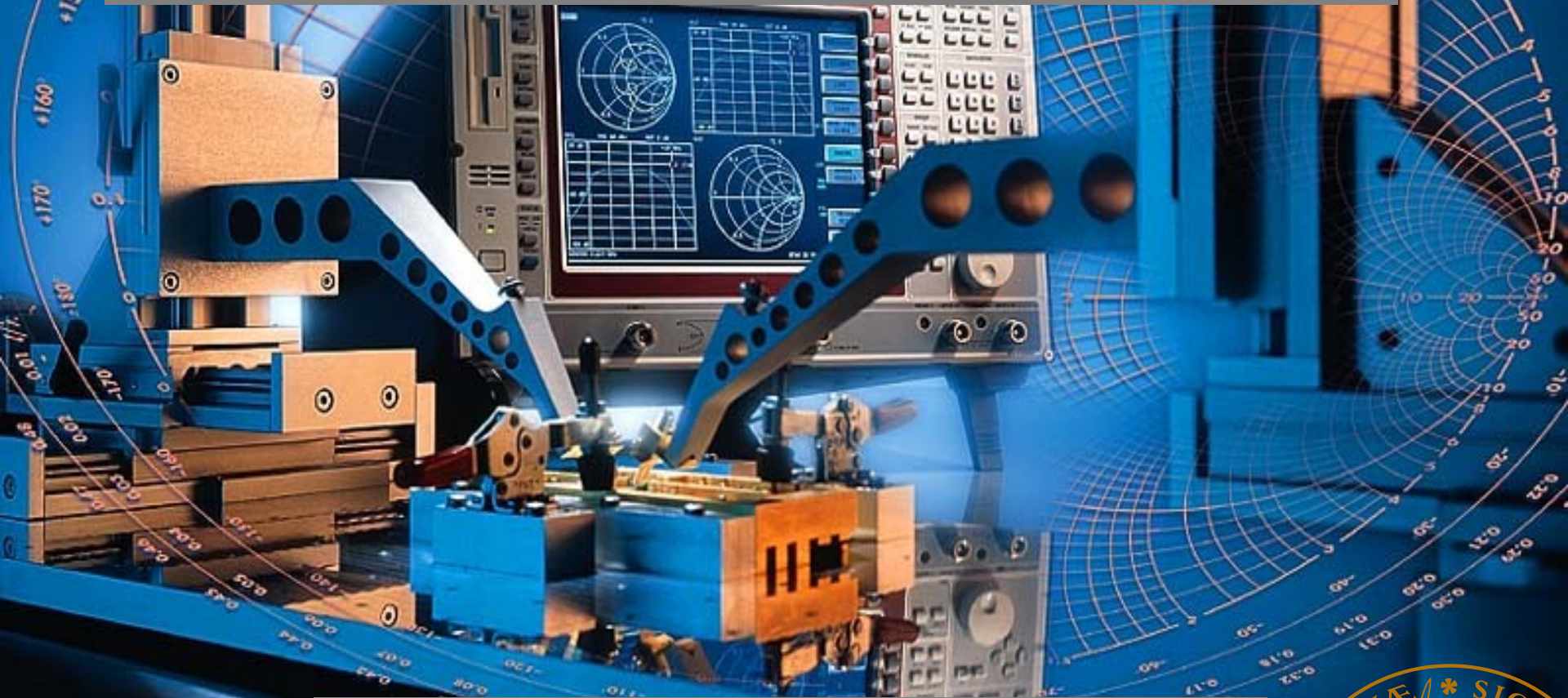


# RF Amplifier Design



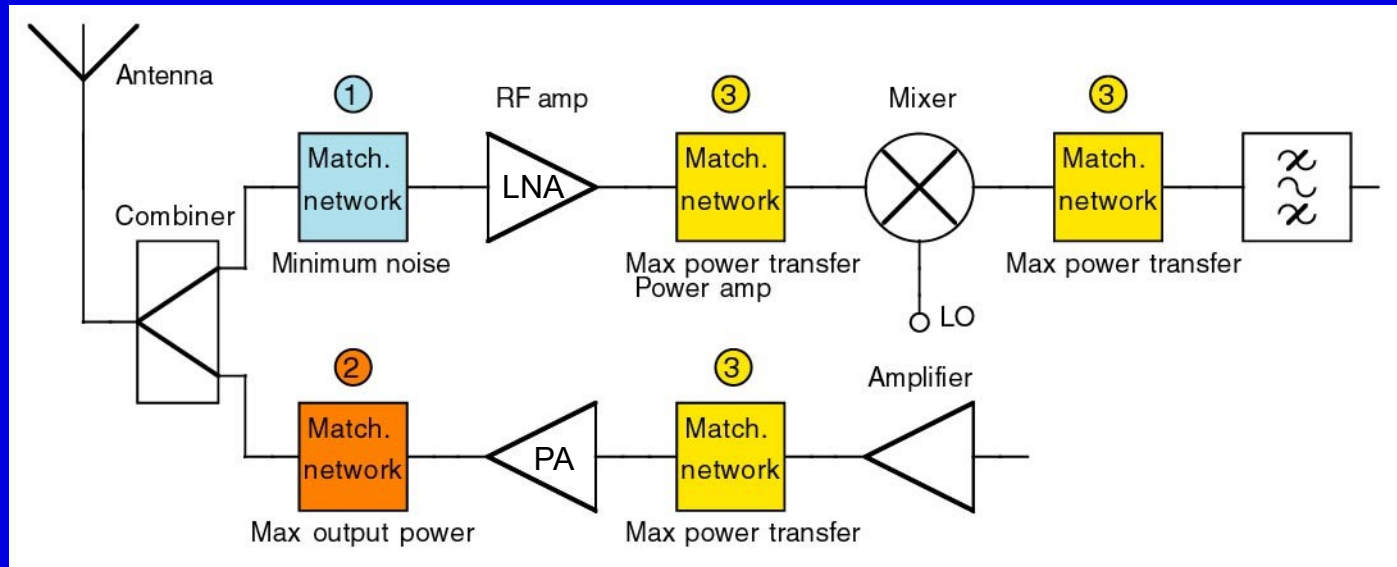
Lars Ohlsson Fhager  
Electrical and Information Technology



# Lecture 4

- Design of Matching Networks
  - Various Purposes of Matching
    - Voltage-, Current- and Power Matching
  - Design by Lumped Circuit Elements
    - L, Pi and T Networks
    - Design by using the Smith Chart
  - Design by Line Structures
    - Transformation by a Transmission Line
    - Quarter-Wave Transformer
    - Line Section with Optimised Length and  $Z_0$
    - Stubs
- Passive Components
  - Lumped Components
    - Resistors
    - Capacitors
    - Inductors
    - Transformers
  - Substrate and Conductor Materials
  - Transmission Lines
    - Coaxial Line
    - Microstrip
    - Stripline
    - Discontinuities
      - Bends, corners

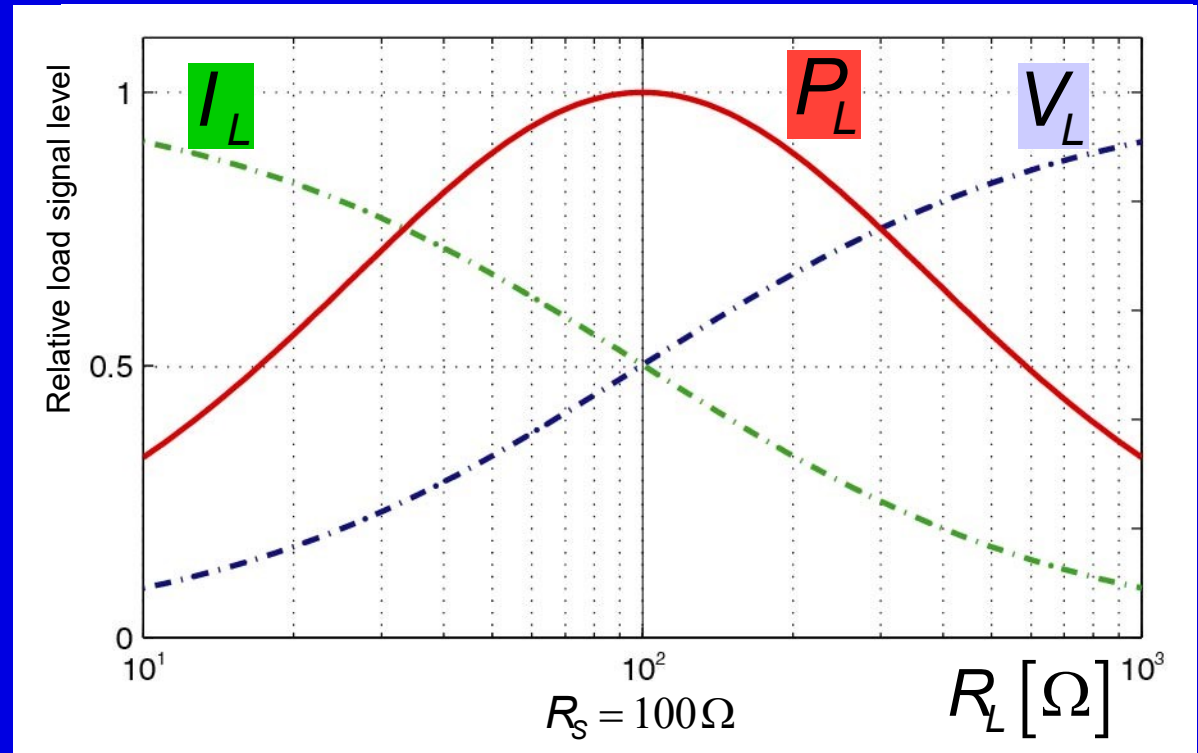
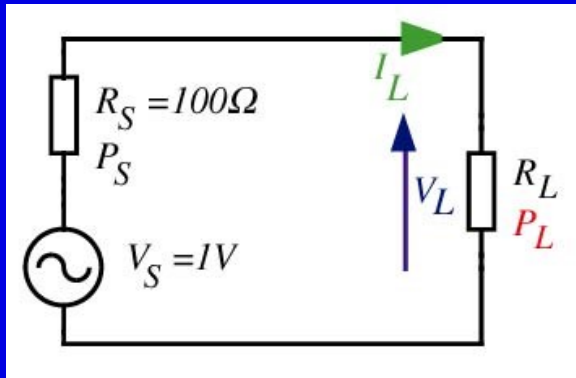
# The Purpose of Matching



- ① Minimum noise power
- ② Maximum output power
- ③ Maximum transfer of power

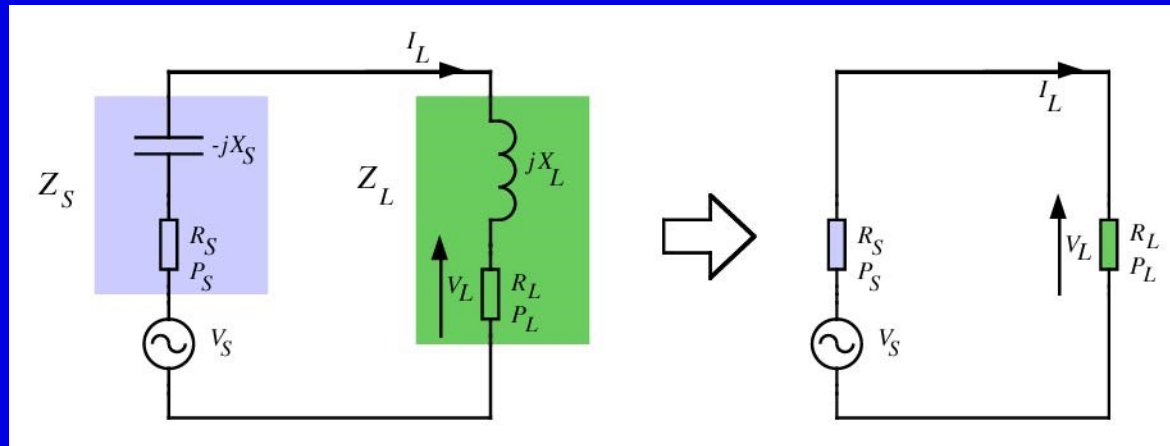
*In most cases bandwidth and filtering are important parameters too*

# Maximum Transfer of Power



- Current matching
- Voltage matching
- Power matching

# Complex Conjugate Matching

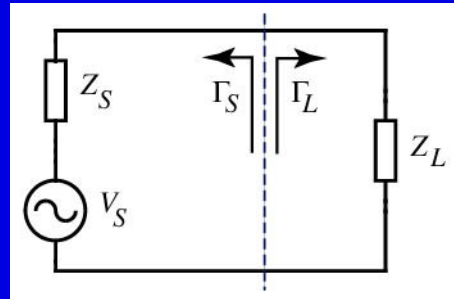


- Maximum transfer of power when

$$Z_L = Z_S^* \Rightarrow \begin{cases} R_L = R_S \\ X_L = -X_S \end{cases}$$

resonance

# Z or $\Gamma$ Representation



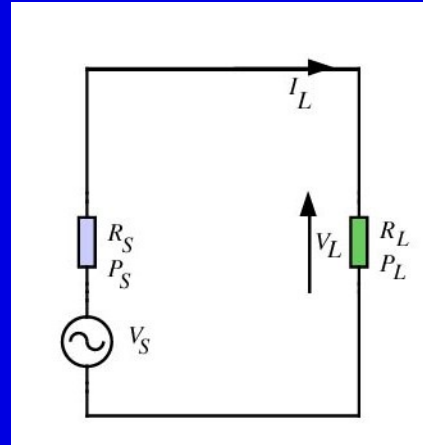
- Impedance

$$Z_L = Z_S^* \Rightarrow \begin{cases} R_L = R_S \\ X_L = -X_S \end{cases}$$

- Reflection coefficient

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

# Available Power from Source, $P_{AVS}$



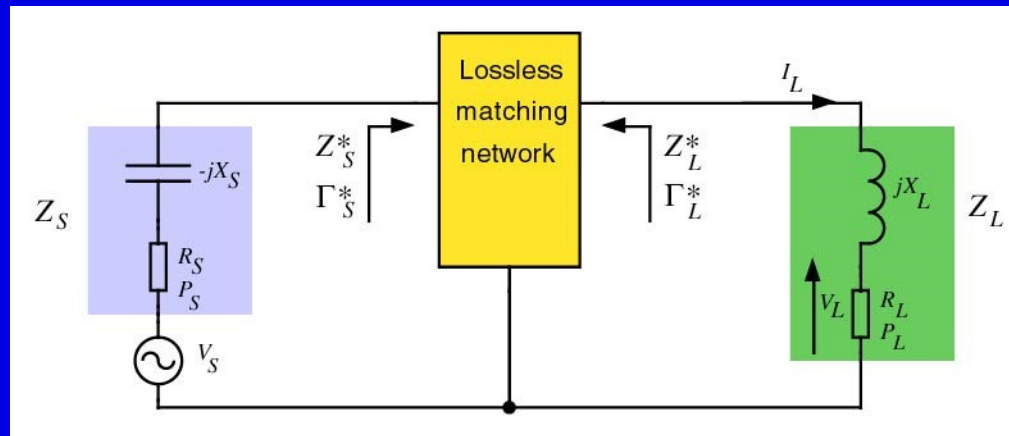
- The maximum amount of power that can be delivered from the source is defined as:

$$P_{AVS} = \frac{V_L^2}{R_L} = V_S^2 \left( \frac{R_L}{Z_S + Z_L} \right)^2 \frac{1}{R_L} \Big|_{Z_L = Z_S^*} = \frac{V_S^2}{4R_S}$$

**NOTE!**  
The unit of  $V_S$  is here  $V_{RMS}$ !

Active power

# The Need of a Matching Network

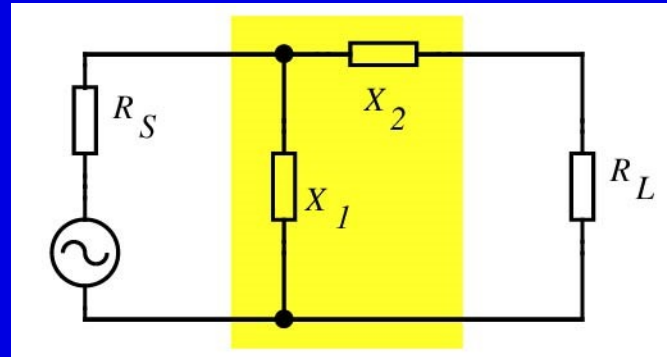


- If  $Z_L$  and/or  $Z_S$  are fixed a matching network is needed to ensure proper matching

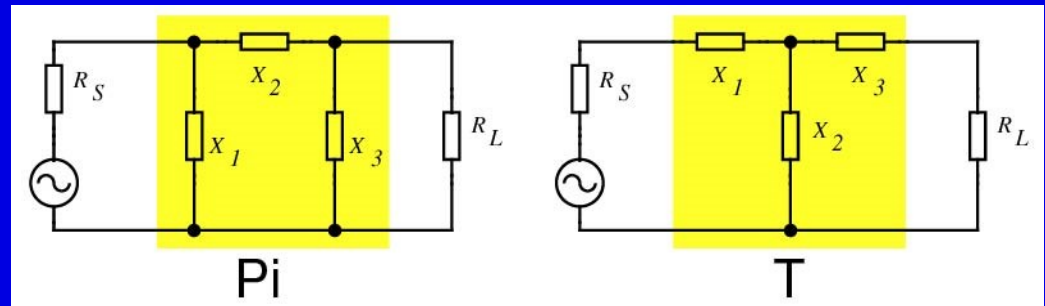


# Network Design by Lumped Circuits

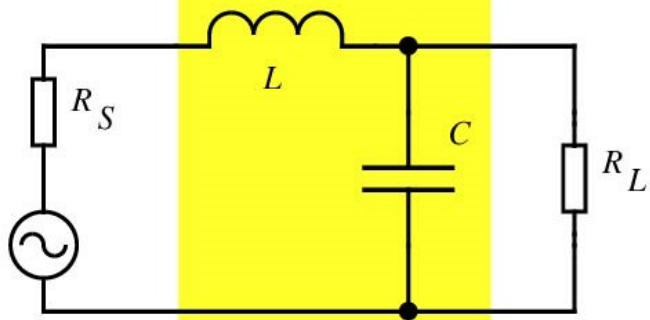
- L network
  - low-pass
  - high-pass
  - fixed circuit  $Q$



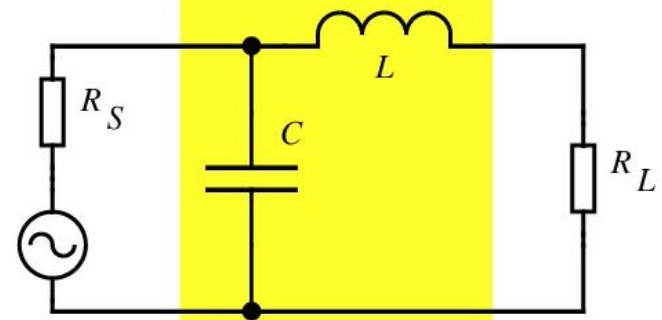
- Pi and T network
  - low-pass
  - high-pass
  - desired circuit  $Q$



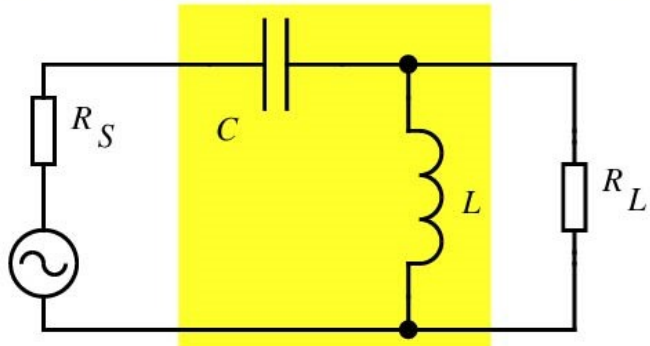
# L Network, Four Different Variations



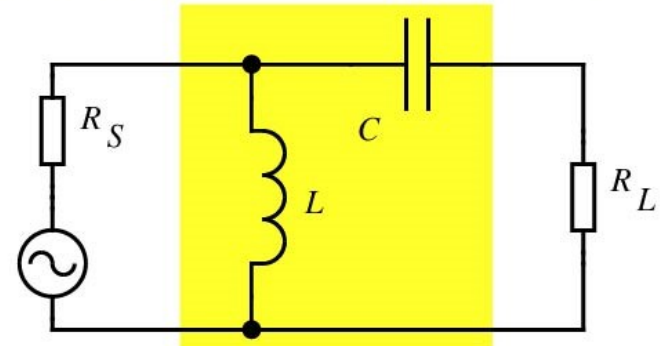
Low-pass,  $R_L > R_S$



Low-pass,  $R_L < R_S$



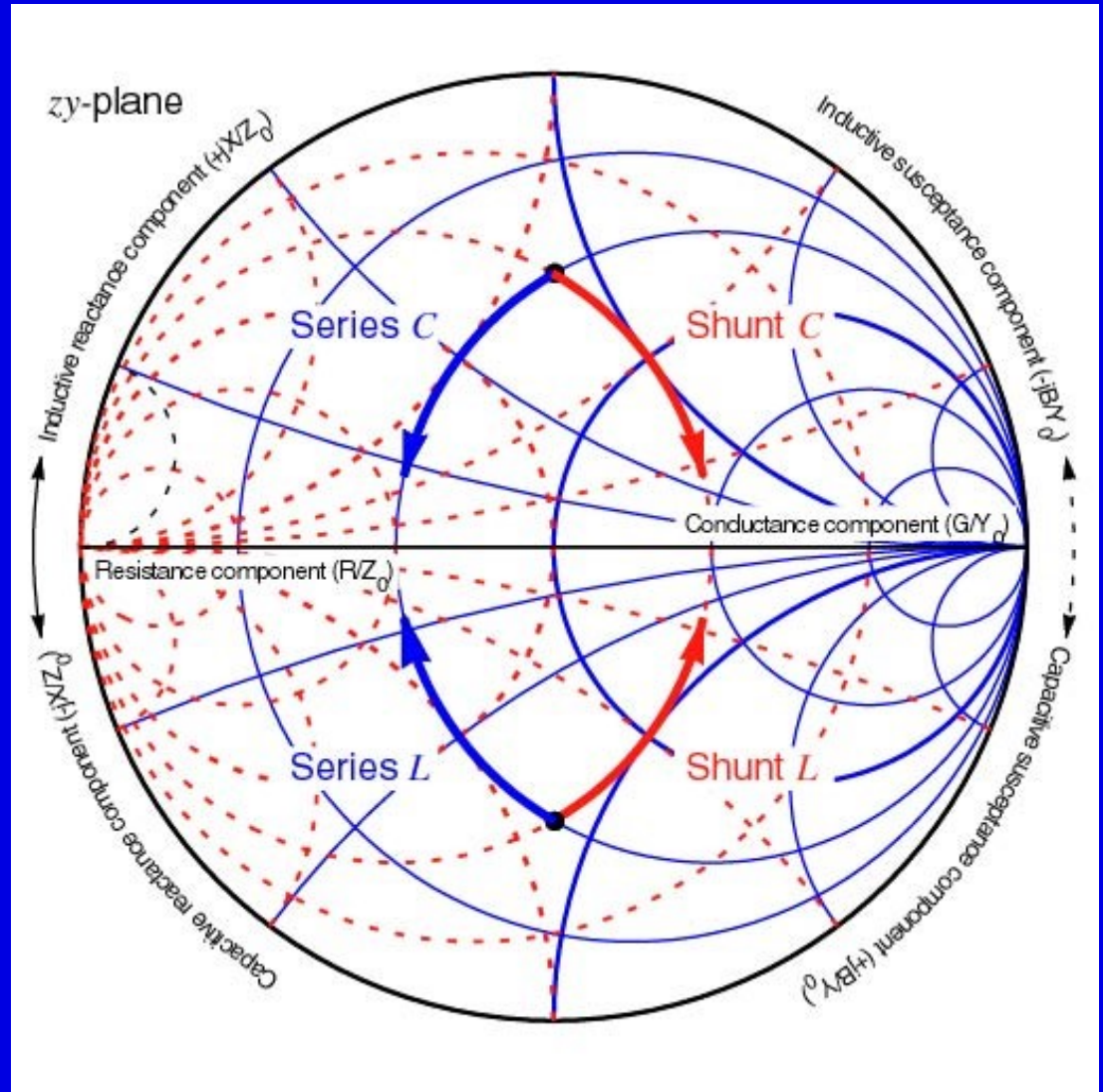
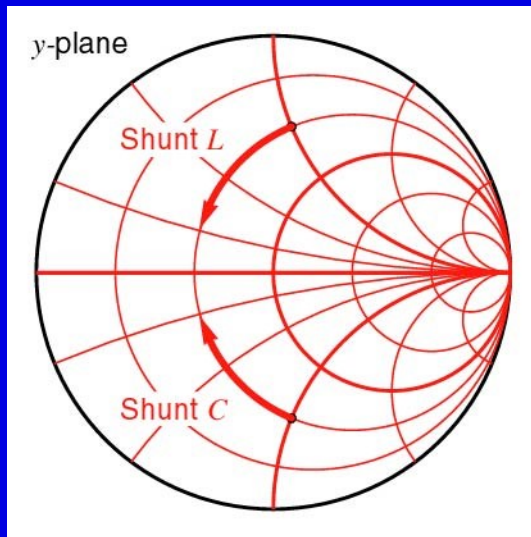
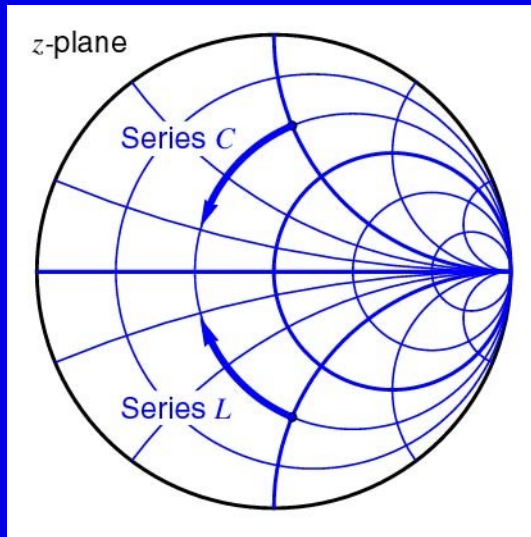
High-pass,  $R_L > R_S$



High-pass,  $R_L < R_S$

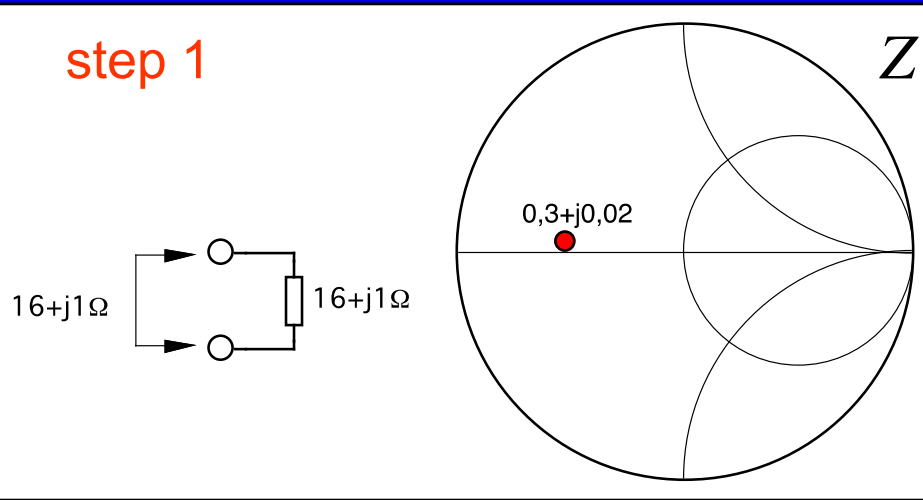
- Series component in series with the smallest impedance
- Shunt component in parallel with the largest impedance

# Designing Reactive Circuit Elements in the Smith Chart



# Experiment: Design a matching network by using the Smith chart and the VNA @ 775 MHz

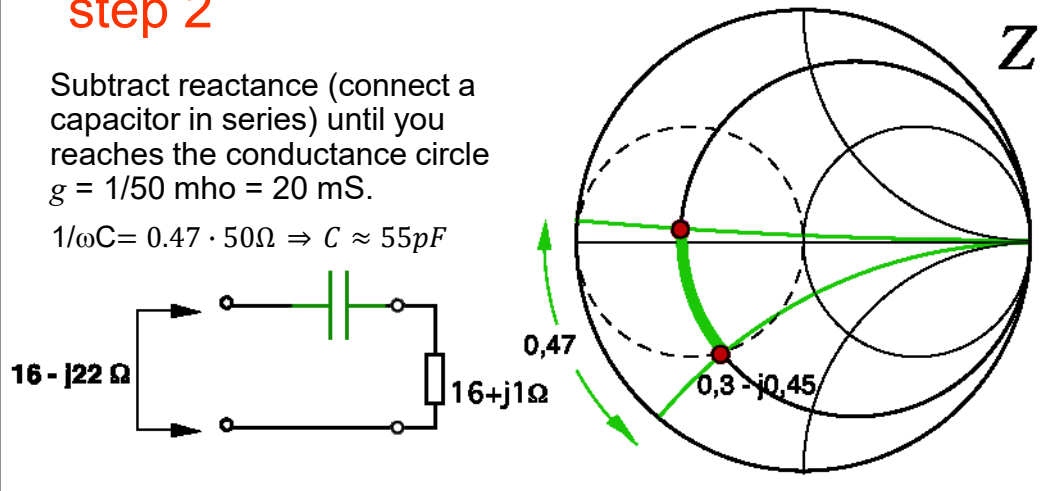
## step 1



## step 2

Subtract reactance (connect a capacitor in series) until you reach the conductance circle  $g = 1/50 \text{ mho} = 20 \text{ mS}$ .

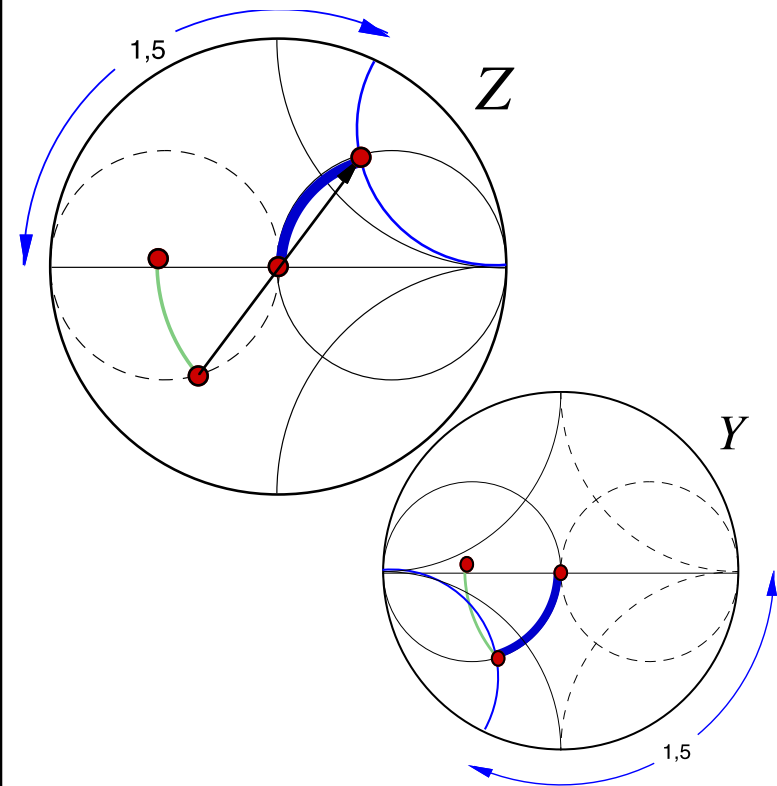
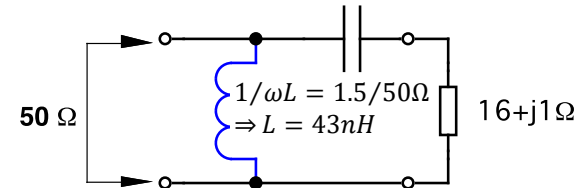
$$1/\omega C = 0.47 \cdot 50 \Omega \Rightarrow C \approx 55 \text{ pF}$$



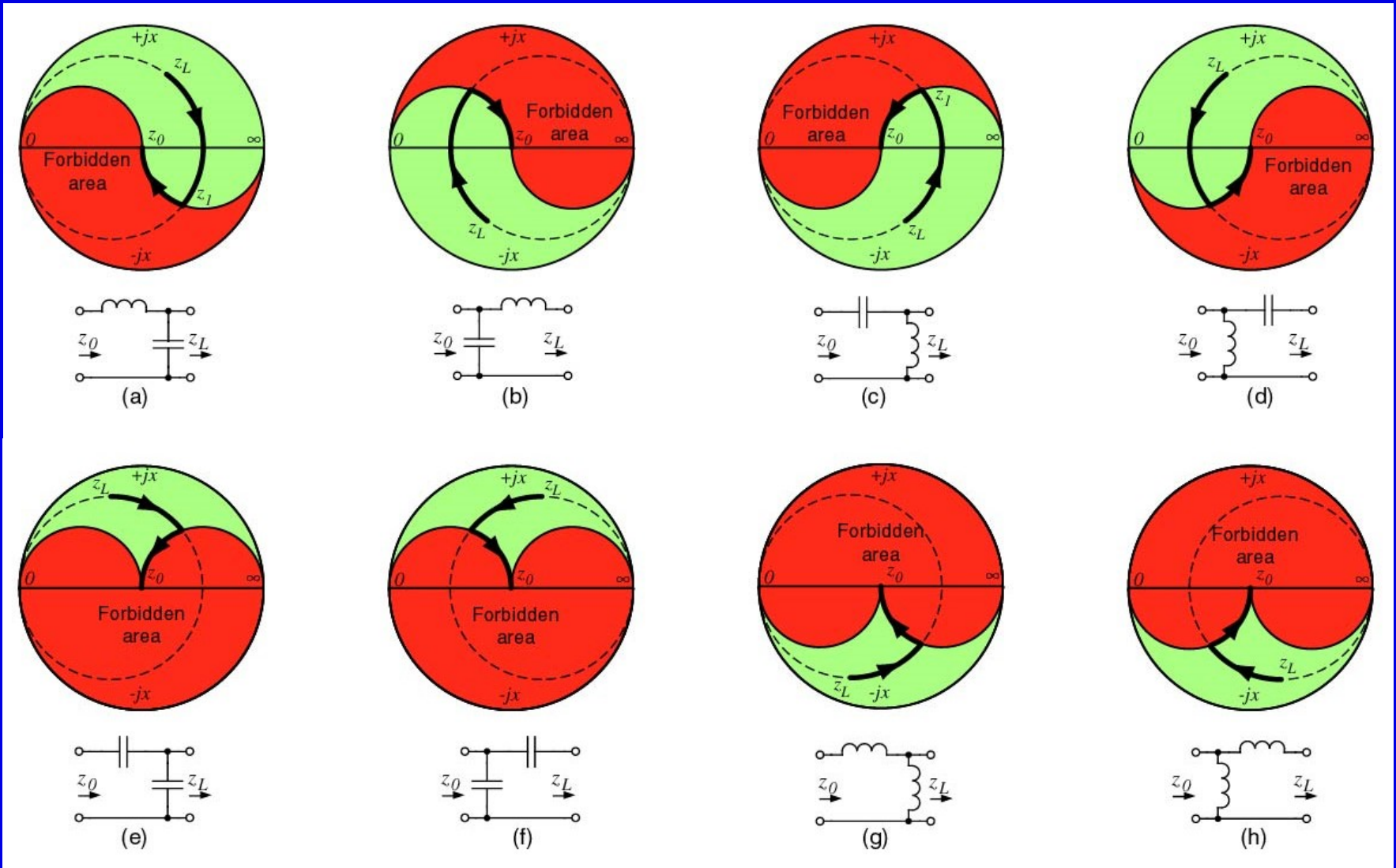
## step 3

Add susceptance (connect an inductor in parallel) until you end up in  $Y = Y_0 = 1/50 \text{ mho} = 20 \text{ mS}$ .

$$1/\omega C = 0.47 \cdot 50 \Omega$$



# Eight possible L-type Networks

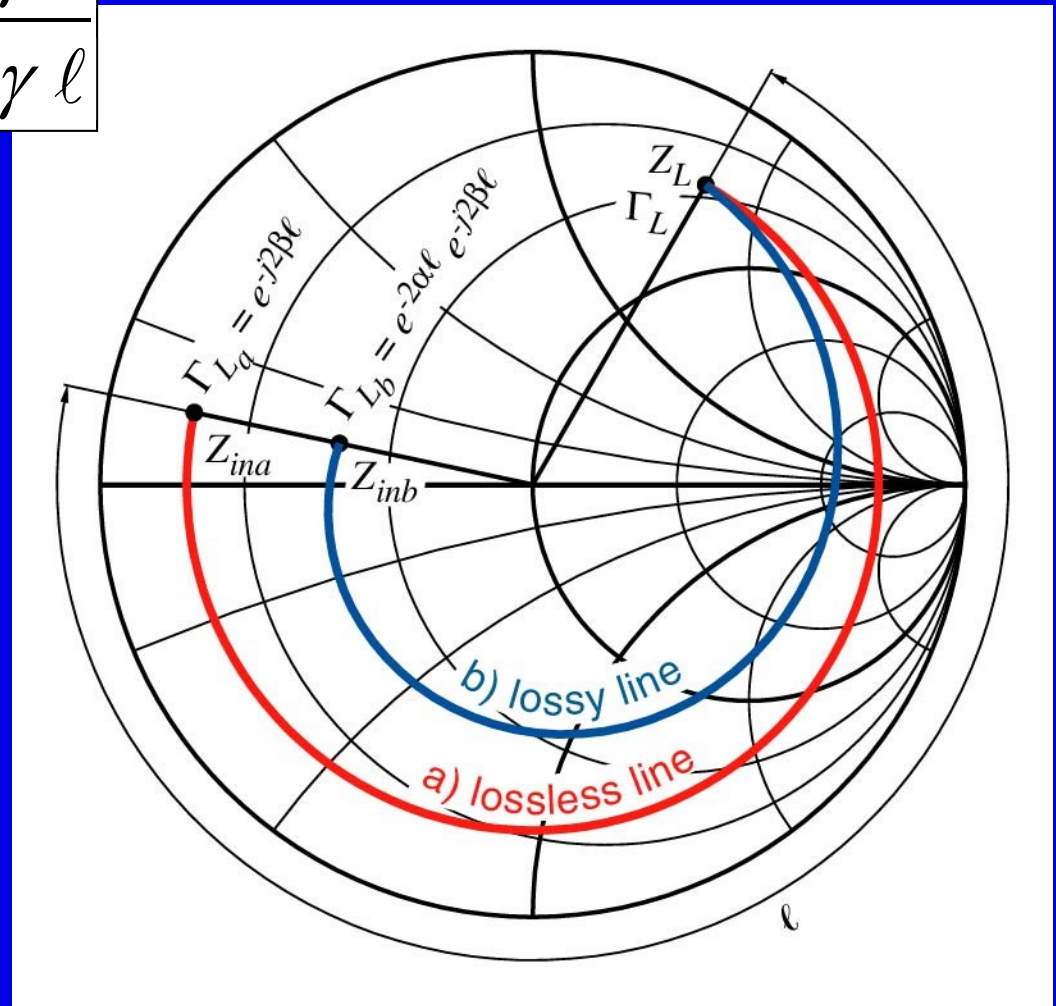
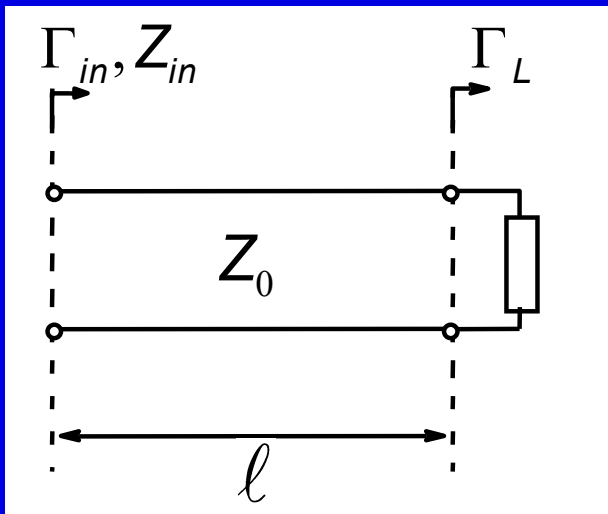


# Matching Networks by Line Structures

- Transformation by a cascaded transmission line
  - line section with optimised length and  $Z_0$
  - quarter-wave transformer
  - matching by multiple sections
- Stubs
  - short-circuited and open-circuited stubs
  - symmetrical stubs

# Impedance Transformation by a Single Serial Line

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma \ell}{Z_0 + Z_L \tanh \gamma \ell}$$



a) lossless line

b) lossy line

# The Quarter-Wave Transformer

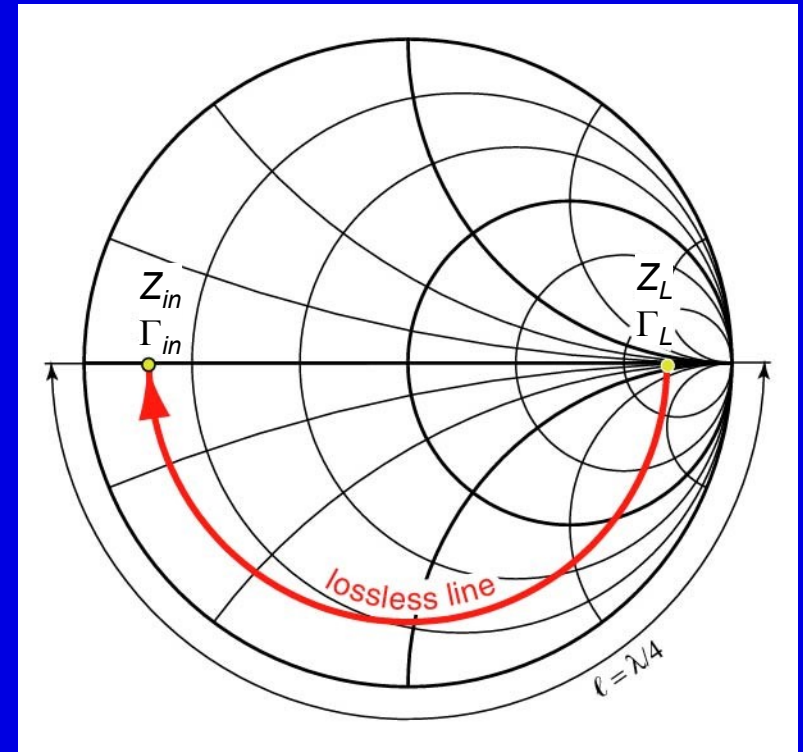
- For a line at the length  $\lambda/4$  is

$$Z_{in} = \frac{Z_0^2}{Z_L} \text{ if } Z_L \text{ is resistive}$$

- may be used for matching between arbitrary resistive source and load impedances.

$$Z_0 = \sqrt{Z_L Z_{in}}$$

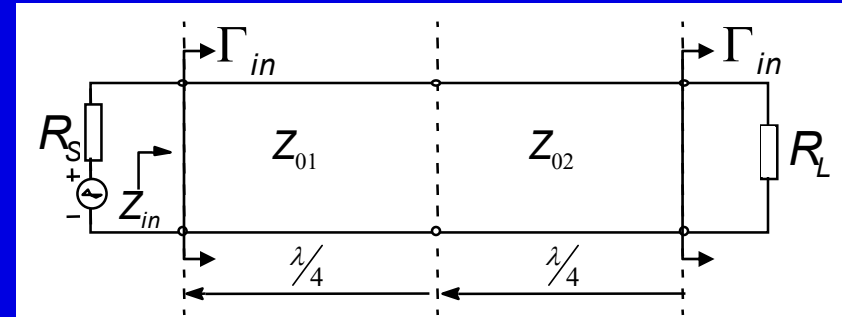
Geometric mean value  
of source and load!





# Quarter-Wave Transformer at Multiple Sections

- Transformation in several and minor impedance steps may provide a larger bandwidth
- If the characteristic impedances are distributed according to binomial coefficients maximum-flatness is achieved.



Ex. three sections:

$$Z_{01} = R_S \left( \frac{R_L}{R_S} \right)^{\frac{1}{8}}$$

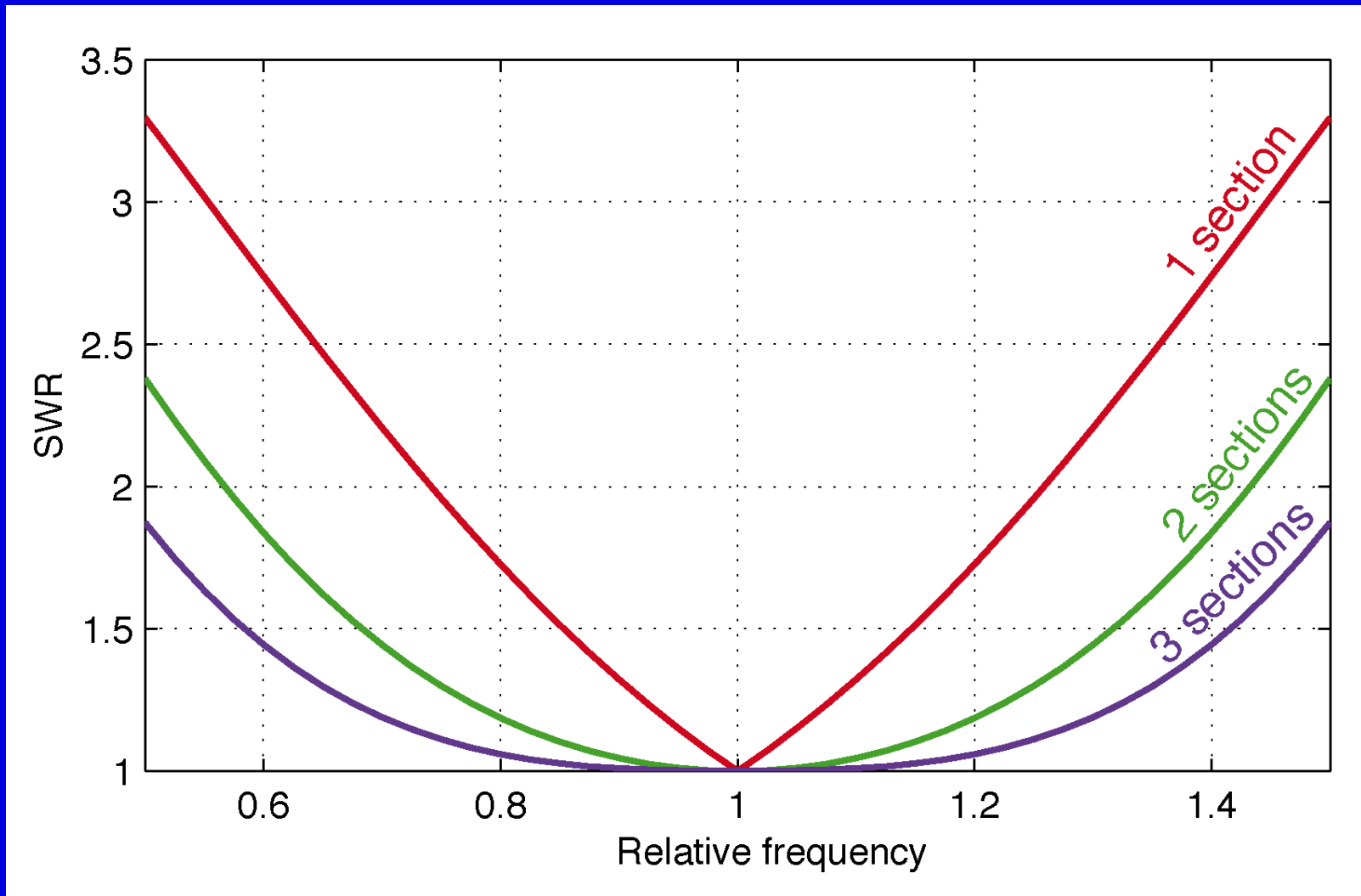
$$Z_{02} = Z_{01} \left( \frac{R_L}{R_S} \right)^{\frac{3}{8}} = R_S \left( \frac{R_L}{R_S} \right)^{\frac{4}{8}}$$

$$Z_{03} = Z_{02} \left( \frac{R_L}{R_S} \right)^{\frac{3}{8}} = R_S \left( \frac{R_L}{R_S} \right)^{\frac{7}{8}}$$

$n \downarrow k \rightarrow$	0	1	2	3	4	5
0	1					
1	1	1				
2	1	2	1			
3	1	3	3	1		
4	1	4	6	4	1	
5	1	5	10	10	5	1

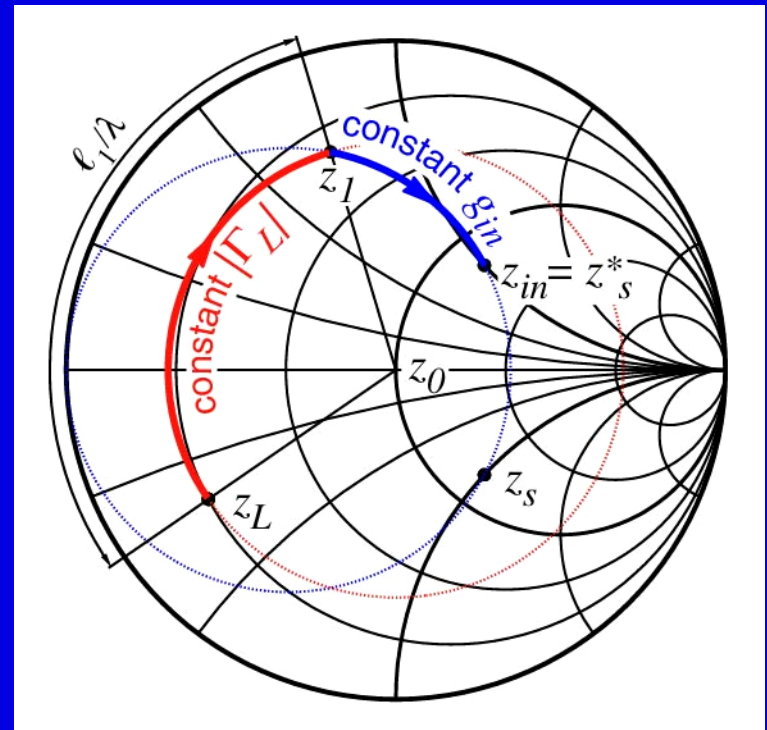
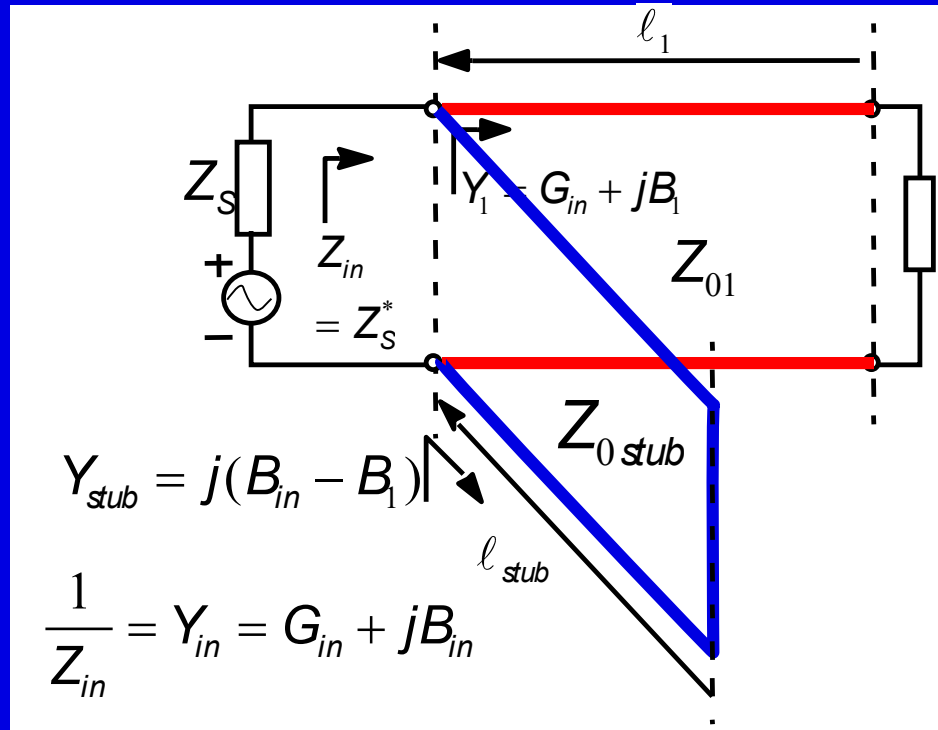
Binomial coefficients

# Quarter-Wave Transformer at Multiple Sections (cont.)

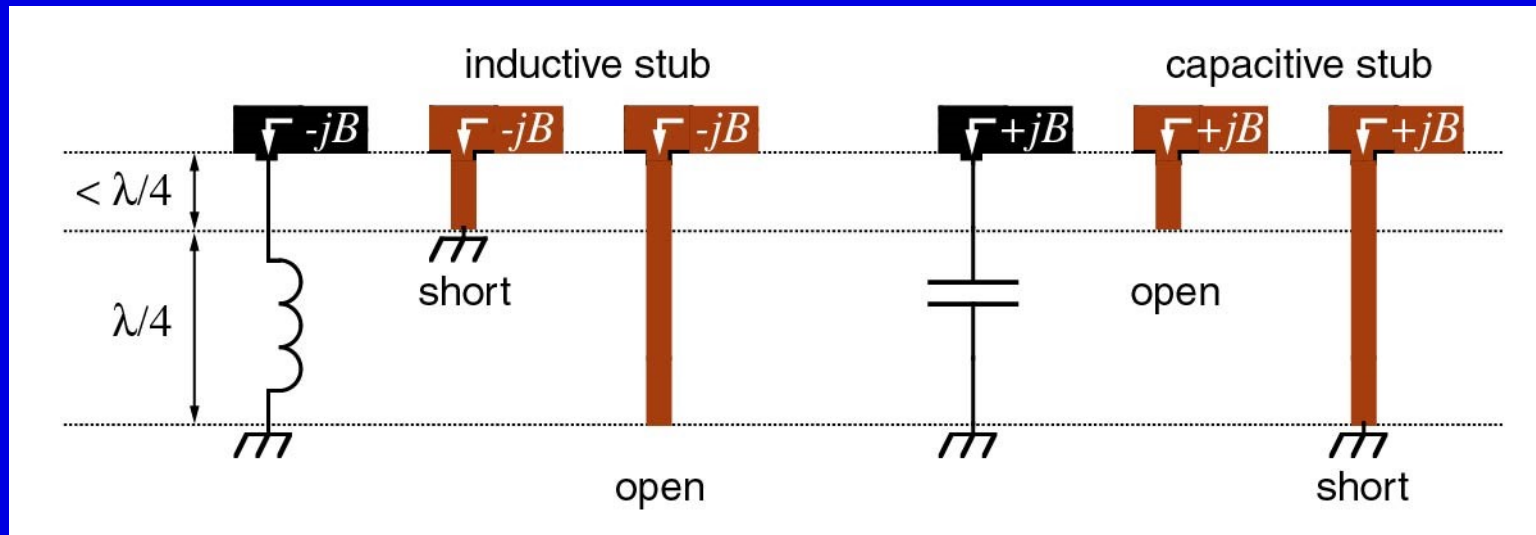


# Matching by Stubs and Cascaded Lines

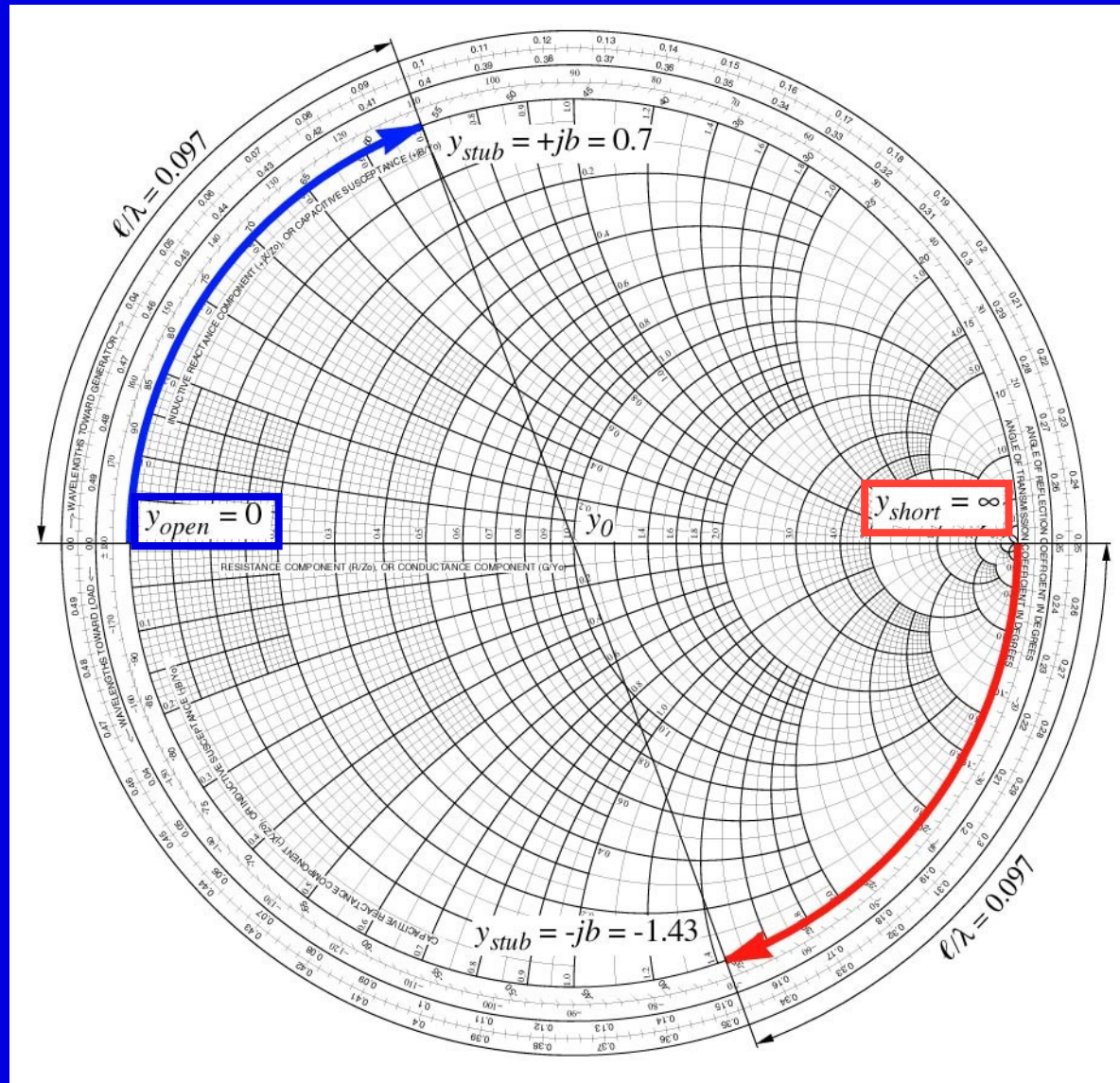
- Short-circuited or open line sections may be used as reactive shunt elements.
- Combined with cascaded line, matching can be achieved between arbitrary loci in the Smith chart.



# Length and Termination of the Stubs

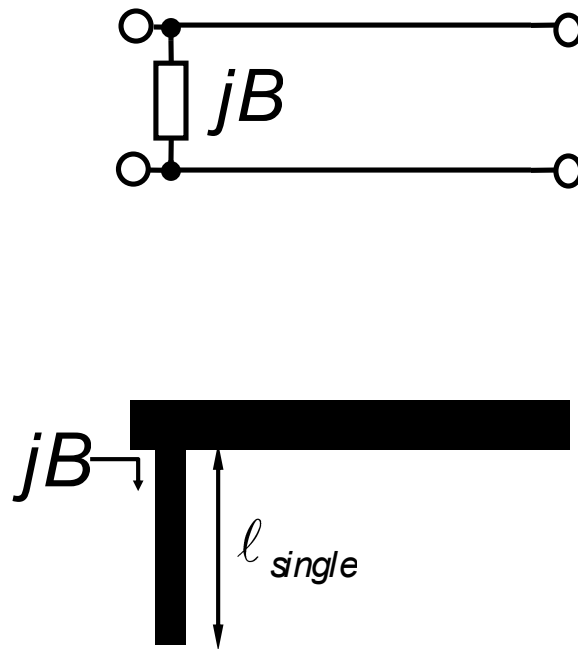


# Designing the Length of the Stub

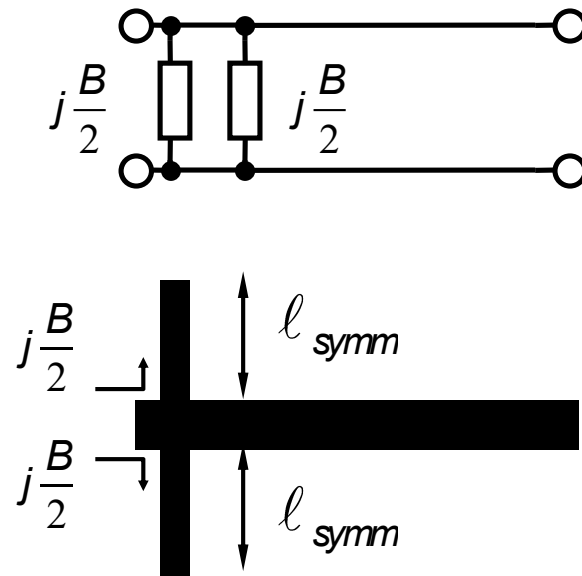


# Symmetrical Stubs

Single stub



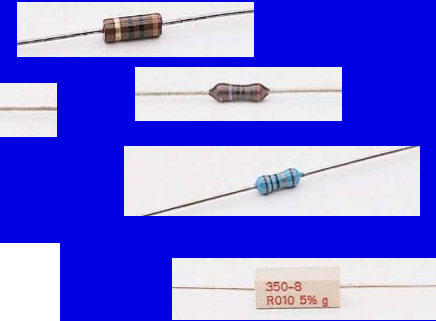
Symmetrical stubs



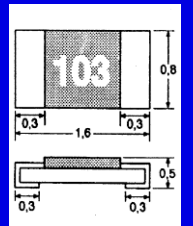
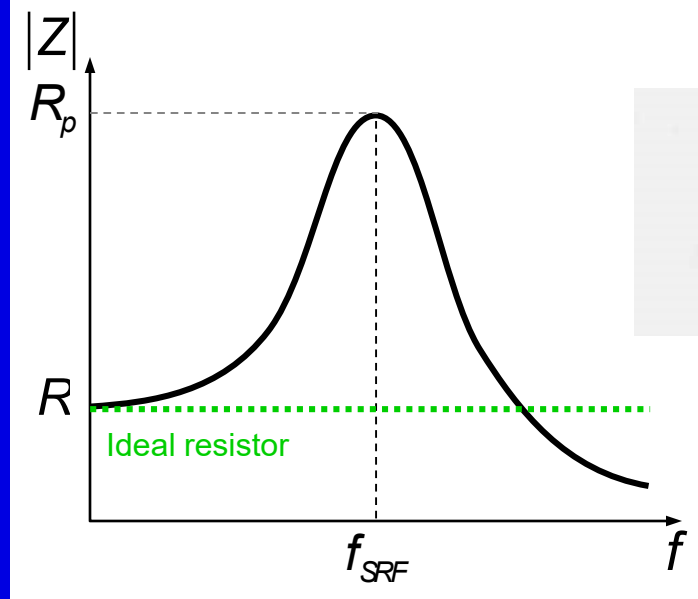
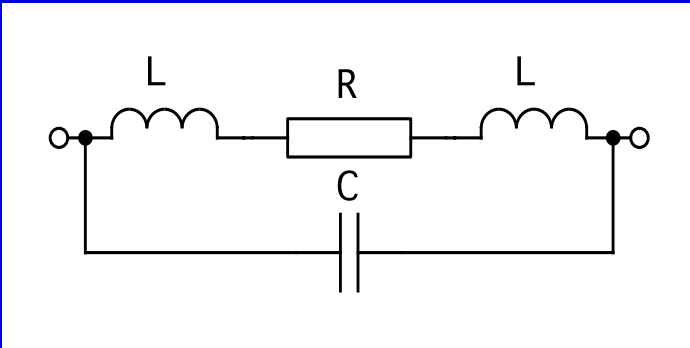
- Note:  $l_{single} \neq 2 \cdot l_{symm}$

- The length of the symmetrical stubs is designed to **individually provide the half value of the requested susceptance.**

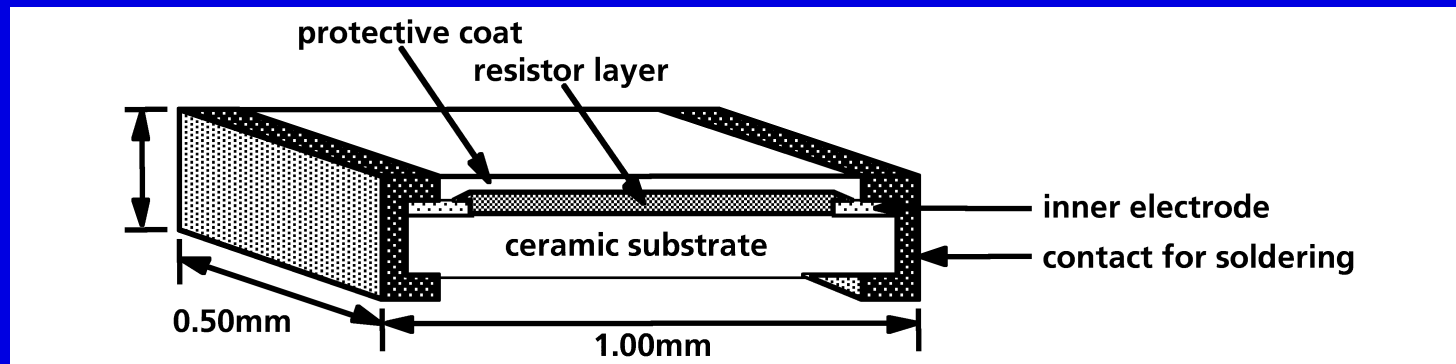
# Resistors



## high-frequency model

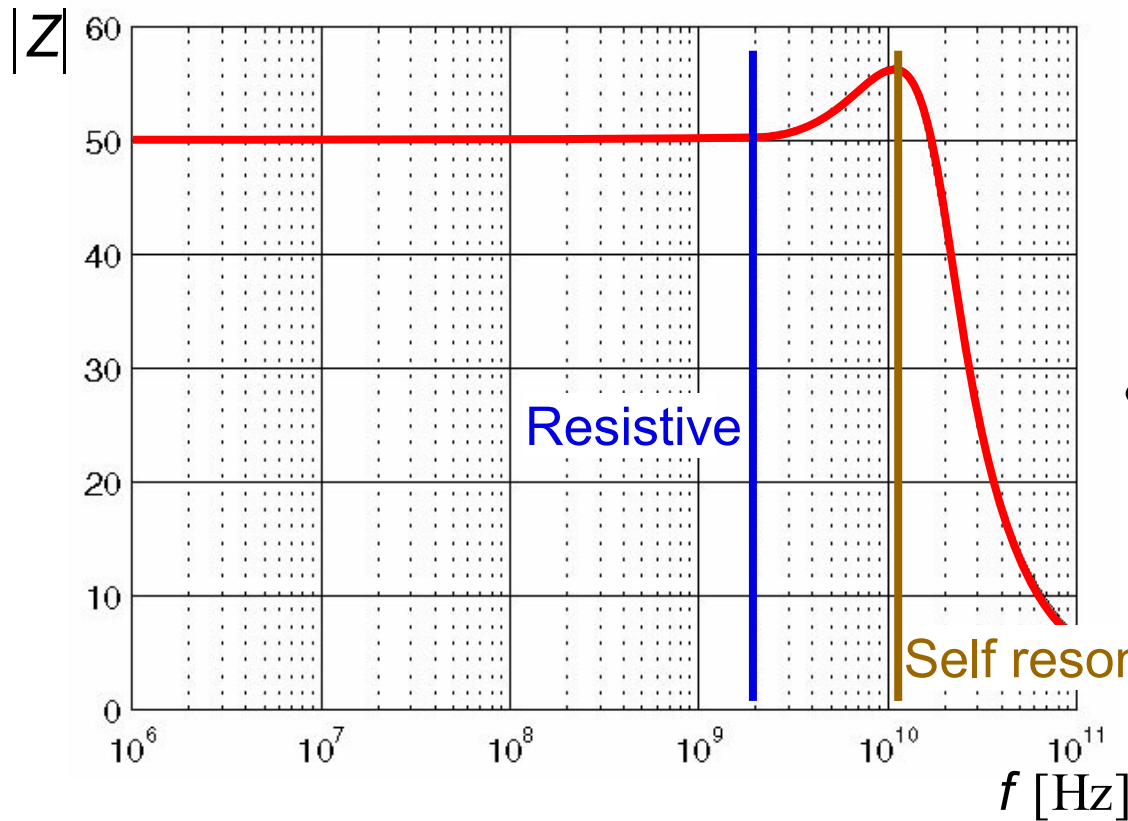


$f_{SRF}$  = Self Resonance Frequency

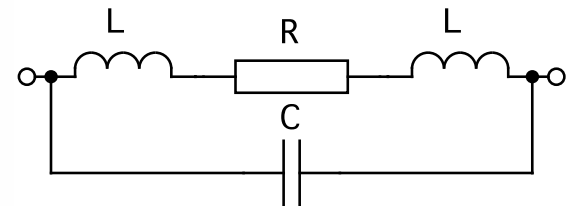


# Resistor

## Frequency characteristics - example



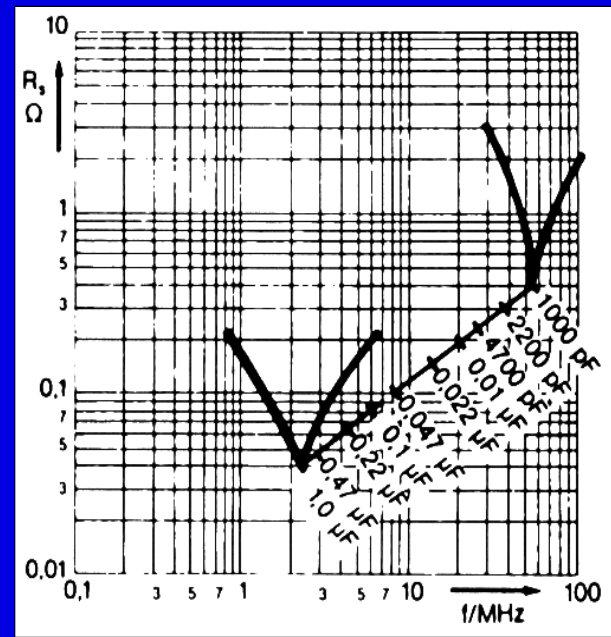
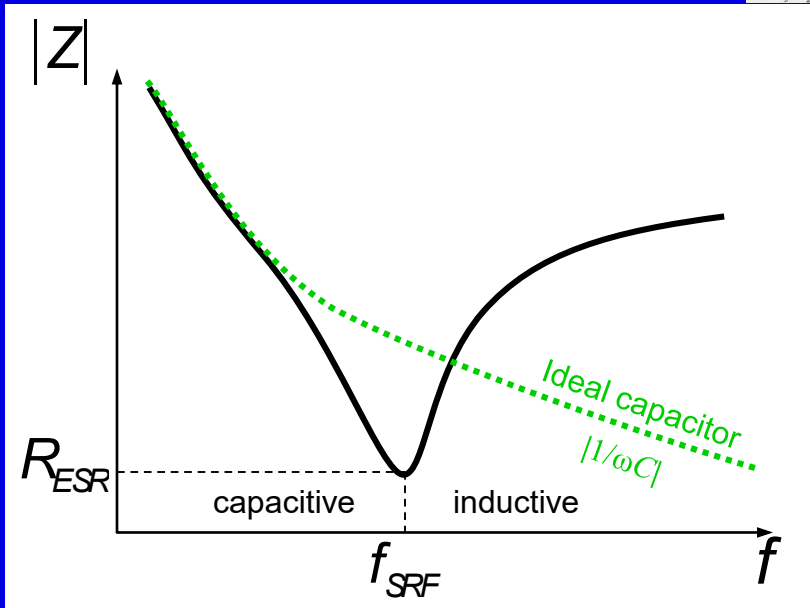
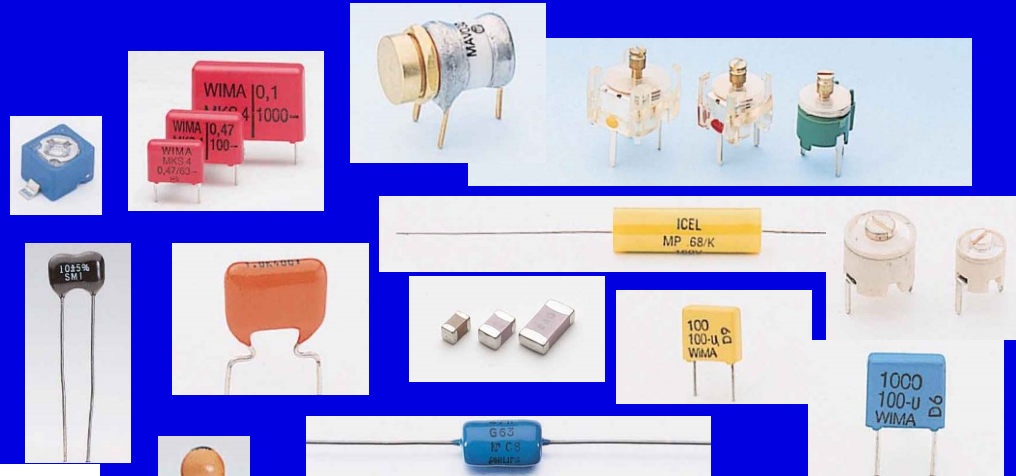
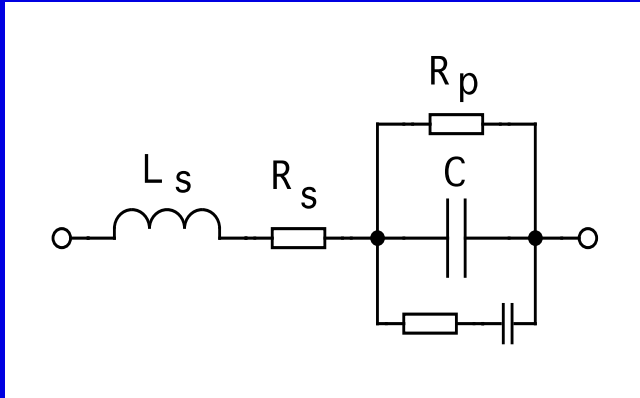
$R = 50\Omega$   
 $L = 0.25\text{nH}$   
 $C = 200\text{fF}$





# Capacitors

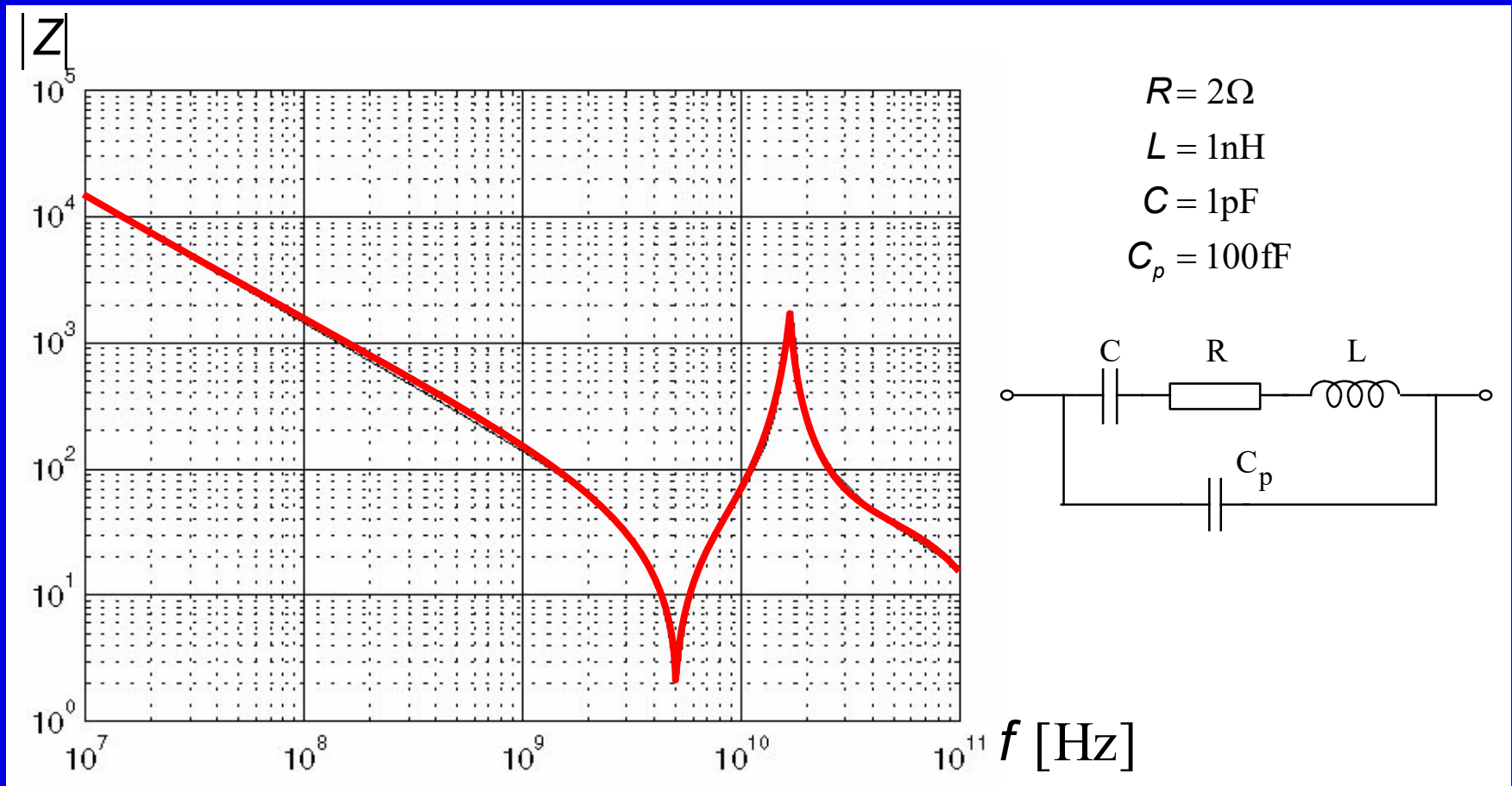
## high-frequency model



$R_{ESR}$  = equivalent series resistance

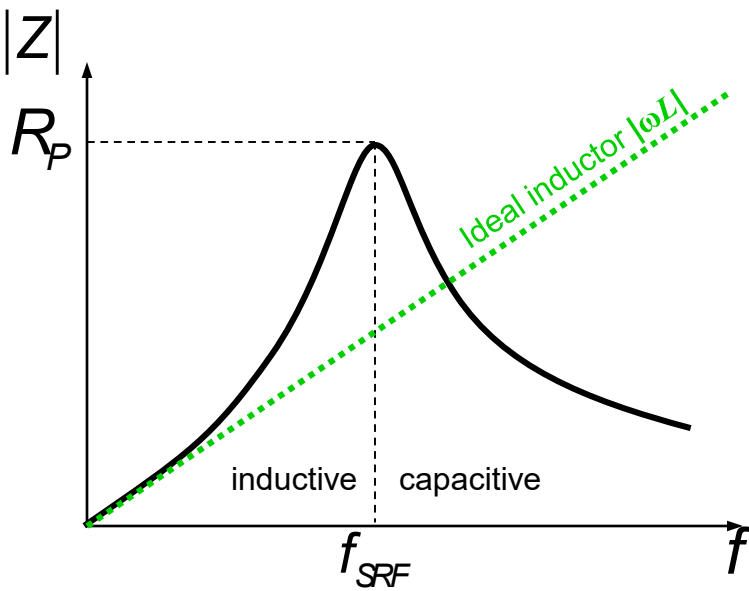
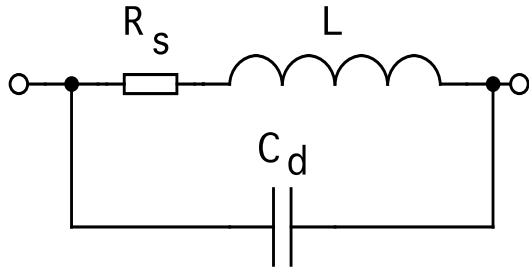
# Capacitor

## Frequency characteristics - example

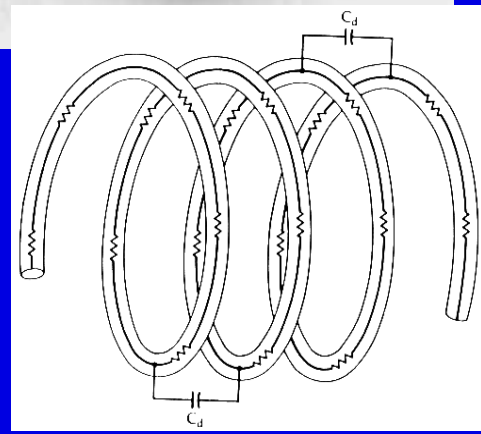
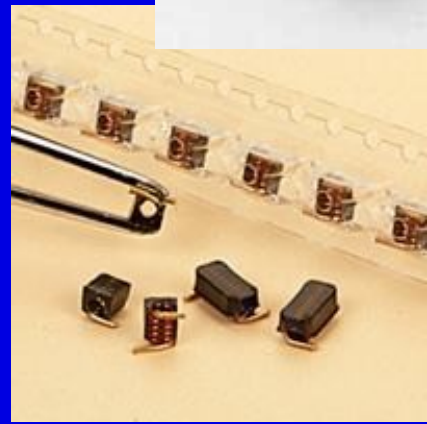


# Inductors

## high-frequency model

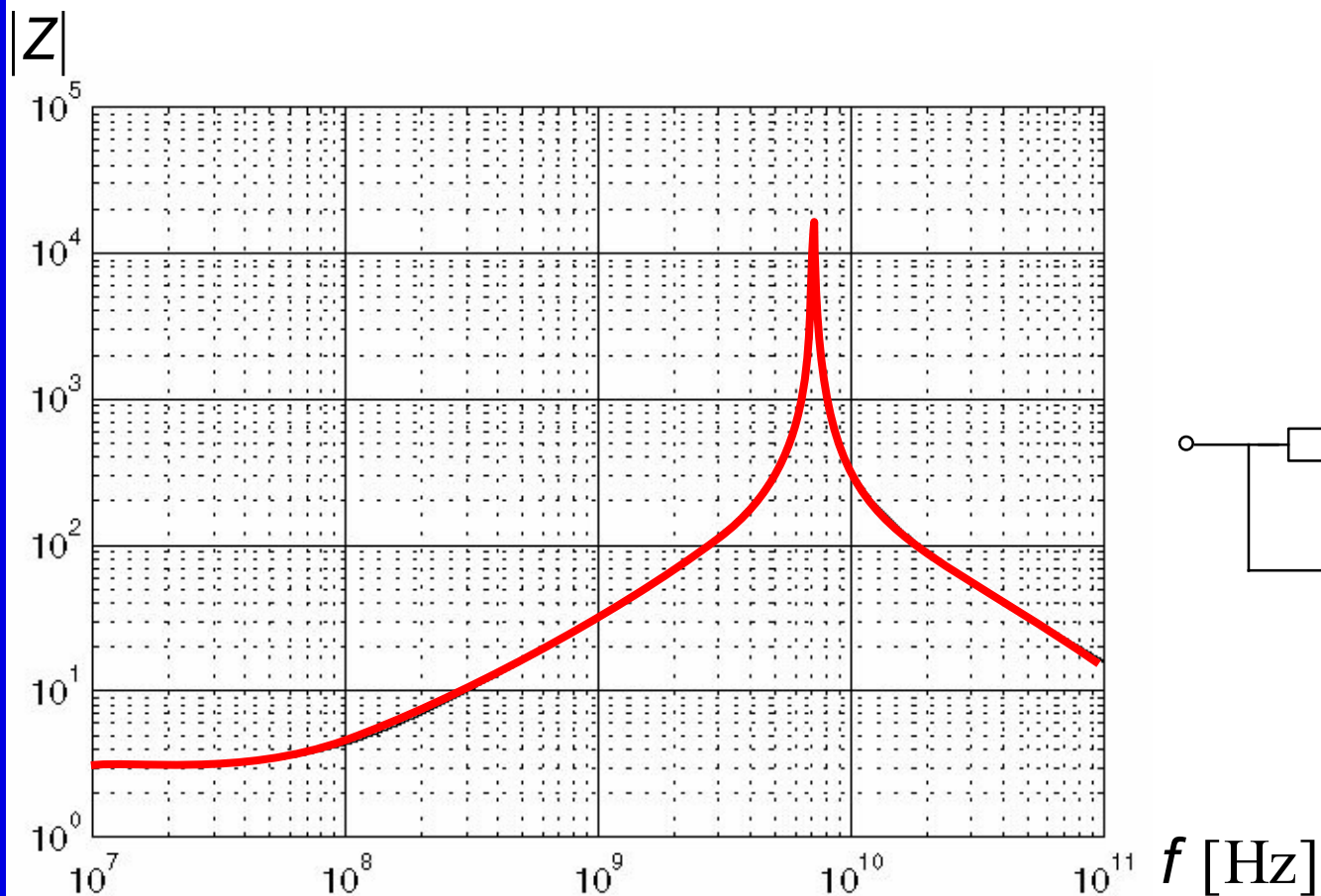


$R_p$  = equivalent parallel resistance

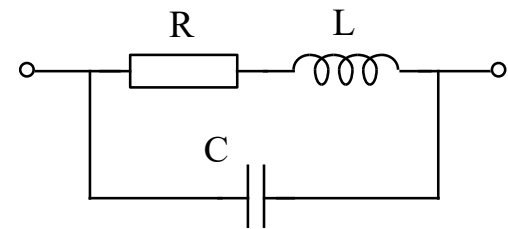


# Inductor

## Frequency characteristics - example



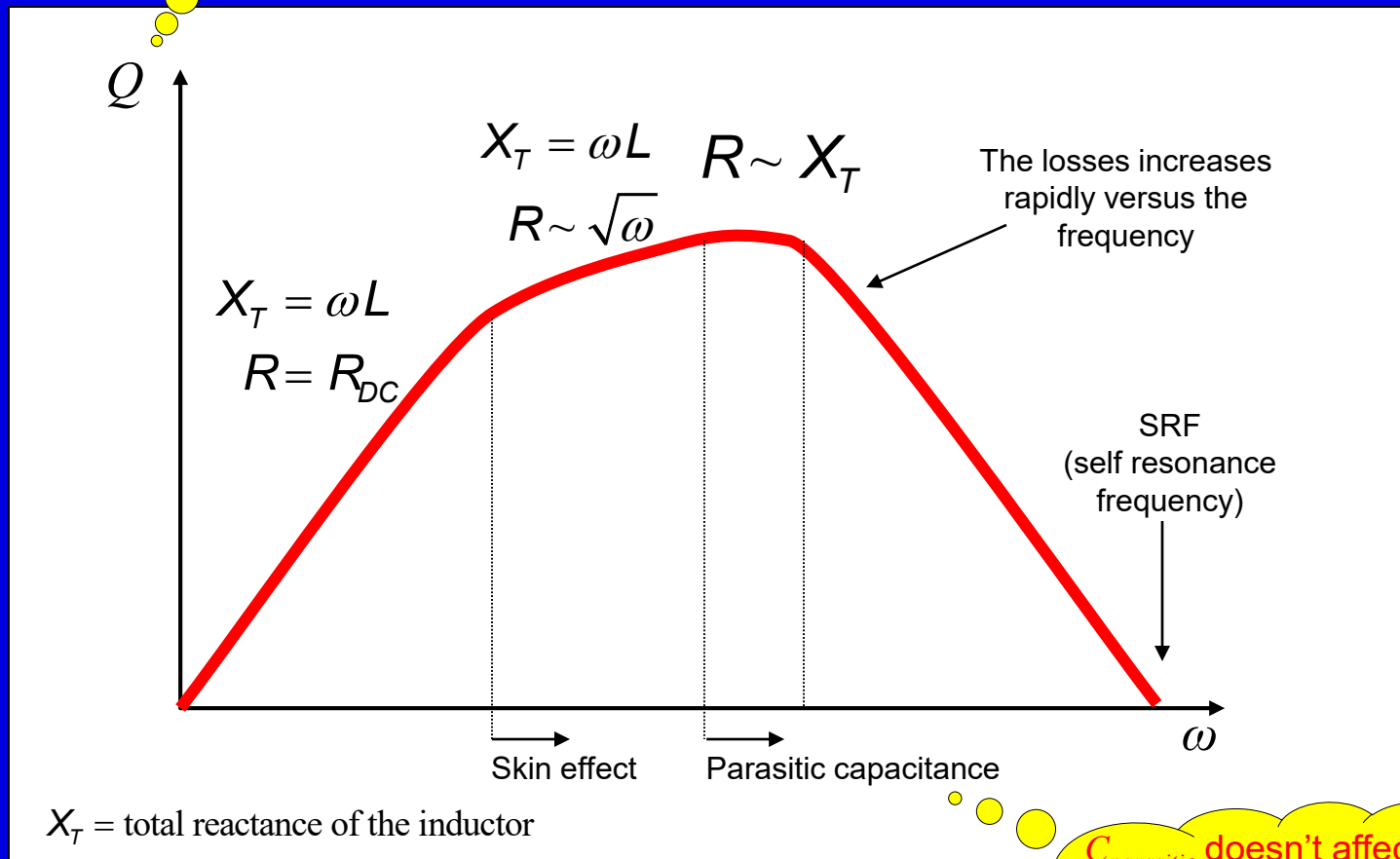
$$R = 3\Omega$$
$$L = 5\text{nH}$$
$$C_p = 100\text{fF}$$



# Inductor

$$Q = \frac{X}{R_s}$$

The  $Q$ -factor frequency dependence



$C_{parasitic}$  doesn't affect the  $Q$  if the inductor is used in a resonant circuit!

# Transformer



- Windings on a ferrite rod or toroid core
- These materials provides a high permeability that unfortunately decreases at higher frequencies
- Usable at best up to a few GHz

# Loss in Substrate Materials

- For dielectric materials often the **loss tangent** used to specify the losses:

$$\tan \delta = \frac{\sigma}{\omega \epsilon}$$

- for material with low loss  $\tan \delta \approx \delta$  where  $\delta$  is the loss angle

- The loss tangent is also related to the quality factor or  $Q$ -factor of the material:  $\tan \delta = \frac{1}{Q_d}$

# Properties of some Substrate Materials

<b>Material</b>	$\epsilon_r$	<b><math>\tan\delta</math></b>
Ceramic	10	0.004
Beryllium oxide	6	0.0003
Duorid	2.6	0.0001
Teflon fibre-glass	2.3	0.0015
Silicon	11.7	0.004
Quarz	3.8	0.0001
Epoxy fibre-glass	4	0.02
Aluminium oxide	10	0.0003

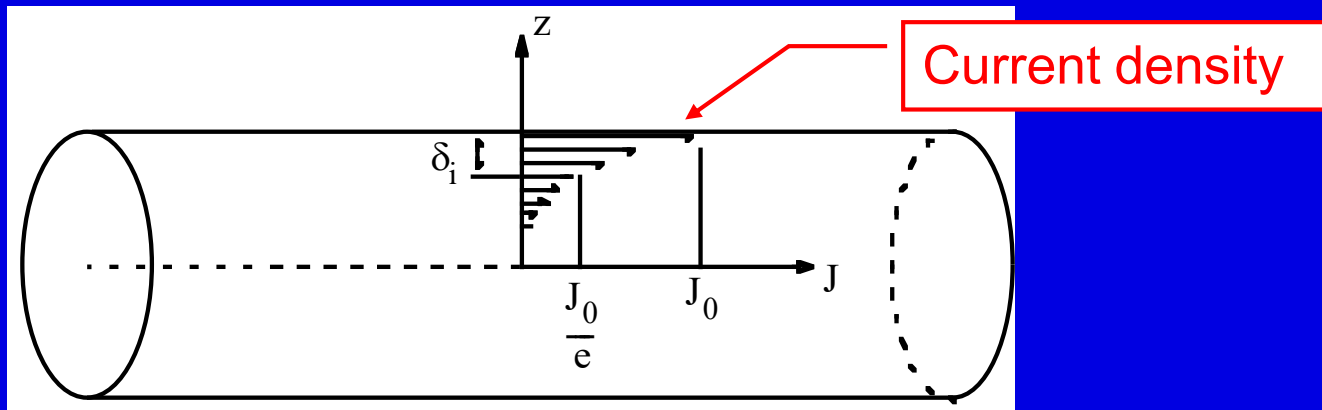


# Properties of some Metals

<b>Metal</b>	<b><math>\sigma</math> [S/m]</b>
Aluminium	$3.816 \cdot 10^7$
Copper	$5.813 \cdot 10^7$
Gold	$4.098 \cdot 10^7$
Iron	$1.03 \cdot 10^7$
Silver	$6.173 \cdot 10^7$

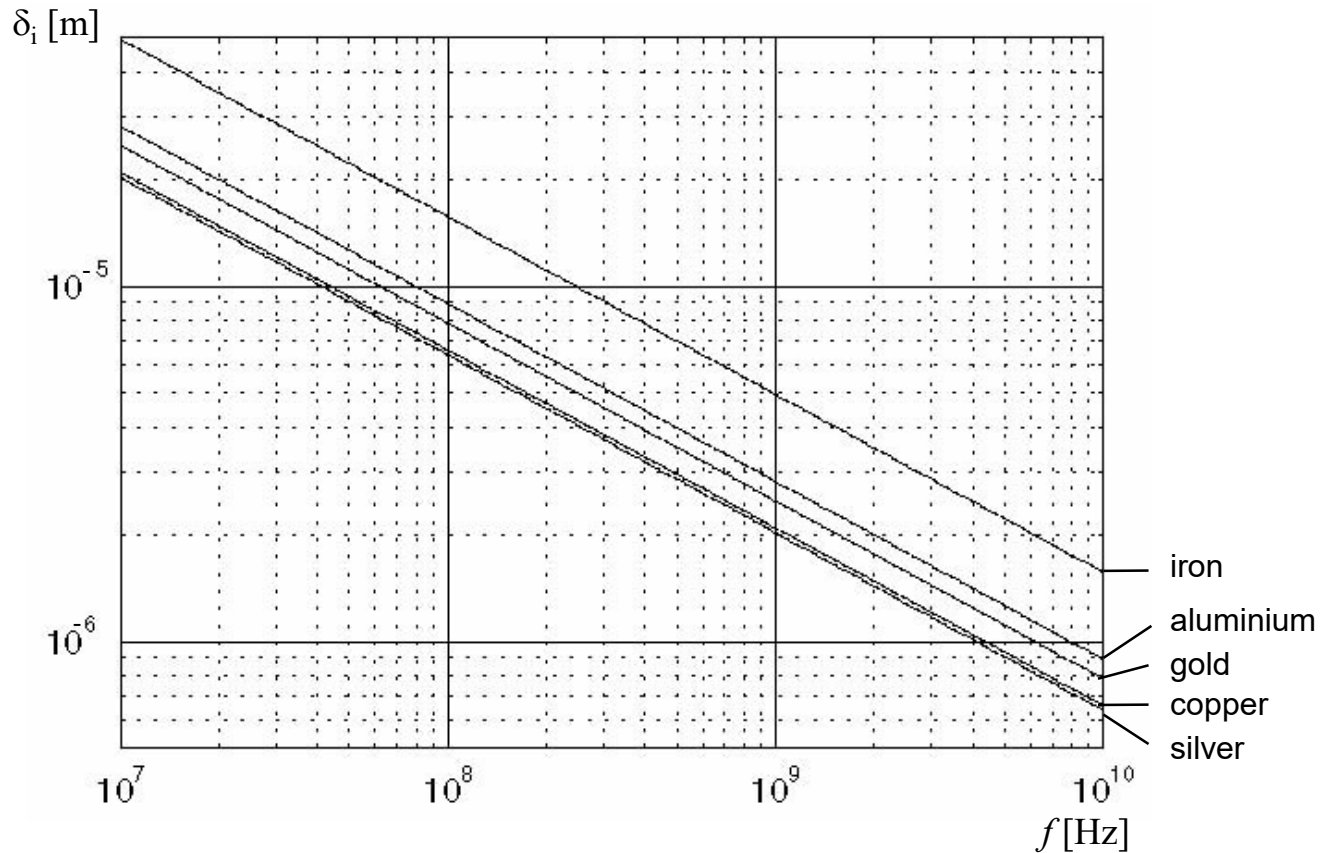
# Skin Effect

- The magnetic flux inside the conductor will effectively push the current to a narrow region close to the surface

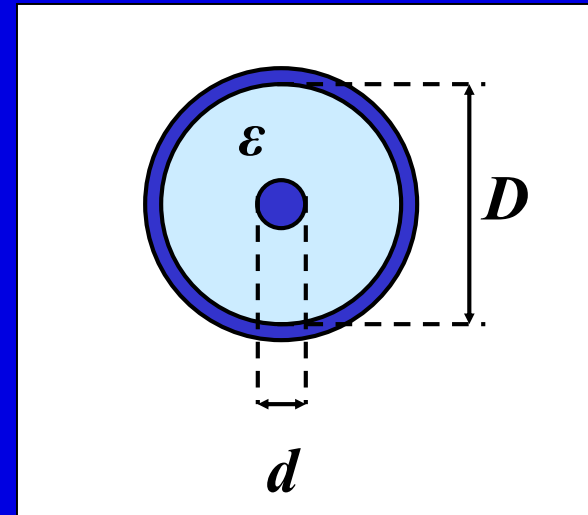
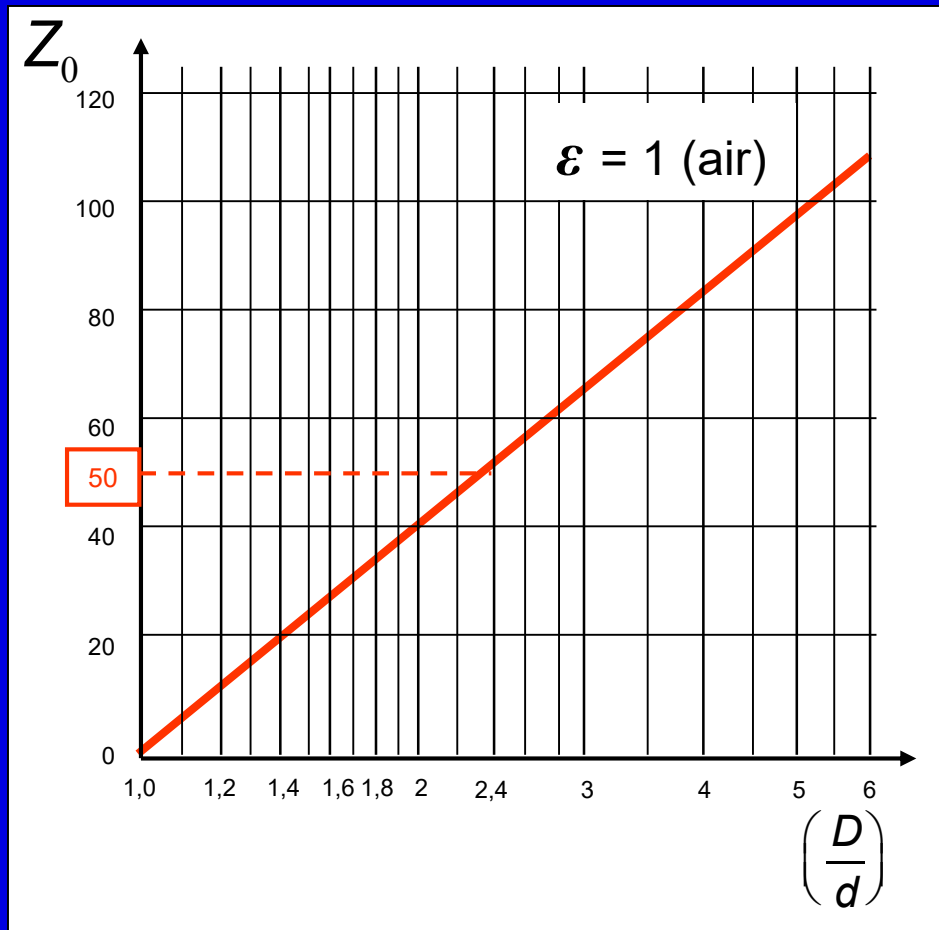


- Skin depth:  $\delta_i = \sqrt{\frac{2}{\omega\mu\sigma}}$  = the distance were the current density has decreased by a factor  $e$
- A large circumference is more essential than a large cross section area of the conductor!

# Example of Skin Depth



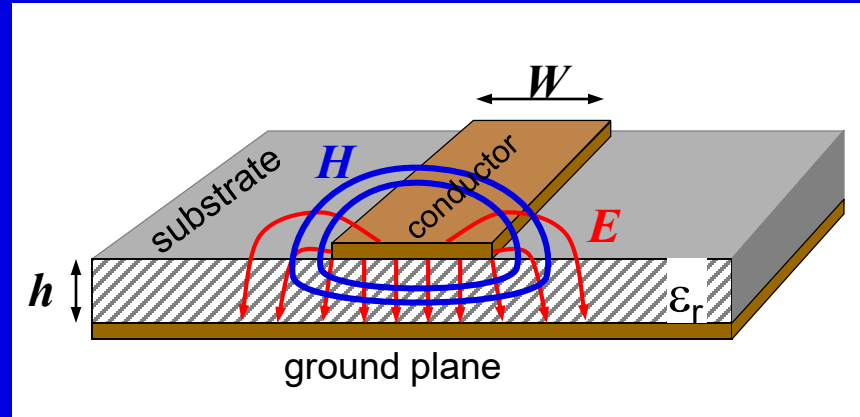
# Coaxial Cable Design: Char. Impedance



$$\begin{aligned} Z_0 &= \frac{138}{\sqrt{\epsilon_r}} \log^{10} \left( \frac{D}{d} \right) = \\ &= \frac{60}{\sqrt{\epsilon_r}} \ln \left( \frac{D}{d} \right) \end{aligned}$$

# Microstrip

Cross section of a microstrip structure



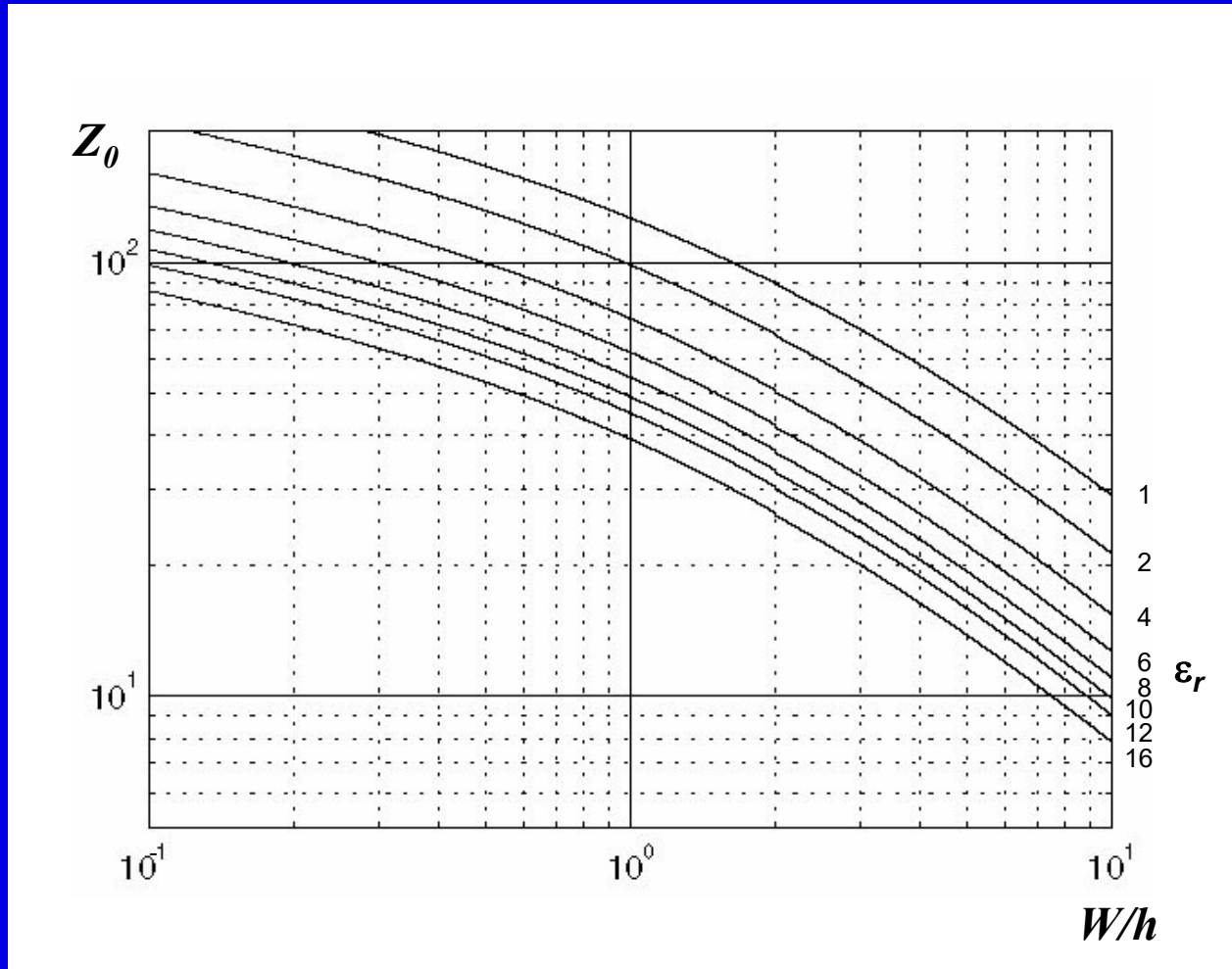
Complex geometry and non-uniform fields makes a complicated flux image.

$\epsilon_r$  in the substrate is therefore not usable for calculation of for example electrical length in the line structure.

Instead the **effective permittivity**,  $\epsilon_{eff}$ , based on the dimensions of the microstrip structure, may be used.

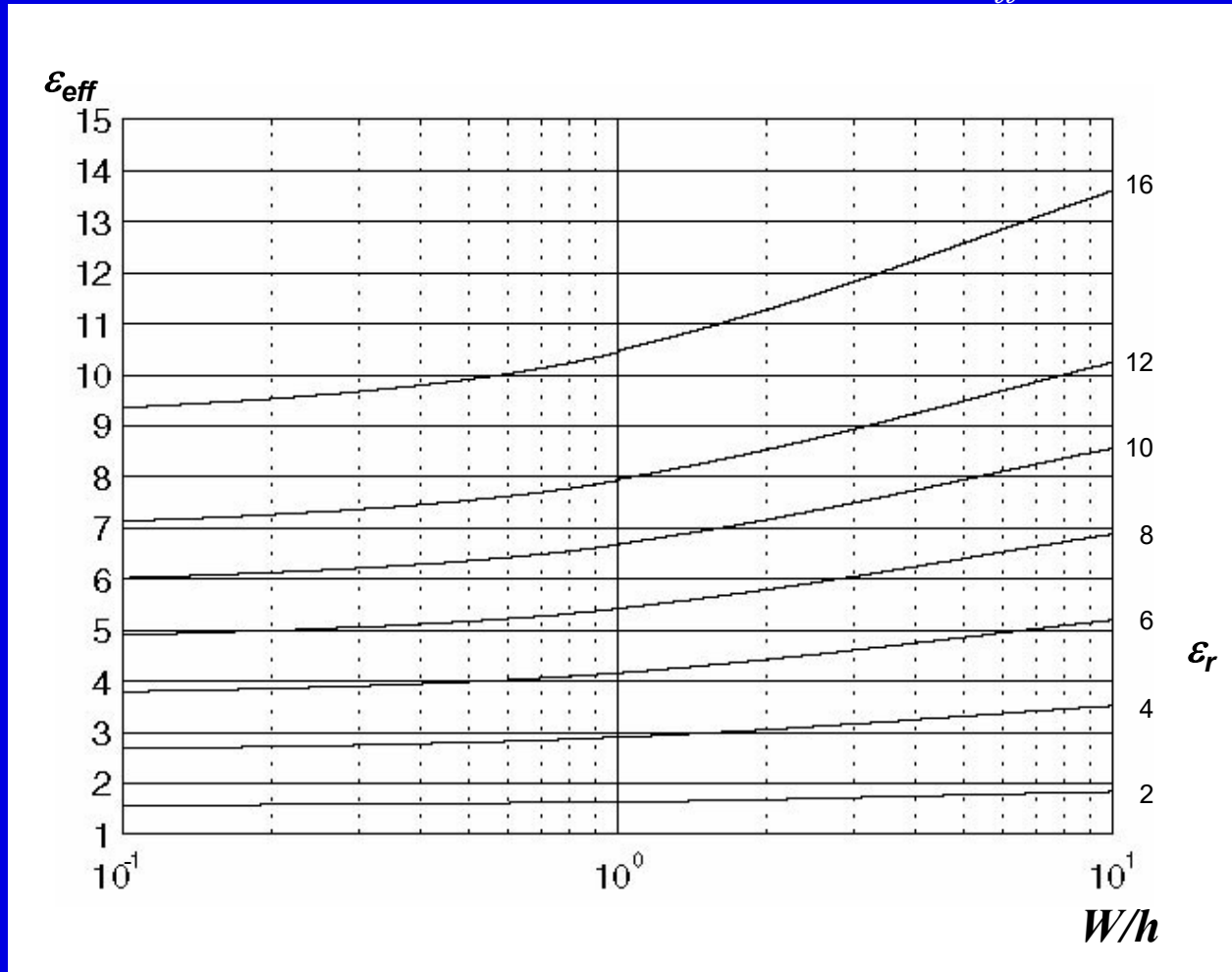
# Microstrip Design: Char. Impedance

Determine the proper  $W$  for a specified  $Z_0$  and  $h$



# Microstrip Design: Eff. Permittivity

Determine the effective permittivity  $\epsilon_{eff}$



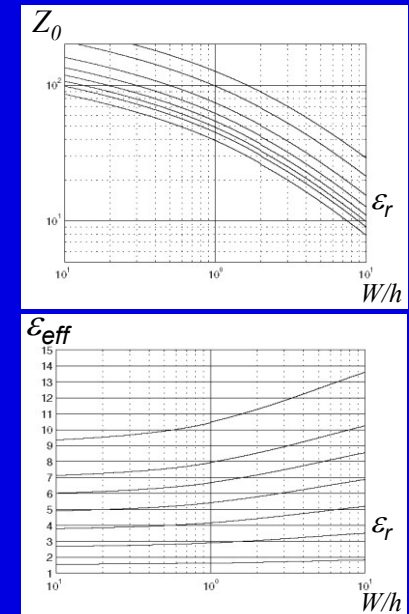
# Microstrip Design: Summary

Specified  $Z_0$ , substrate height  $h$  and permittivity  $\epsilon_r$ :

1. Select the width  $W$  in diagram 1
2. Select the effective permittivity  $\epsilon_{eff}$  in diagram 2

3. Calculate the effective wavelength

$$\lambda_{eff} = \frac{\lambda_0}{\sqrt{\epsilon_{eff}}}$$

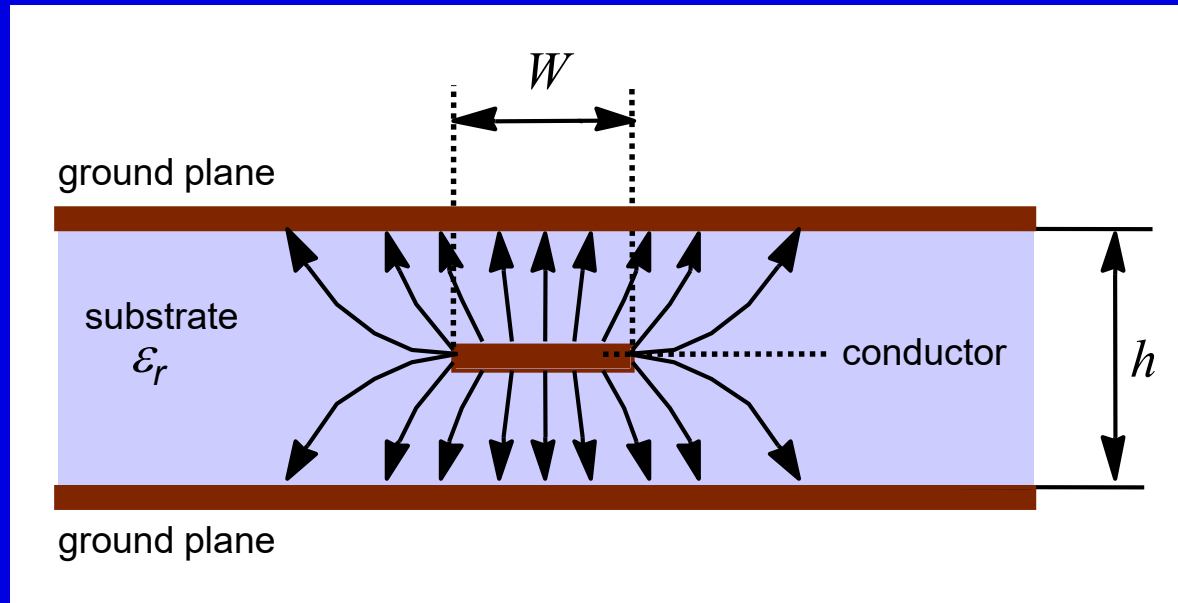


Specified electrical length  $l_{el}$

4. Calculate the physical length  $l = l_{el} \lambda_{eff}$

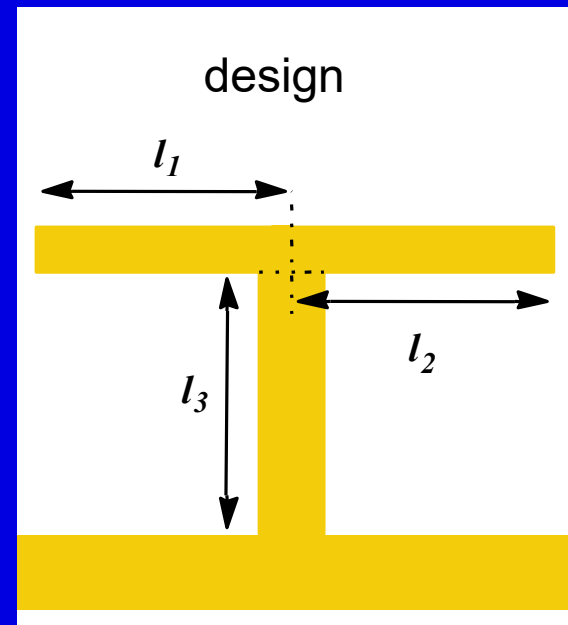
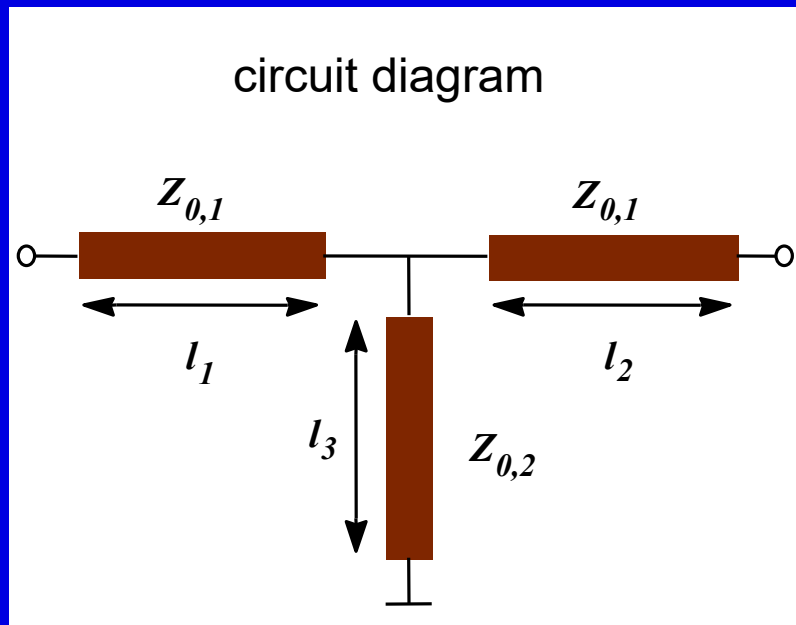


# Stripline



# Discontinuities

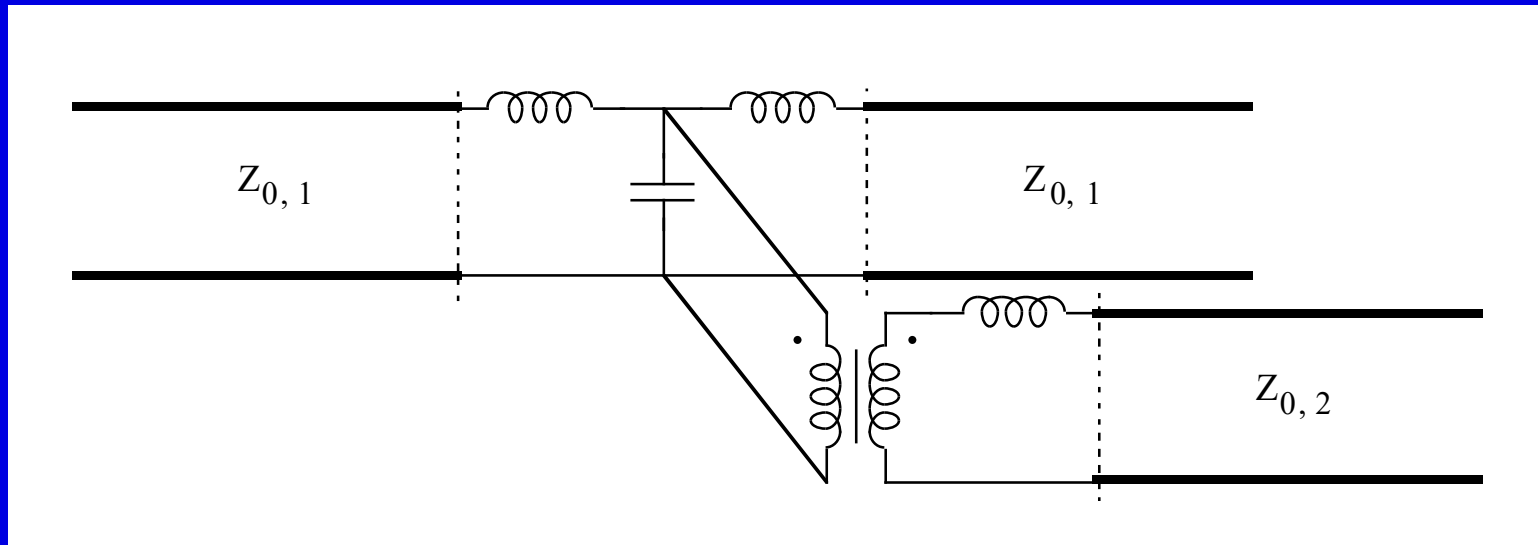
- The design of circuits containing transmission lines must take into account the impact of junctions and transitions between different line widths.



- How to model such a geometry?

# Discontinuities (cont.)

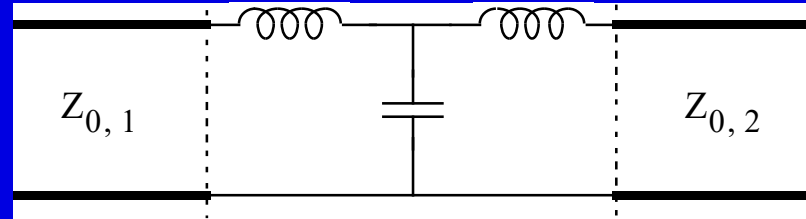
- T-junction, equivalent model



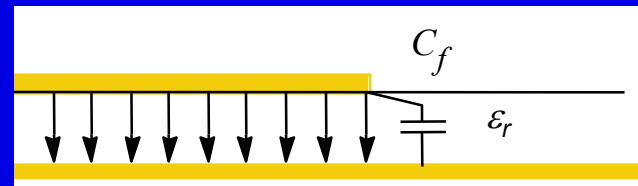
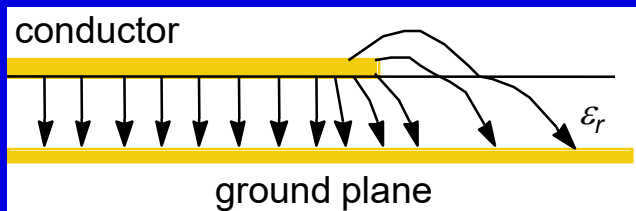
- Methods for manual calculation of the circuit elements in the equivalent model may be found in handbooks.

# Discontinuities (cont.)

- Symmetrical step, an abrupt transition between two different line widths



- Transmission line, open-circuit termination



# Discontinuities (cont.)

- Some other cases are
  - corners and bends
  - serial gap (to realize a serial capacitance)
  - undesired coupling between nearby structures
    - Crosstalk
- CAD tools are necessary for analysis and design of more complex circuits
  - ADS and others
- In worst case a complete structure may be simulated by the finite element method and Maxwell's equations which is both time and memory consuming