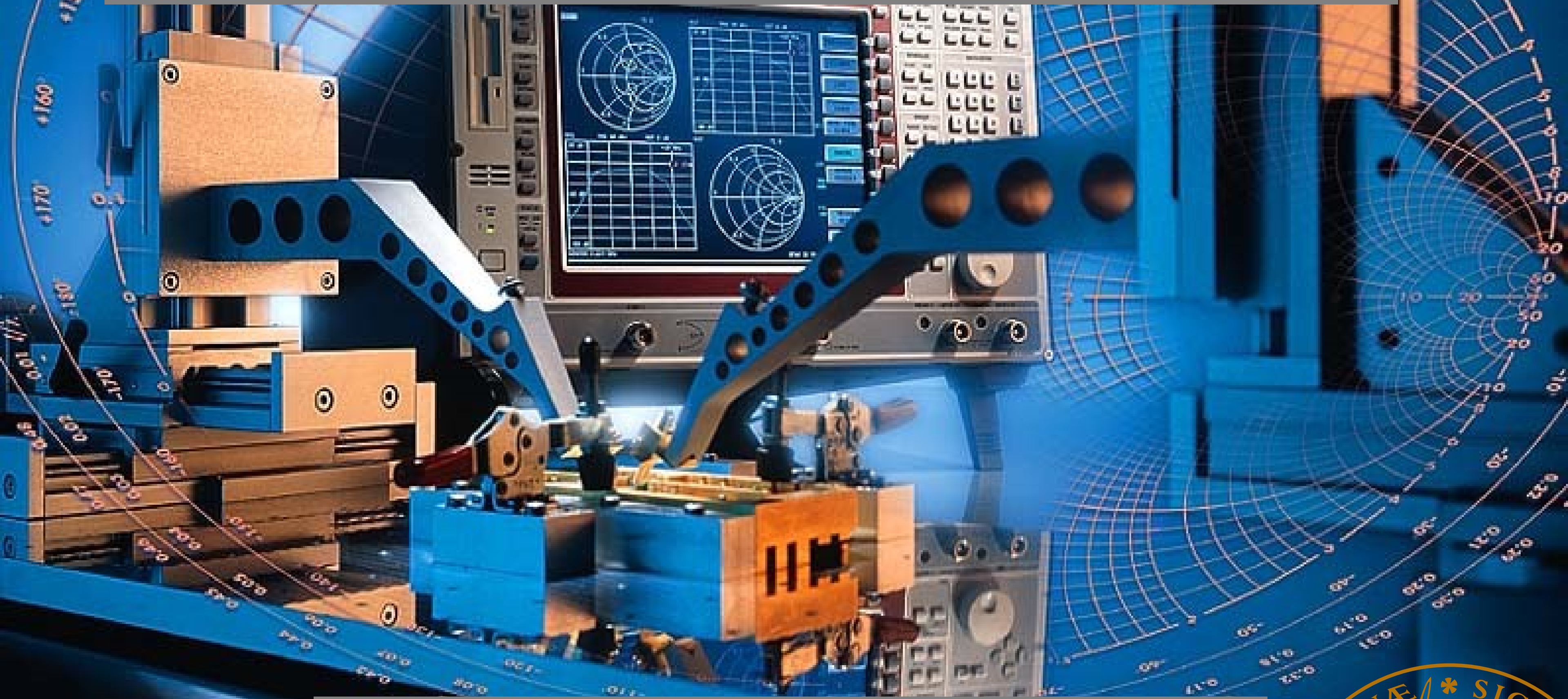
2019-11-06

Lecture 2

RF Amplifier Design



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Lecture 2

- Transmission lines
  - Concepts
    - waves
    - propagation constant
    - modelling transmission lines
    - characteristic impedance
    - phase velocity
    - reflection coefficient
    - impedance transformation
    - normalised impedance
    - standing-wave ratio
    - transmission line resonator
    - transmission factor
    - multiple reflection

From Source to Load

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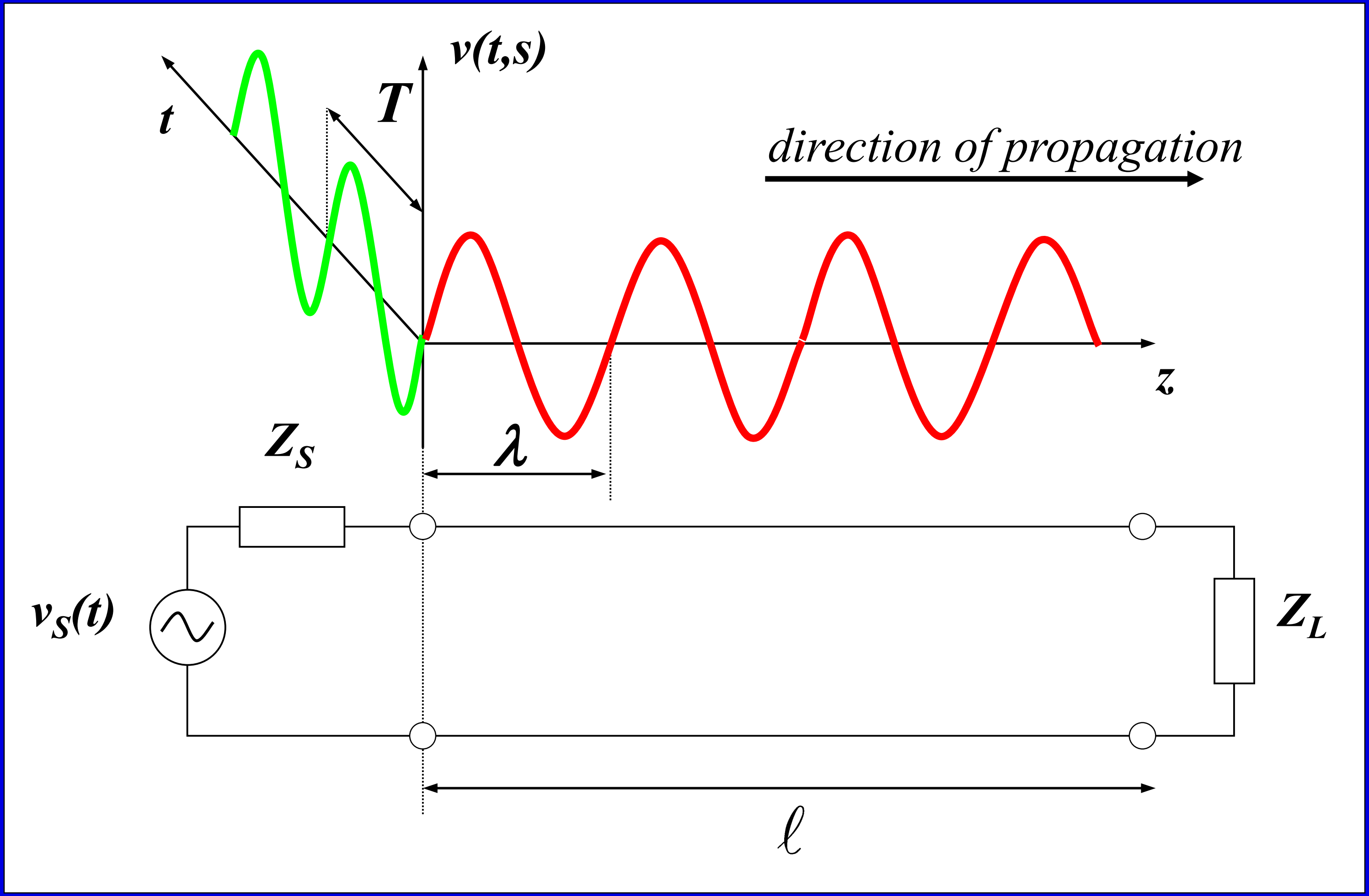


Waves on Lines

- Voltage/current is expressed by both time and distance:

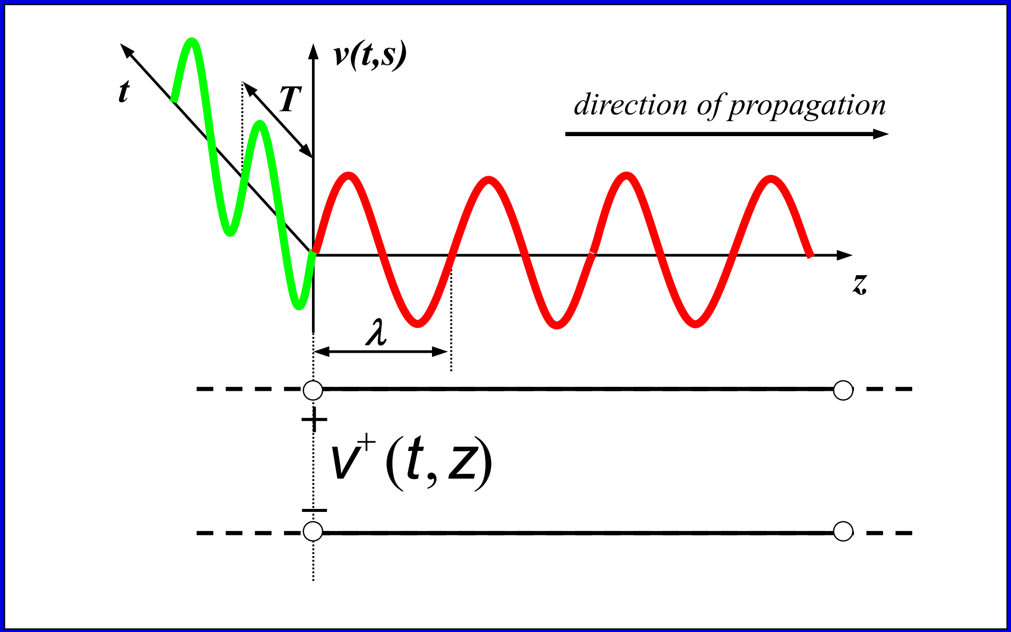
$$v^{+}(t,z) = |V_0^{+}| \exp(-\alpha z) \cos[\omega t - \beta z + \phi_0^{+}] =$$
$$= \mathrm{Re}(V_0^{+} \exp[j\omega t - \gamma z])$$

  - where $\gamma = \alpha + j\beta$ is the propagation constant

→ α is the attenuation constant

→ β is the phase constant $\beta = \frac{2\pi}{\lambda}$

Travelling Voltage Wave on a Lossless Line

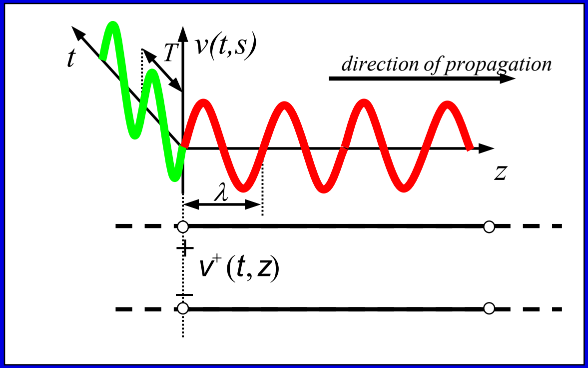


$$v^{+}(t,z) = |V_0^{+}|\cos[\omega t - \beta z + \phi_0^{+}] = \\ = \mathrm{Re}(V_0^{+}\exp[j(\omega t - \beta z)])$$

– where $V_0^{+} = |V_0^{+}|\exp(j\phi_0^{+})$ is the complex amplitude of the signal $v^{+}(t,z)$ at $z = 0$

Travelling Voltage Wave on a Lossless Line (cont.)

$$v^+(t,z) = |V_0^+|\cos[\omega t - \beta z + \phi_0^+] = \\ = \mathrm{Re}(V_0^+ \exp[j(\omega t - \beta z)])$$



– The phase constant β may be expressed in other parameters

– $v^+(t,z)$ shows maximum amplitude when $\omega t - \beta z + \phi_0^+ = 2n\pi$

– the phase at a fixed point at the wave front is shifted by 2π if

- z is increased by one wavelength or
- the time is increased by T

$$2\pi = \omega T = \beta\lambda$$

$$\beta = \frac{2\pi}{\lambda}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$v_p = \frac{\lambda}{T} = \lambda f = \frac{\omega}{\beta}$$

Generalization

- A wave travelling in negative direction may also be defined:

$$v^{-}(t,z) = |V_0^{-}|\cos[\omega t + \beta z + \phi_0^{-}] =$$
$$= \mathrm{Re}(V_0^{-}\exp[j(\omega t + \beta z)])$$

Note sign of phase constant

  - normally there are one incident and one reflected wave on the line
  - for this reason the total voltage is

$$v(t,z) = v^{+}(t,z) + v^{-}(t,z)$$

The Time Dependence is Often Less Important

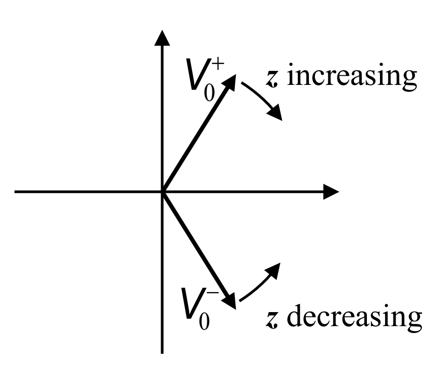
- the signal is normally a sinusoidal wave at fixed frequency
  it is therefore often sufficient to express the wave by the complex amplitude

- it is however important to study how the complex amplitude changes along the line:

$$V^{+}(z) = V_0^{+} e^{-j\beta z} \qquad V^{-}(z) = V_0^{-} e^{j\beta z}$$

- generally the complex amplitude of the total voltage in the position z at the line is:

$$V(z) = V^{+}(z) + V^{-}(z)$$



Current Wave

- There is always an associated current wave related to the voltage wave:

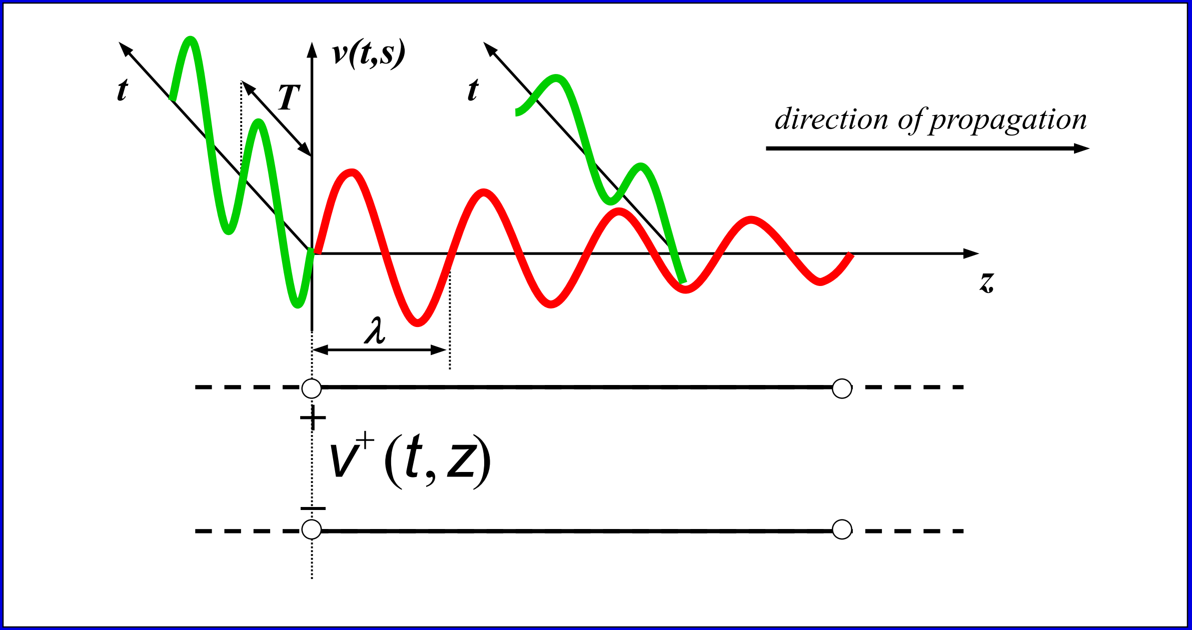
$$I^{+}(z) = I_0^{+} e^{-j\beta z} \qquad I^{-}(z) = I_0^{-} e^{j\beta z}$$

- generally the complex amplitude of the total current through the node at the position z is:

$$I(z) = I^{+}(z) - I^{-}(z)$$

- the ratio between voltage and current is determined by the electrical characteristics of the line. This will be discussed later on.

Travelling Voltage Wave on a Lossy Line



$$V^+(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-(\alpha + j\beta)z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

the propagation constant $\gamma = \alpha + j\beta$

where α is the attenuation constant expressed in Neper/m

1 Neper/m = 8.69 dB/m www.sizes.com/units/neper.htm

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neper=20*log10(exp(-1))
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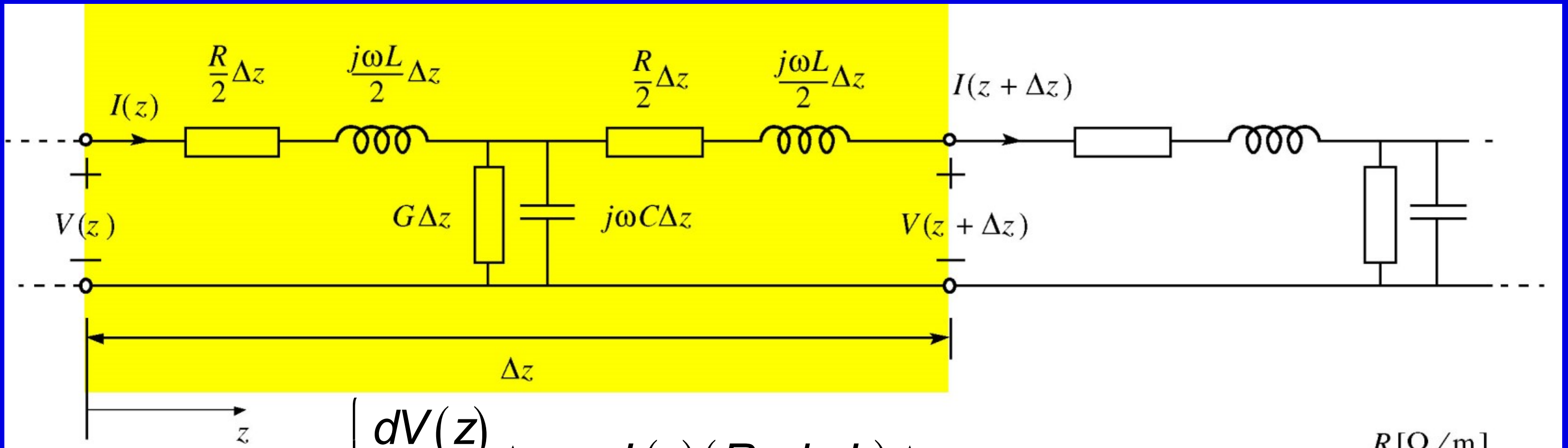
A Lumped Circuit Model for a Transmission Line



$$\begin{cases} \frac{dV(z)}{dz}\Delta z = -I(z)(R + j\omega L)\Delta z \\ \frac{dI(z)}{dz}\Delta z = -V(z)(G + j\omega C)\Delta z \end{cases}$$

$R[\Omega/\mathrm{m}]$
$G[\Omega^{-1}/\mathrm{m}]$
$L[\mathrm{H/m}]$
$C[\mathrm{F/m}]$

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

"wave description" "line description"

RLGC per unit length

The Relation Current - Voltage

- is expressed by the characteristic impedance Z_0

$$\frac{V_0^+}{I_0^+} = \frac{V_0^-}{I_0^-} = Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = \cdots \approx$$

$$\approx \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \cdots \approx \sqrt{\frac{L}{C}}$$

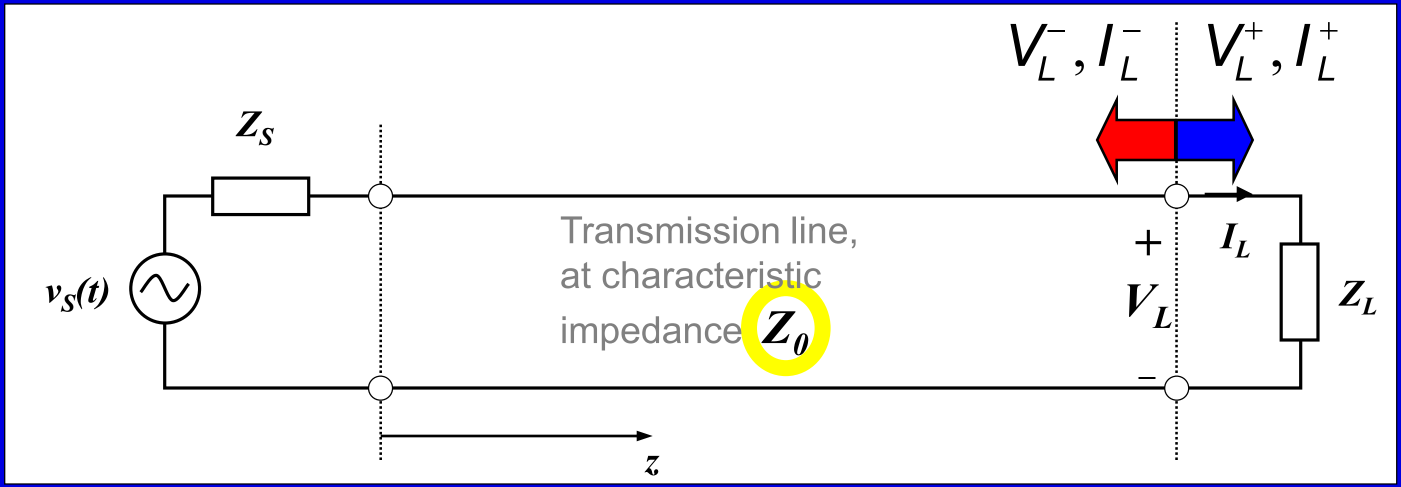
if dielectric is assumed to be air:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Reflection

(PAUSE)

- Study the voltage and the current at Z_L



$$V_L = V_L^+ + V_L^- = \left(I_L^+ + I_L^-\right) Z_0 \qquad V_L = I_L Z_L = \left(I_L^+ - I_L^-\right) Z_L$$

$$I_L^- = I_L^+ \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{or} \quad V_L^- = V_L^+ \frac{Z_L - Z_0}{Z_L + Z_0}$$

Important

- Definition of the reflection coefficient

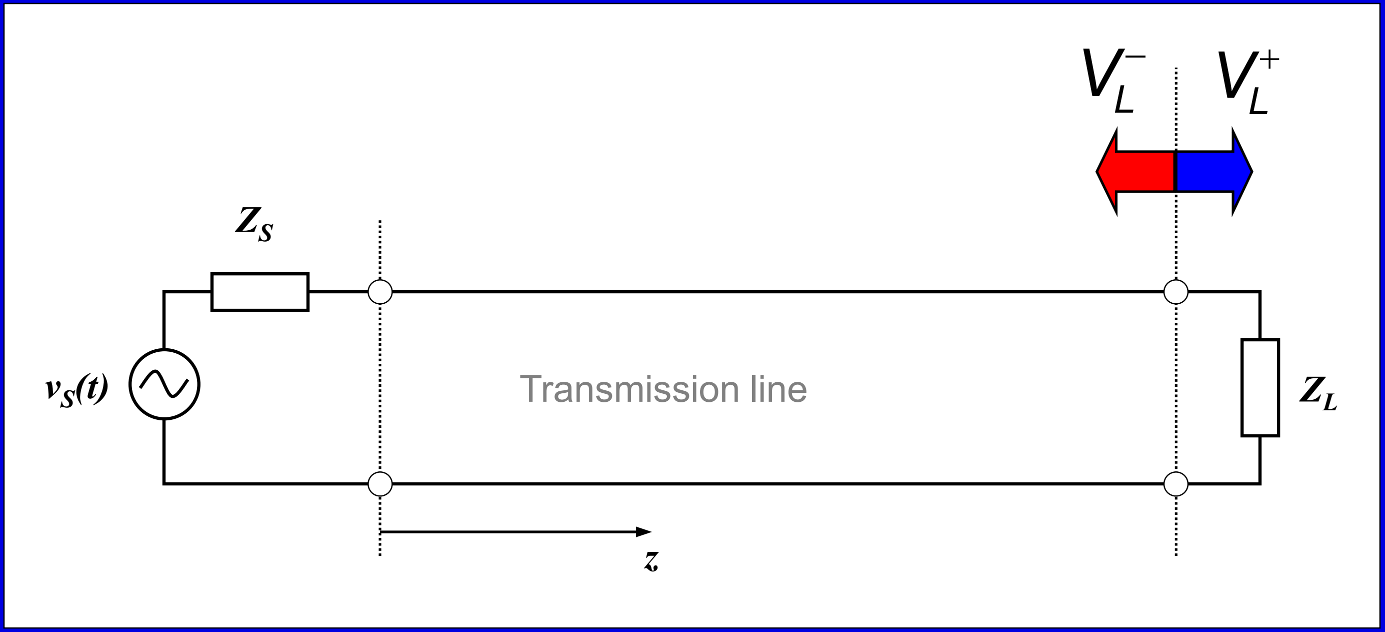
$$\Gamma_L = \frac{I_L^-}{I_L^+} = \frac{V_L^-}{V_L^+}$$

$$\Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \Leftrightarrow Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

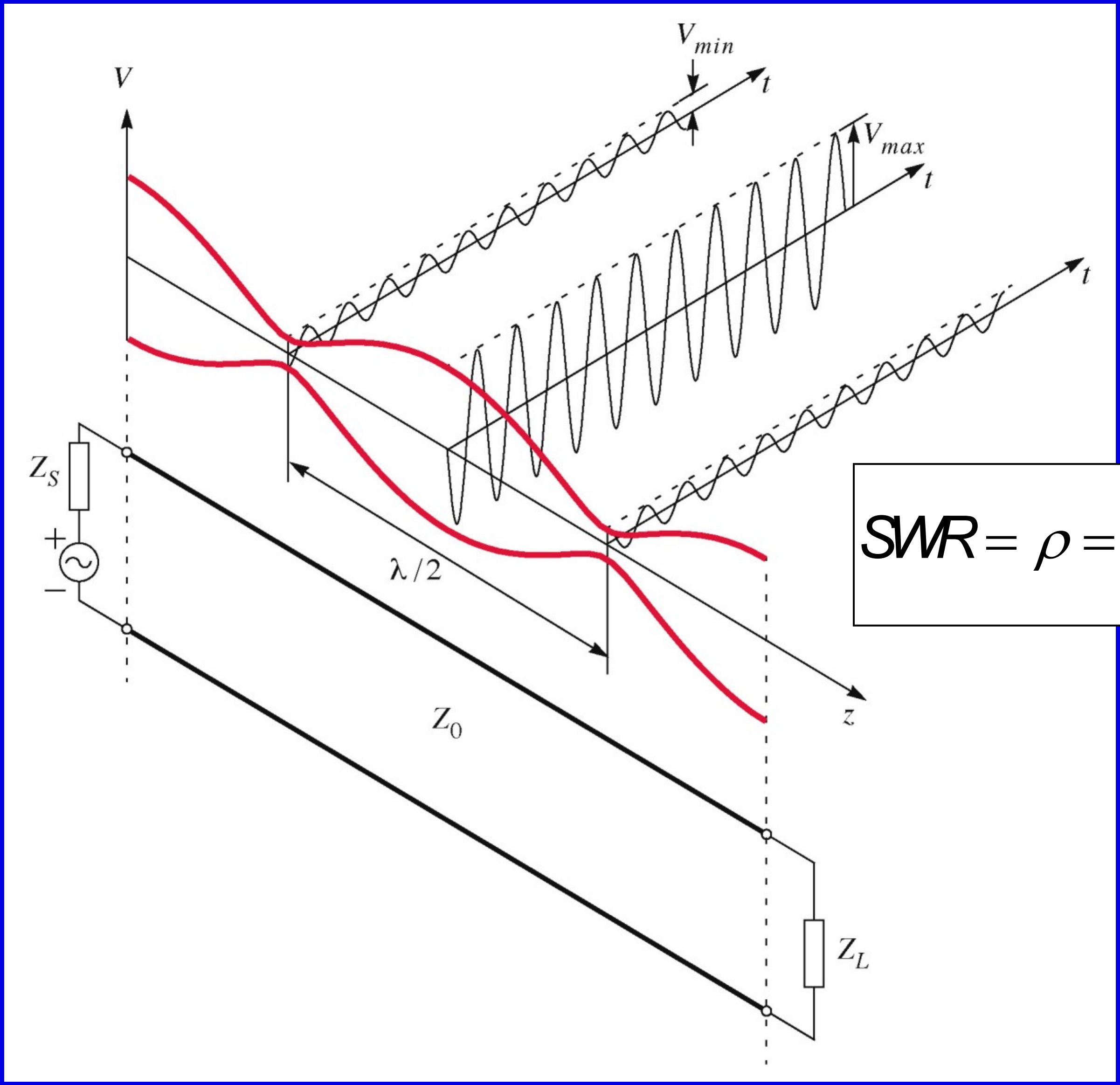
Reflection Coefficient

- Definition:

$$\Gamma = \frac{\text{reflected voltage wave}}{\text{incident voltage wave}} = \frac{V^- e^{\gamma z}}{V^+ e^{-\gamma z}}$$



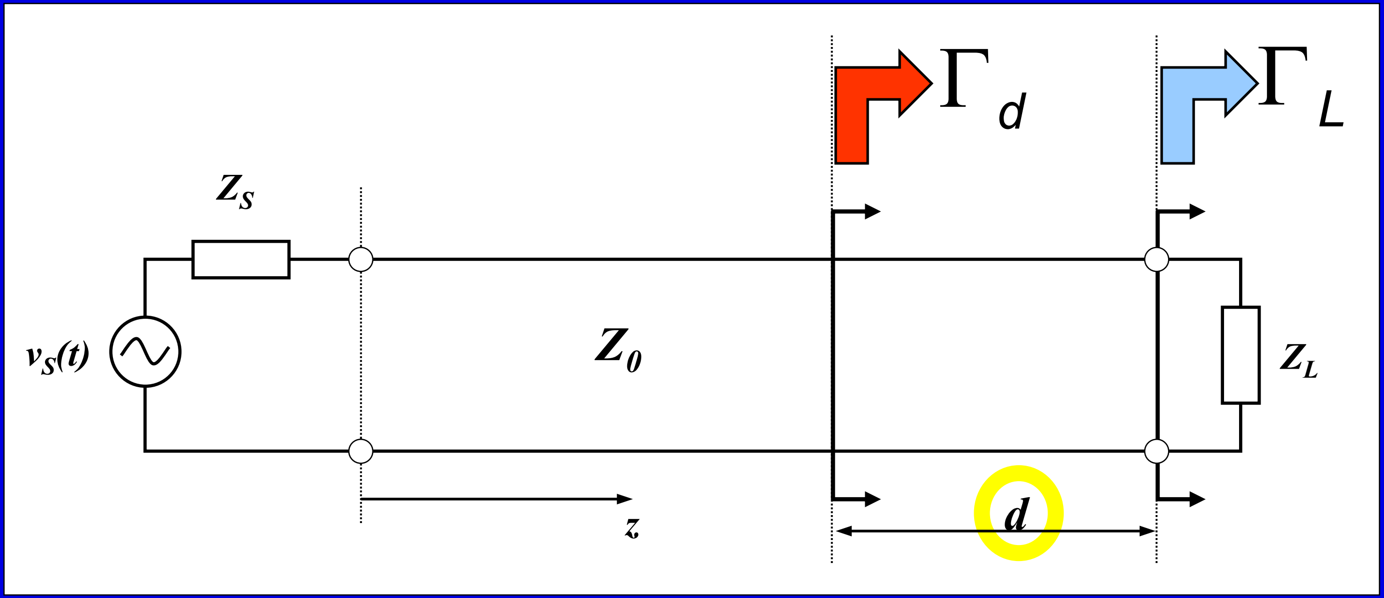
Standing-Wave Ratio



$$SWR = \rho = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Transformation of the Reflection Coefficient

- Determine the reflection coefficient at an arbitrary location d at the line:



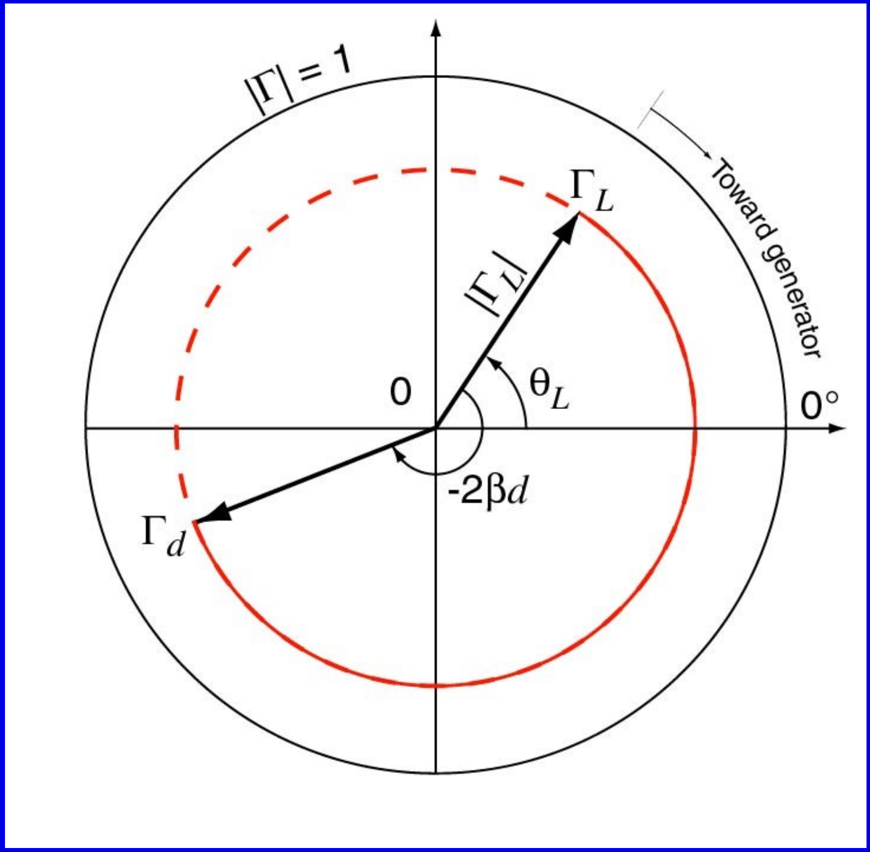
$$\Gamma_d = \Gamma_L e^{-2\gamma d} = \Gamma_L e^{-2\alpha d} e^{-2j\beta d}$$

Transformation of the Reflection Coefficient

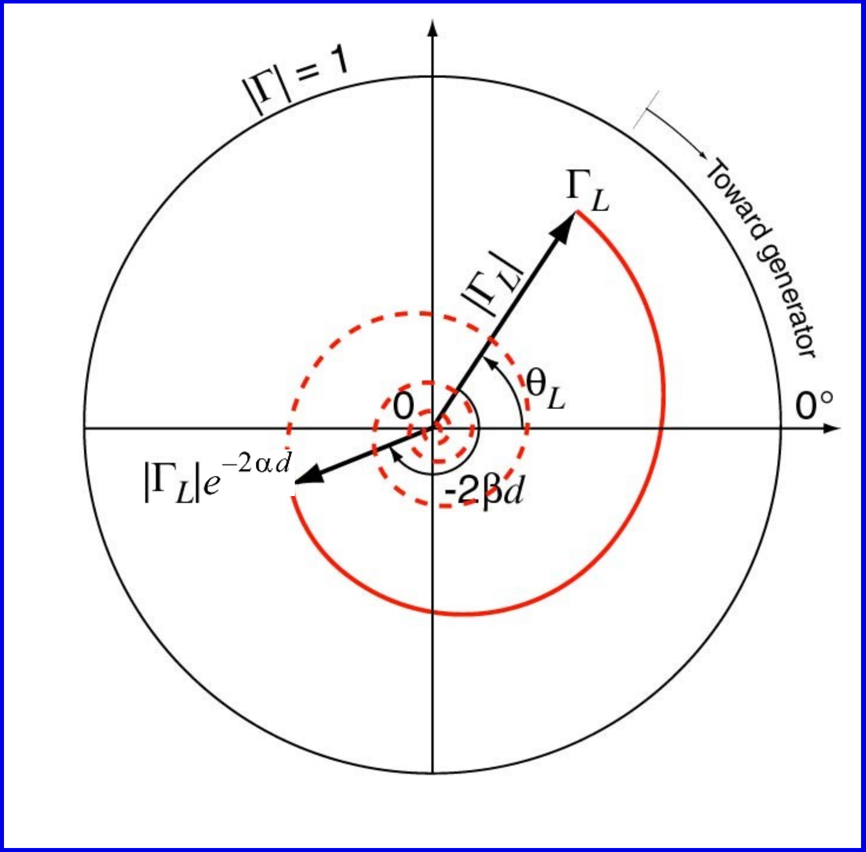
Implies a rotation in the polar Γ-plane

- Polar diagram $\Gamma_d = \Gamma_L e^{-2\gamma d} = \Gamma_L e^{-2\alpha d} e^{-2j\beta d}$

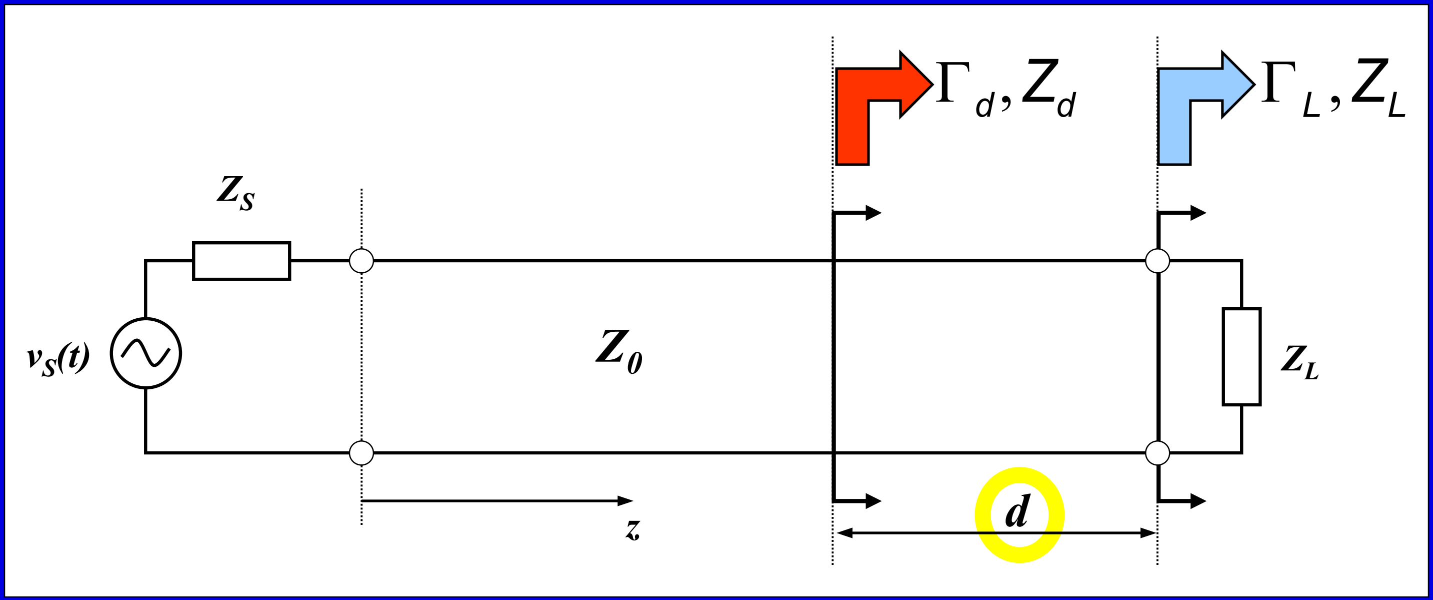
Lossless transmission line



Lossy transmission line

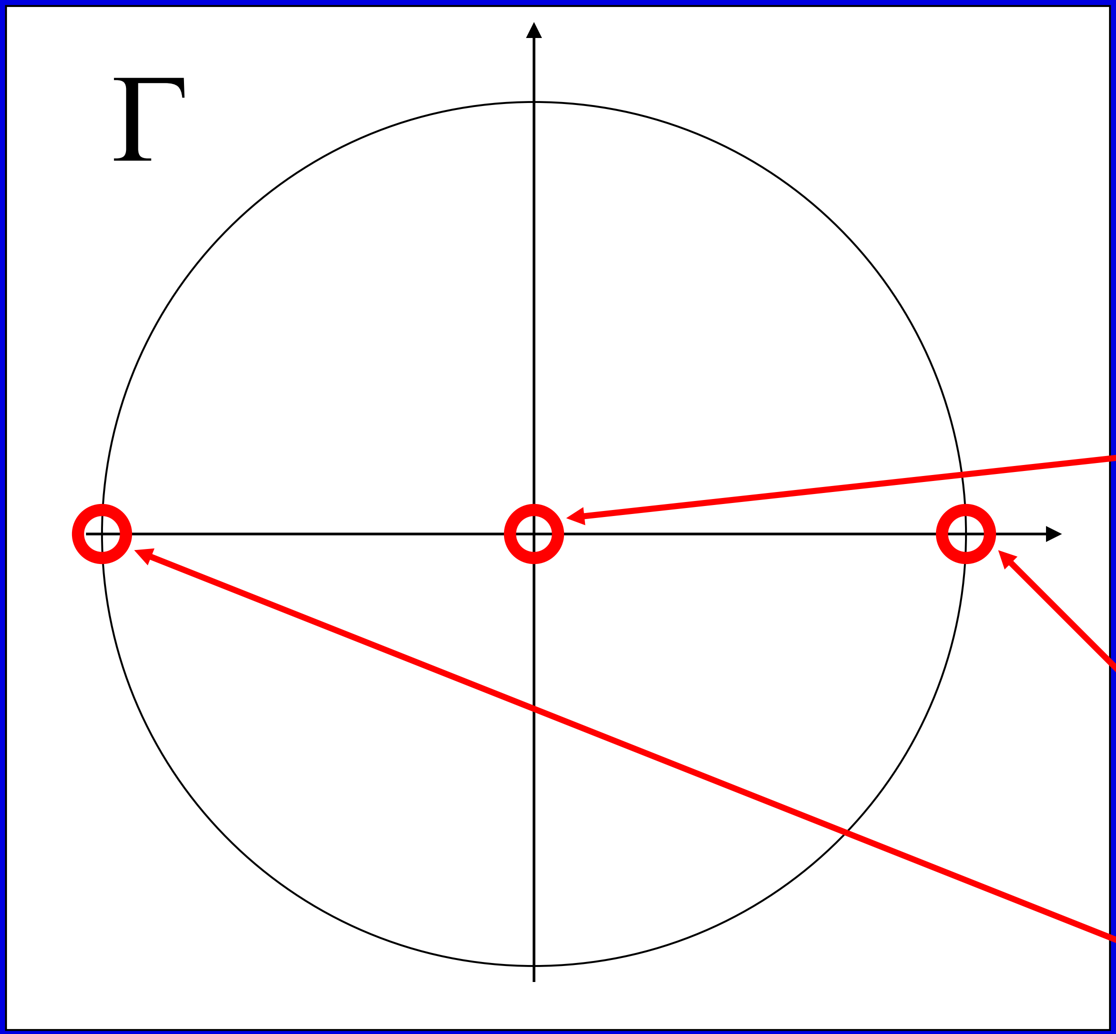


Conversion of Reflection Coefficient to Impedance



$$\Gamma_d = \frac{Z_d - Z_0}{Z_d + Z_0} \Leftrightarrow Z_d = Z_0 \frac{1 + \Gamma_d}{1 - \Gamma_d}$$

Reflection Coefficient ⇔ Load Impedance



$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

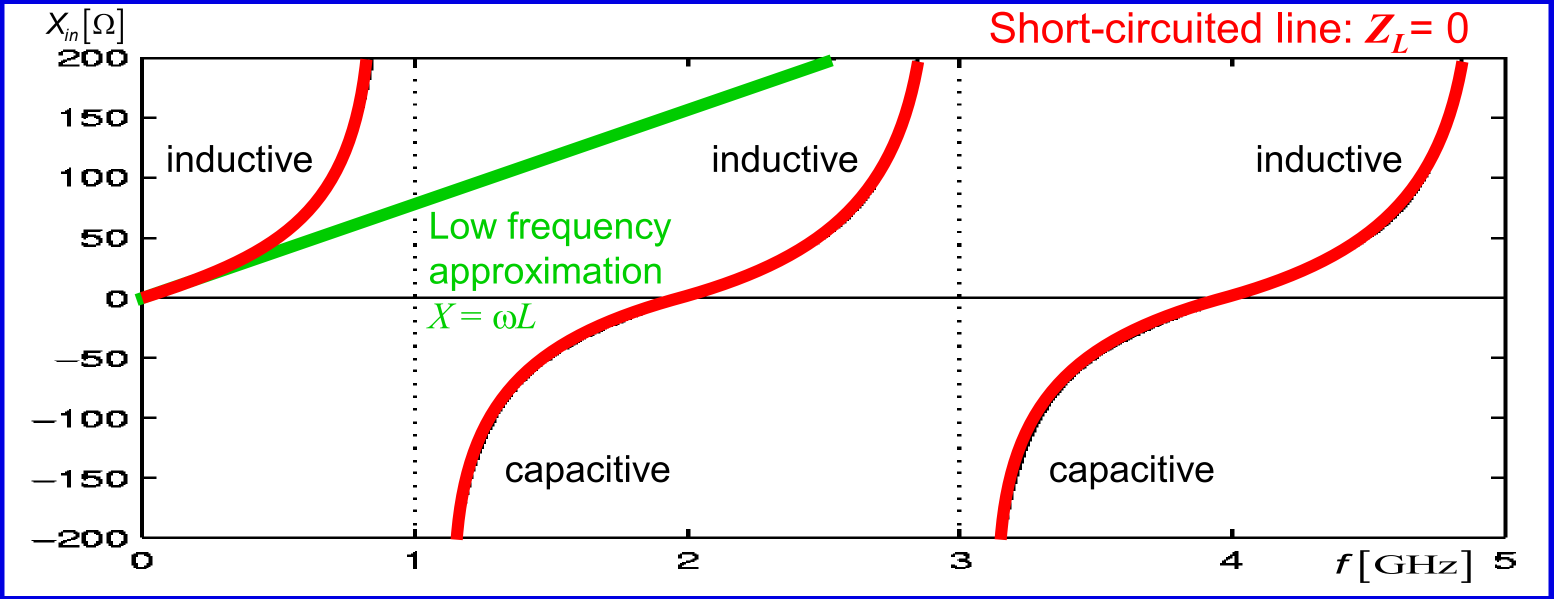
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

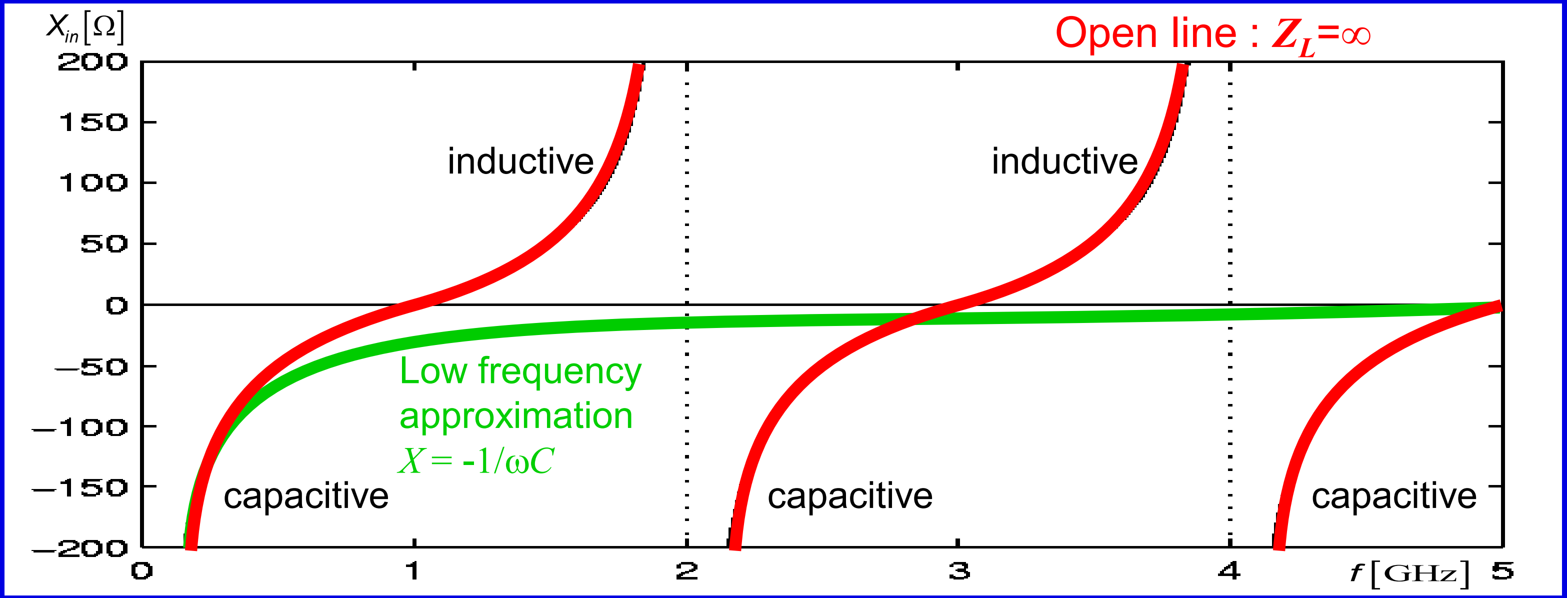
$\Gamma_L = 0 \Leftrightarrow Z_L = Z_0$

$\Gamma_L = 1 \Leftrightarrow Z_L = \infty$

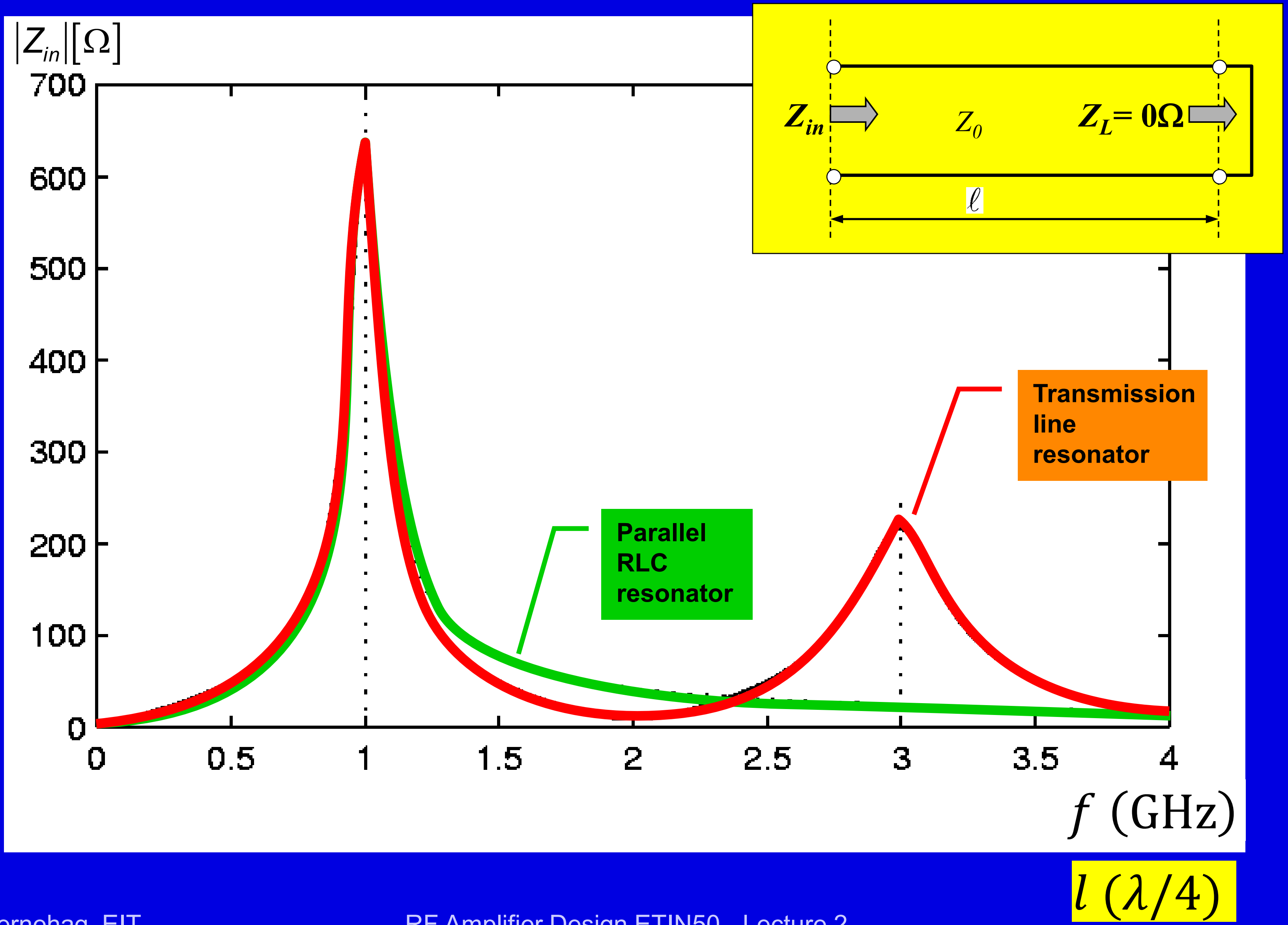
$\Gamma_L = -1 \Leftrightarrow Z_L = 0$

Reactance of 1 GHz quarter-wavelength line resonator

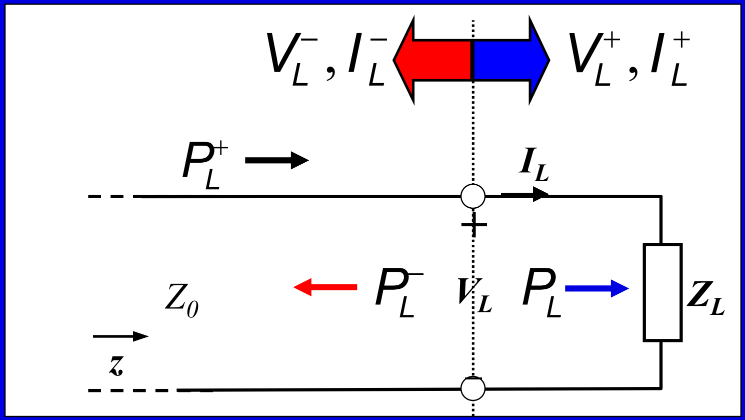




Impedance of 1 GHz quarter-wavelength line resonator



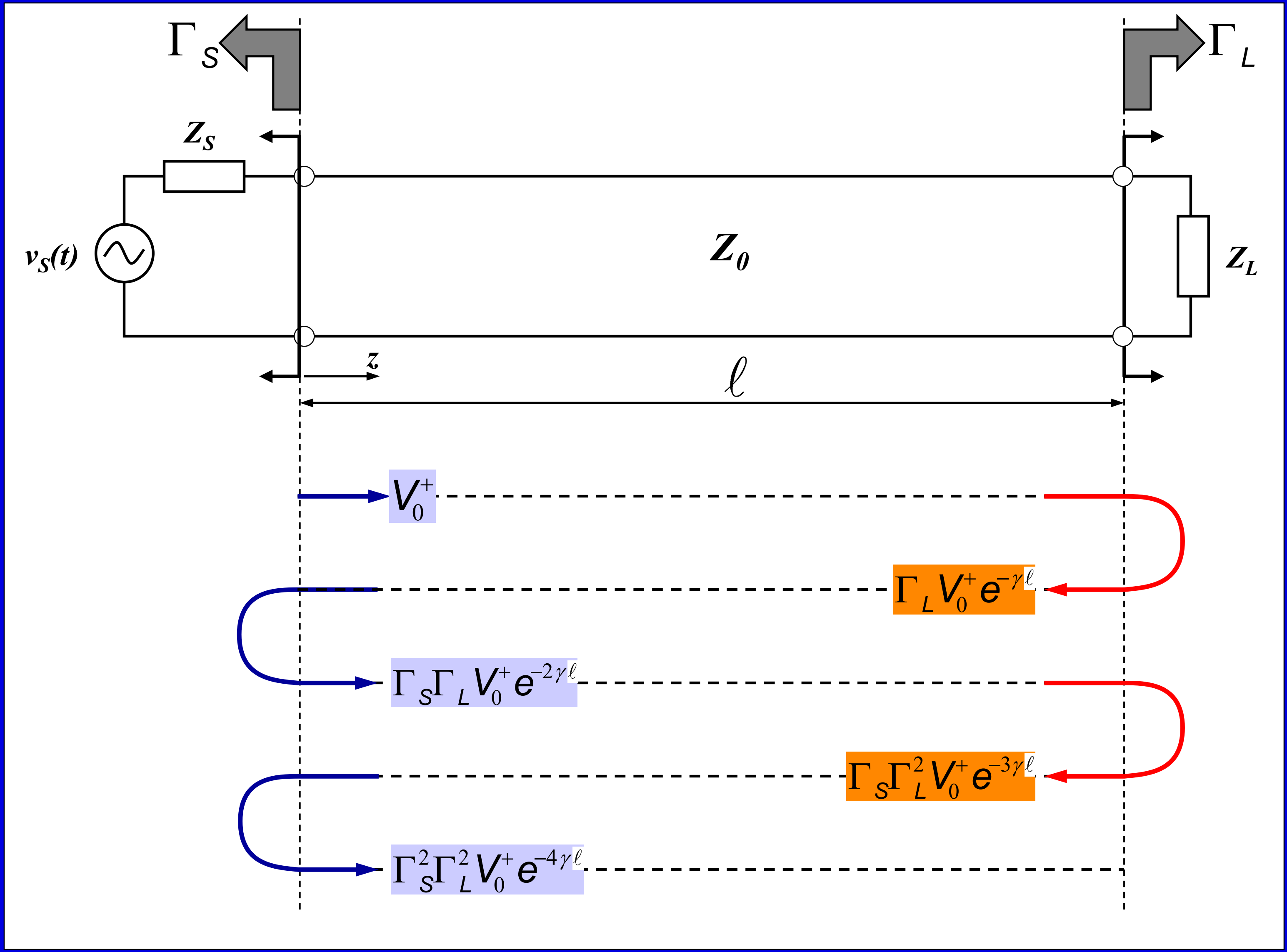
Waves and Power



$$V^+, I^+ \Rightarrow P^+$$
$$V^-, I^- \Rightarrow P^-$$

- Power carried by the incident wave: $$P_L^+ = \frac{1}{2} V_L^+ \left(I_L^+\right)^* = \frac{\left|V_L^+\right|^2}{2Z_0}$$
- Power of the from Z_L reflected wave: $$P_L^- = \frac{1}{2} V_L^- \left(I_L^-\right)^* = \frac{\left|V_L^-\right|^2}{2Z_0}$$
- Power transmitted to the load Z_L: (power continuity) $$P_L = P_L^+ - P_L^- = T_p P_L^+$$
- Transmission factor: $$T_p = 1 - \frac{\left|V_L^-\right|^2}{\left|V_L^+\right|^2} = 1 - \left|\Gamma_L\right|^2$$
- Mismatch loss: $$L_m = -10\log_{10}\left(T_p\right) = -10\log_{10}\left(1 - |\Gamma_L|^2\right)$$

Multiple Reflection



Multiple Reflection

- Power transmitted to the load Z_L :

$$P_L = P_S e^{-2\alpha\ell} \frac{\left(1-\left|\Gamma_S\right|^2\right)\left(1-\left|\Gamma_L\right|^2\right)}{\left|1-\Gamma_S\Gamma_L e^{-2\gamma\ell}\right|^2}$$

- where P_S denotes the *available power from source:*

$$P_S = \frac{\left|V_S\right|^2}{8\,\mathrm{Re}\left[Z_S\right]}$$