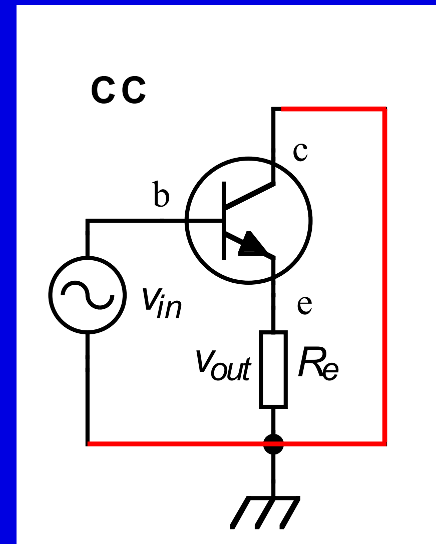
$$C_{in} = C_\pi$$

- High output resistance and low output capacitance

$$R_{out} = r_o || R_c$$

$$C_{out} = C_\mu$$

Common Collector (CC) Transistor Configuration



- Low voltage gain

$$A_v \approx 1$$

- High input resistance and low input capacitance

$$R_{in} = \beta\left(\frac{1}{g_m} + R_e\right) \approx \beta(r_e + R_e)$$

$$C_{in} = C_\mu$$

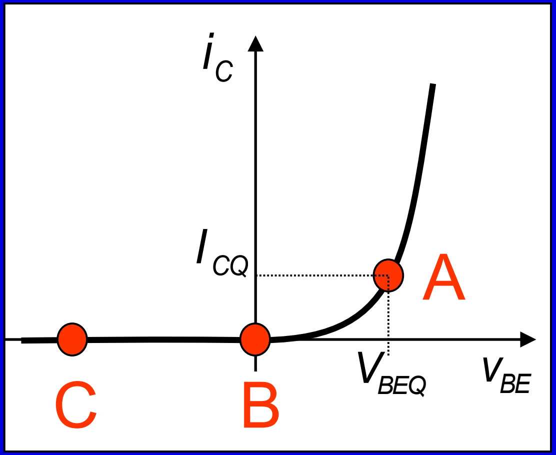
- Low output resistance and capacitance

$$R_{out} = r_e || R_e$$

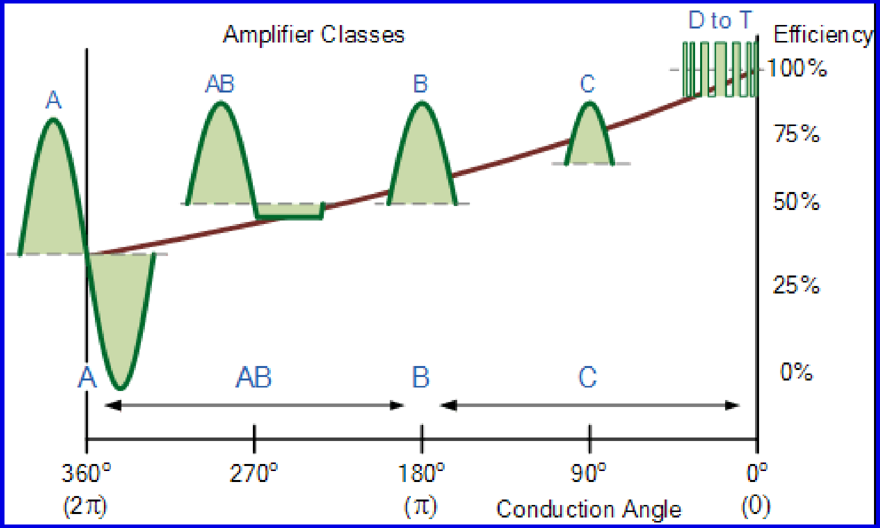
$$C_{out} \approx 0$$

Amplifier Classes

- Class A
  - linear
  - suitable for all signals
  - efficiency $\boxed{\eta \leq 50\%}$



- Class B
  - non-linear
  - only signals at high and constant amplitude
  - efficiency $\boxed{\eta \leq 78\%}$
- Class C
  - non-linear
  - only signals at high and constant amplitude (e.g. FM)
  - efficiency $\boxed{\eta \leq 90\%}$



When the Frequency Increases Further...

- Only simple neutralization (inductive peaking) is not efficient
  - more reactive elements affects the properties of the transistor
  - more elements generally affect the properties and therefore the rough hybrid-π-model must be further refined, or other data required

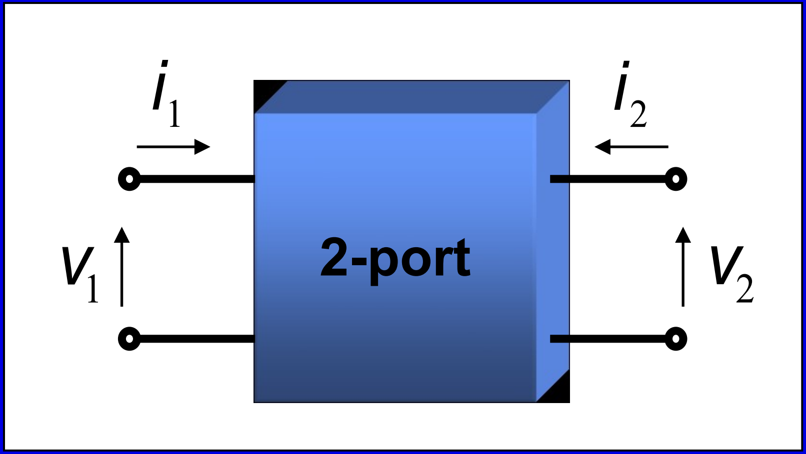
- Important to consider power to “preserve” available performance

- Design at high frequencies by lumped components therefore utilizes two-port models of devices, described by S-parameters – sometimes easier to measure than to calculate...

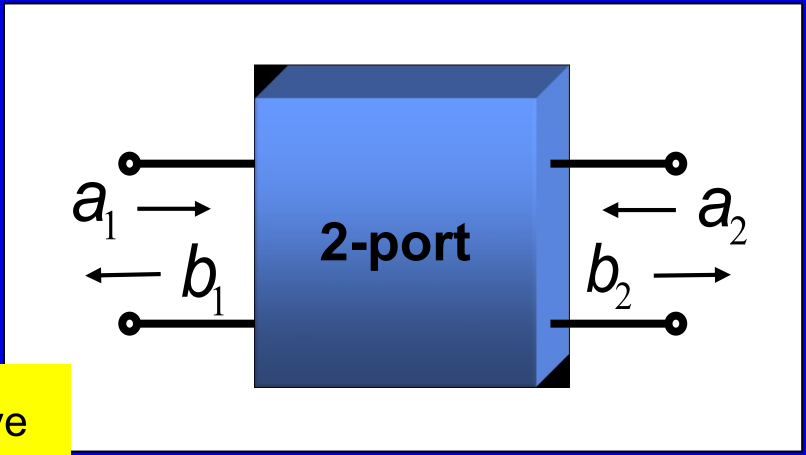
- Two-port models describe total response of the device
  - Careful measurements and modelling needed

Two-Port Network Representations

- voltage and current
  - z, impedance parameters
  - y, admittance parameters
  - ABCD, chain parameters
  - ...

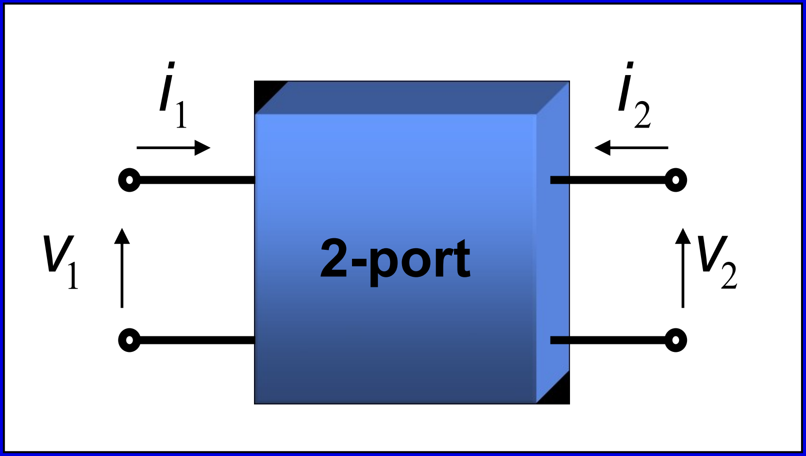


- waves
  - S, scattering parameters
  - T, transmission parameters
  - X, large signal scattering parameters



a_x = incident wave
b_x = reflected wave

Two-Port



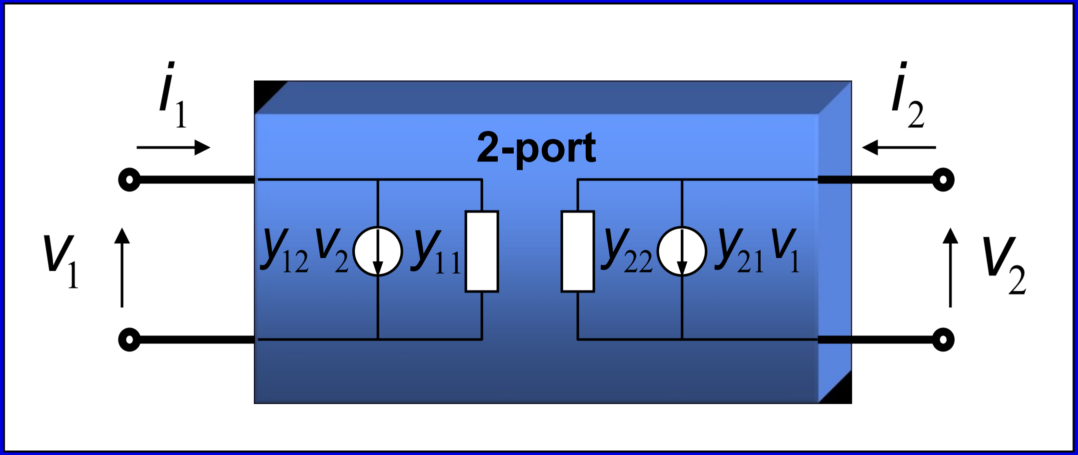
- If
  - the two-port is linear and
  - port signals (voltages and currents) are <u>complex</u> quantities (phasors)

the small signal properties must be characterized by four <u>complex</u> parameters

  - these are only valid at the specified frequency and
  - at the specific bias conditions

Admittance (y) Parameters

Two-port schematic (transadmittance, controlled Norton)



Definition

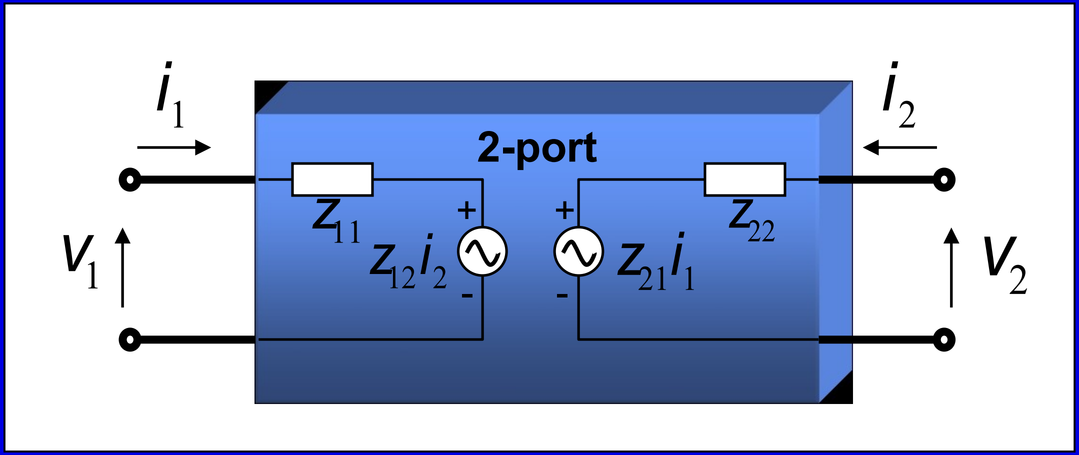
$$\begin{cases} i_1 = y_{11} \cdot v_1 + y_{12} \cdot v_2 \\ i_2 = y_{21} \cdot v_1 + y_{22} \cdot v_2 \end{cases}$$

or in matrix format:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Impedance (z) Parameters

Two-port schematic (transimpedance, controlled Thevenin)



Definition

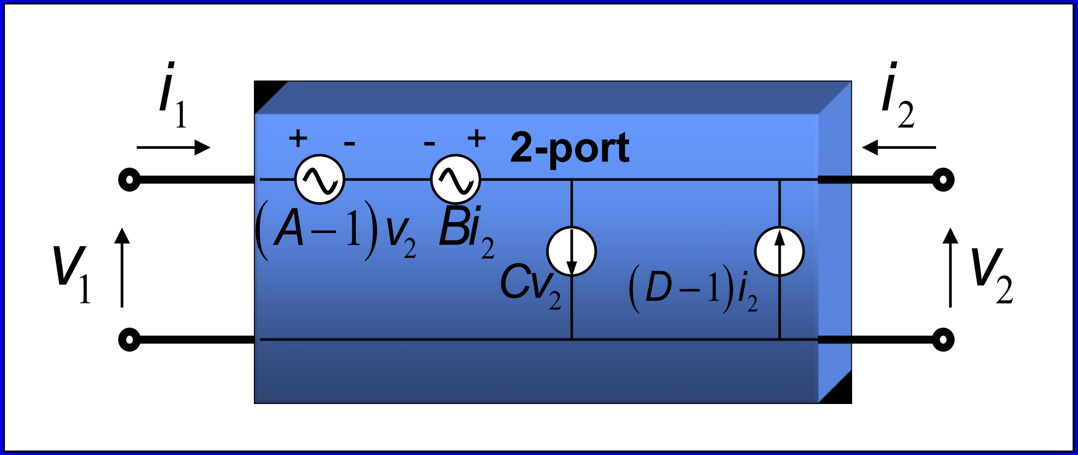
$$\begin{cases} v_1 = z_{11} \cdot i_1 + z_{12} \cdot i_2 \\ v_2 = z_{21} \cdot i_1 + z_{22} \cdot i_2 \end{cases}$$

or in matrix format:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Chain (ABCD, Cascade) Parameters

Two-port schematic
(just ideal sources)



or in matrix format:

Definition

$$\begin{cases} v_1 = A \cdot v_2 + B \cdot (-i_2) \\ i_1 = C \cdot v_2 + D \cdot (-i_2) \end{cases}$$

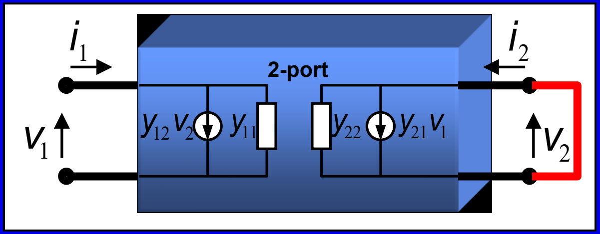
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

Measuring Two-Port Parameters

ex. y parameters

$$\begin{cases} i_1 = y_{11} \cdot v_1 + y_{12} \cdot v_2 \\ i_2 = y_{21} \cdot v_1 + y_{22} \cdot v_2 \end{cases}$$

- Parameters are simply derived when a port voltage is turned to zero, i.e. a port is short-circuited:



- at the same time as port 2 is short-circuited a signal is applied to port 1. The current voltage ratio at port 1 is then:

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$$

$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$

- repeat from other side for full matrix

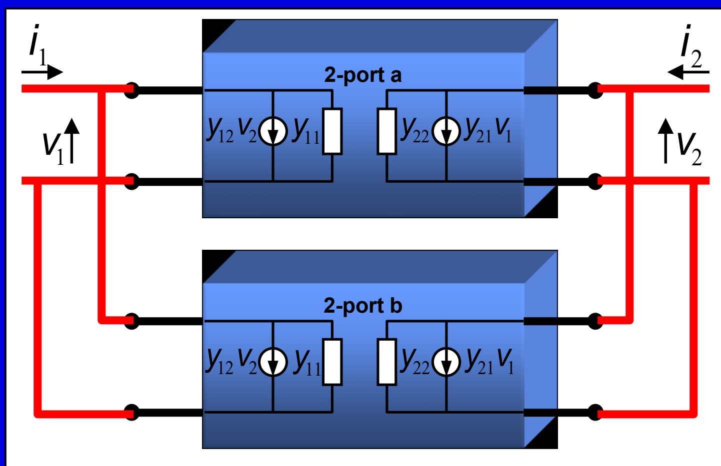
Measuring Two-Port Parameters (cont.)

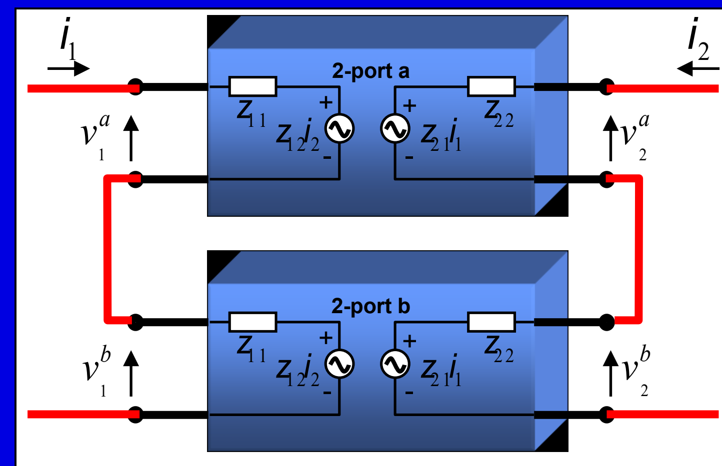
- Two-port parameters in z/y/ABCD-format are generally (mathematically) measured by means of open $(i = 0)$ or short-circuited $(v = 0)$ ports.
- Open or short-circuited terminations are however hard to achieve in reality and more so at high frequencies, especially close to the device.
- Open or short-circuited terminations may also cause instability and self-oscillation when measurements are performed on active components.
- At high frequency there is obviously a need for a model (e.g. S parameters) that doesn't require open or short-circuited device ports.
- y, z and ABCD parameters are still valid and useful for calculations at higher frequencies as well, but cannot be directly measured.

Circuit Analysis by Two-Port Parameters

- parallel connection:
  use y parameters

- series connection:
  use z parameters





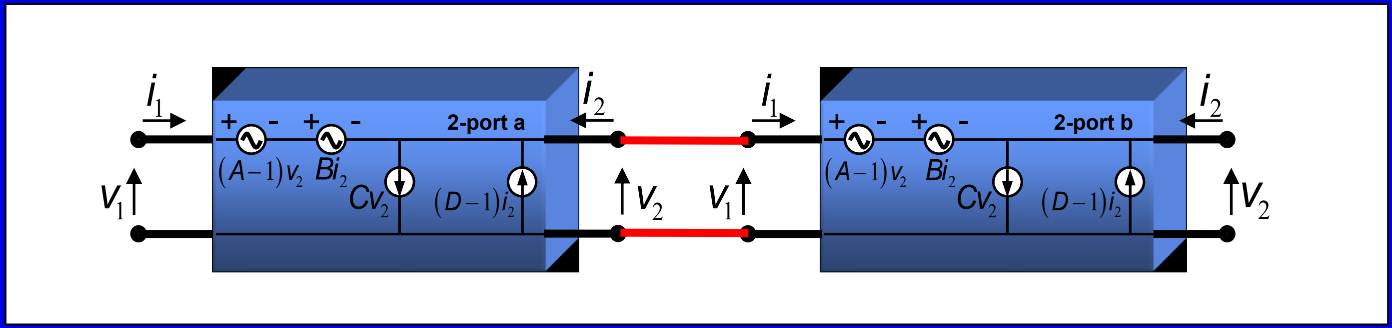
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1^a + i_1^b \\ i_2^a + i_2^b \end{bmatrix} = \begin{bmatrix} y_{11}^a + y_{11}^b & y_{12}^a + y_{12}^b \\ y_{21}^a + y_{21}^b & y_{22}^a + y_{22}^b \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1^a + v_1^b \\ v_2^a + v_2^b \end{bmatrix} = \begin{bmatrix} z_{11}^a + z_{11}^b & z_{12}^a + z_{12}^b \\ z_{21}^a + z_{21}^b & z_{22}^a + z_{22}^b \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Be careful, however, because some connections can disrupt the port condition implied for each device, yielding invalid results.

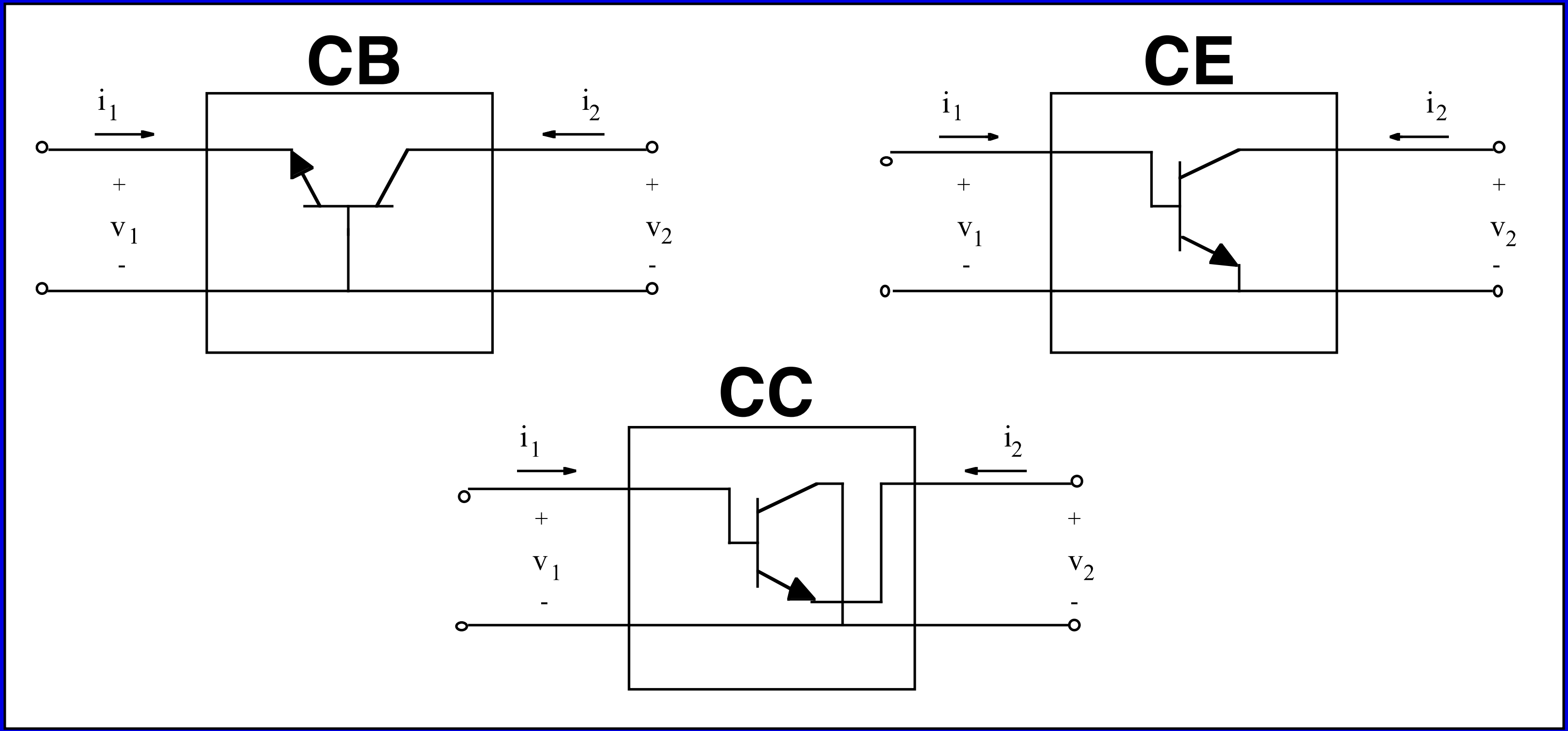
Circuit Analysis by Two-Port Parameters (cont.)

- cascade connection: use ABCD parameters



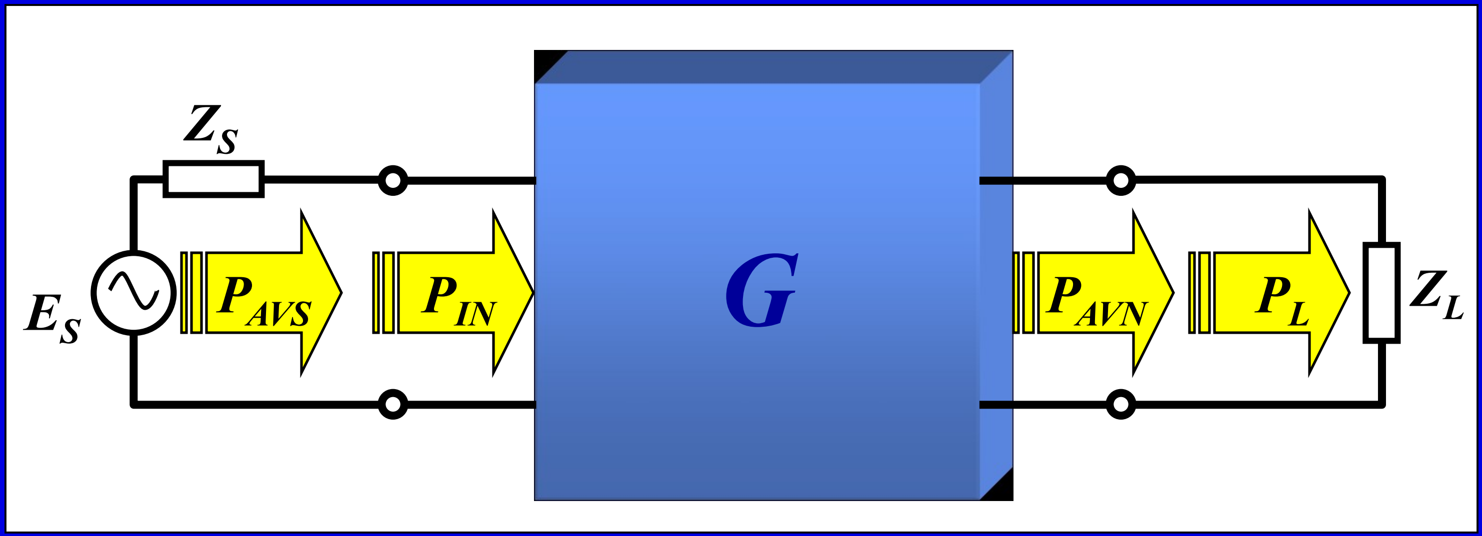
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} v_1^a \\ i_1^a \end{bmatrix} = \begin{bmatrix} A^a & B^a \\ C^a & D^a \end{bmatrix} \begin{bmatrix} v_2^a \\ -i_2^a \end{bmatrix} = \begin{bmatrix} A^a & B^a \\ C^a & D^a \end{bmatrix} \begin{bmatrix} A^b & B^b \\ C^b & D^b \end{bmatrix} \begin{bmatrix} v_2^b \\ -i_2^b \end{bmatrix}$$

Parameters for Various Transistor Configurations



- The parameters may easily be converted between different configurations without any loss of information.
  (Table 8.1 in the textbook)

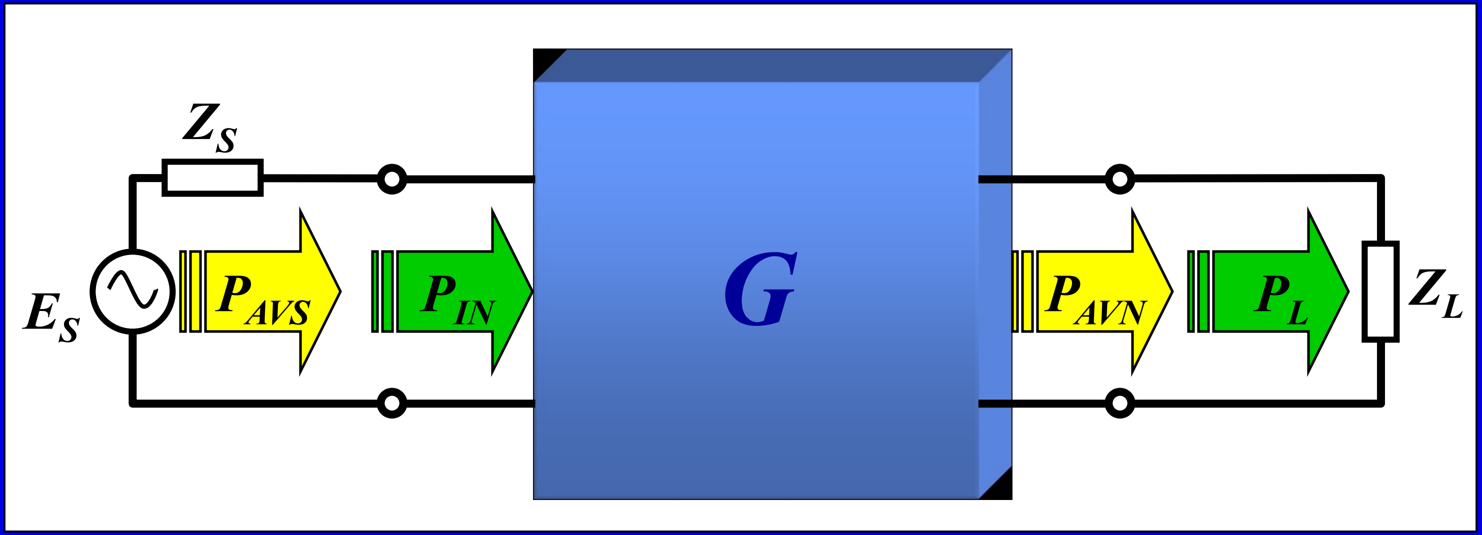
Power Gain Definitions



- P_{AVS} = AVailable power from Source
- P_{IN} = power delivered to the INput of the two-port
- P_{AVN} = AVailable power from Network (the output of the two-port)
- P_L = power delivered to the Load

- How should we define the power gain?

Power Gain Definitions (cont.)



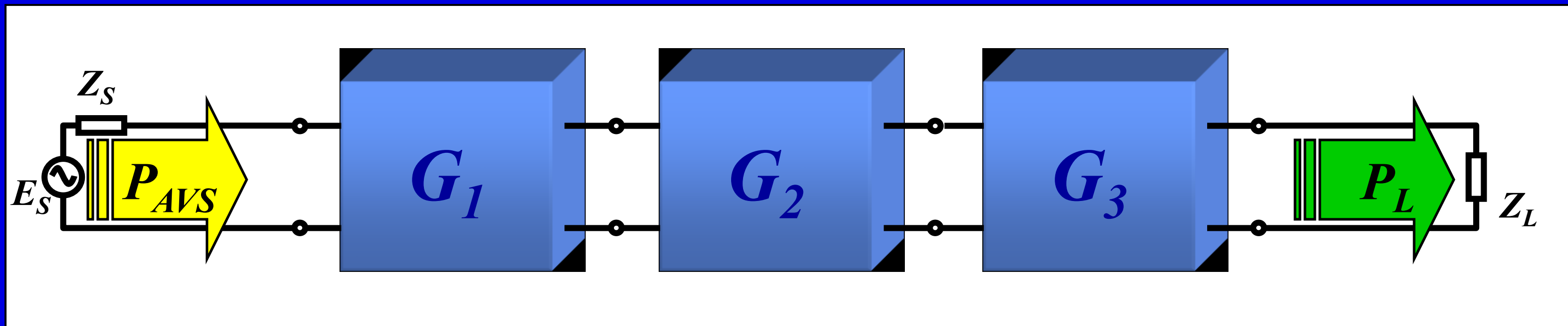
available gain $G_A = \dfrac{P_{AVN}}{P_{AVS}}$

Important definitions!

operating gain $G_P = \dfrac{P_L}{P_{IN}}$

transducer gain $G_T = \dfrac{P_L}{P_{AVS}}$

Power Gain from Several Stages



What is the total power gain?

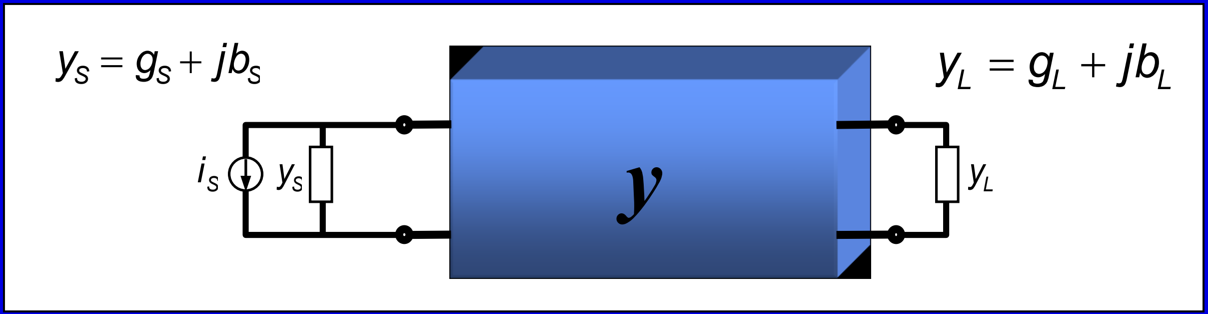
total transducer gain $\qquad G_{T\,tot} = \dfrac{P_L}{P_{AVS}}$

Extract the expression by using the gain definitions:

$$G_{T\,tot} = \frac{P_L}{P_{AVS}} = \frac{P_{OUT1}}{P_{AVS}} \cdot \frac{P_{OUT2}}{P_{IN2}} \cdot \frac{P_L}{P_{IN3}} = G_{T1} \cdot G_{P2} \cdot G_{P3}$$

Can G_T be calculated in alternative ways? How?

The Gain Expressed by y parameters

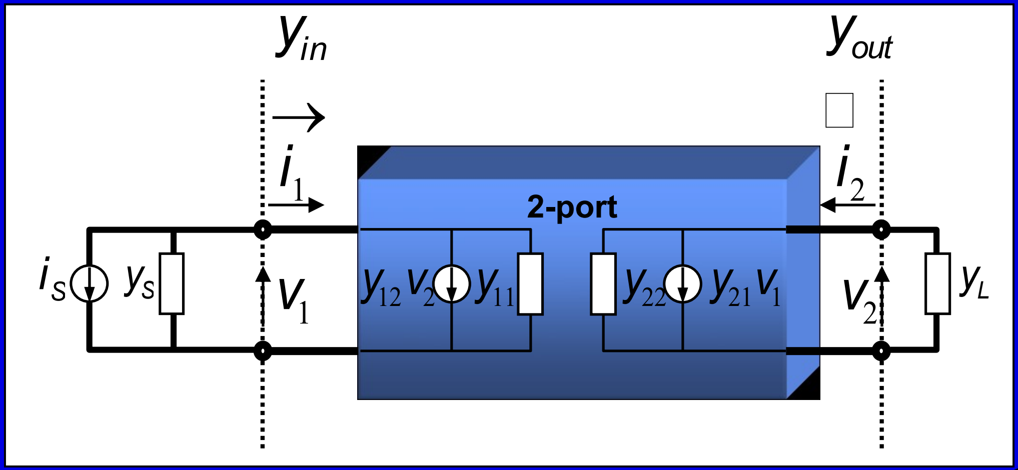


available gain
$$G_A = \frac{g_S |y_{21}|^2}{|y_S + y_{11}|^2 \operatorname{Re}\left[y_{22} - \dfrac{y_{12} y_{21}}{y_S + y_{11}} \right]}$$

operating gain
$$G_P = \frac{g_L |y_{21}|^2}{|y_L + y_{22}|^2 \operatorname{Re}\left[y_{11} - \dfrac{y_{12} y_{21}}{y_L + y_{22}} \right]}$$

transducer gain
$$G_T = \frac{4 g_S g_L |y_{21}|^2}{\left|(y_S + y_{11})(y_L + y_{22}) - y_{12} y_{21}\right|^2}$$

Port Admittances for a Two-Port



y_{12} connects output to input

input admittance $$y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + y_L}$$

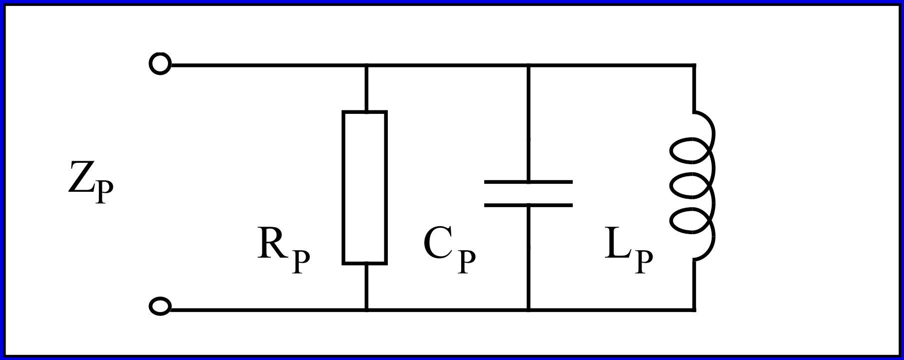
y_{21} connects input to output

output admittance $$y_{out} = y_{22} - \frac{y_{21}y_{12}}{y_{11} + y_S}$$

Stability

- If the circuit isn't able to self-oscillate it is considered to be stable (zero input signal, zero output signal).
- This is not an obvious quality at RF design because
  - several feedback paths (parasitic elements)
  - several reactive circuit elements (parasitic elements)
- Stability analysis based on poles is possible only if the complete circuit is completely known.
- It is not trivial (some may say impossible) to calculate poles and zeros if only the two-port parameters are available.
  There is a need for alternative methods to study stability…

Stability - an Example



Stable when $R_P > 0$ - R_P denotes the circuit losses

Unstable when $R_P < 0$ - R_P supplies power to the circuit

Passive sign convention: positive power is dissipated

Unconditional Stability

$$y_{in} = y_{11} - \frac{y_{12} \cdot y_{21}}{y_{22} + y_L}$$

$$y_{out} = y_{22} - \frac{y_{12} \cdot y_{21}}{y_{11} + y_S}$$

A two-port is defined to be unconditionally stable if no y_S or y_L results in a negative port admittance

Linvill's stability factor

$$C_L = \frac{\left|y_{12} \cdot y_{21}\right|}{2 \cdot g_{11} \cdot g_{22} - \mathrm{Re}\left[y_{12} \cdot y_{21}\right]}$$

The two-port is unconditionally stable if $0 < C_L < 1$

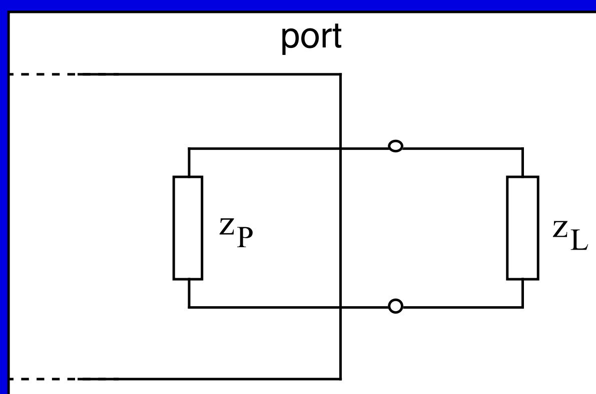
Note! The result is only valid at the actual frequency where the stability test is performed

Conditional Stability

$$y_{in} = y_{11} - \frac{y_{12} \cdot y_{21}}{y_{22} + y_L}$$

$$y_{out} = y_{22} - \frac{y_{12} \cdot y_{21}}{y_{11} + y_S}$$

- If it's not unconditionally stable the two-port may be conditionally stable:
  - some y_S or y_L causes a negative port admittance
  - avoid these y_S and y_L if possible
- If the port impedance still is negative
  - examine the loop impedance:



$$r_{loop} = \mathrm{Re}\left[z_P\right] + \mathrm{Re}\left[z_L\right]$$

$$x_{loop} = \mathrm{Im}\left[z_P\right] + \mathrm{Im}\left[z_L\right]$$

- The two-port is stable if the loop resistance $r_{loop} > 0$
  - Note! The result is only valid at the actual frequency where the calculation is done.