Assignments

1.

2. (a) Resolution = $1/2^6$ V = 0.015625 V

   (b) Full scale dynamic range = 0.1 V $\cdot\ 2^6$ = 6.4 V

3. • 0010.1101 = 2.8125

   • 1011.0011 = -4.8125

   (a) Truncation at the binary point results in 2 and -5, respectively.

   (b) For rounding, add the bit right next to binary point, which yields 3 and -5, as expected.

   (c) DC errror, that is, a mean value not equal 0, appears due to truncation which always rounds towards $-\infty$, hence there is a constant offset.

   (d) Left-extension with MSB does not change a 2's complement number.

4. The calculations are

	0	1	0	1	1	
+	0	1	1	0	1	
	1	1	0	0	0	Outside range
+	1	0	1	1	1	
	0	1	1	1	1	Inside range

   Overflows in a calculation chain are not important as long as the expected result lies inside the number range.

5. Safe scaling is applied to avoid internal overflows in digital filters.

   (a) $\beta = \sum_{i=0}^{4} |h_i| = 1.5$, that is, choose a power of two which is greater than β, for example, $\tilde{\beta} = 2$. Then the multiplication by $1/\tilde{\beta}$ can be done with a left-shift.

   (b) The coefficients for the FIR filter are symmetric and thus multipliers can be reused, see slides from the lecture.

   (c) The number of multiplications is reduced by $\lfloor \frac{m+1}{2} \rfloor$.

   (d) Change signs when superposing the contents of the shift register before multiplication.

6. Let $u(n) = \sigma(n),\ \text{for}\, n = 0, \ldots, \infty$.

$$\begin{aligned}
x(0) &= 1 \\
x(1) &= 1 + a \\
x(2) &= 1 + a(1+a) = 1 + a + a^2 \\
x(3) &= 1 + a(1 + a + a^2) = 1 + a + a^2 + a^3 \\
&\ \ \vdots \\
x(n) &= \sum_{i=0}^{n} a^i \overset{n \to \infty}{=} \frac{1}{1-a}
\end{aligned}$$

For safe scaling, consider the system's impulse response, that is, $u(n) = \delta(n)$.

$$\begin{aligned}
x(0) &= 1 \\
x(1) &= a \\
x(2) &= a^2 \\
&\ \ \vdots \\
x(n) &= a^n
\end{aligned}$$

to yield the scaling factor $\beta = \sum_{i=0}^{\infty} |a^n| = \frac{1}{1-|a|} \geq 1$. Alternatively, one can use the $\mathcal{Z}$-transform to yield

$$\begin{aligned}
X(z) &= U(z) + az^{-1}X(z) \\
X(z) &= \frac{1}{1-az^{-1}}U(z) = \frac{z}{z-a}U(z) \overset{U(z)=1}{=}
\end{aligned}$$

$$= \frac{z}{z-a} \overset{\mathcal{Z}}{\bullet\!\!-\!\!\circ}\ a^n = x(n)$$

7. (a) On the original SFG, change edge directions and exchange input and output.

   (b) Either one simply writes out the equations or one can use the $\mathcal{Z}$-transform on the original convolution. Then

$$\begin{aligned}
Y(z) &= \sum_{i=0}^{4} h_i X(z) z^{-i} \\
&= X(z)[h_0 + z^{-1}(h_1 + z^{-1}(h_2 + z^{-1}(h_3 + z^{-1}h_4)))],
\end{aligned}$$

   which represents the transposed form.

   (c) Again, simply write out the equations and note the increase in latency.

8. (a) The iteration bound T_∞ is the fundamental limit on how fast a recursive DFG can be implemented in HW, $T_\infty = \max\limits_{l \in L}\{\frac{t_l}{w_l}\}$.

(b) There are two loops, **A1-A2-M2** and **A1-A3-M1-A2-M2**, with loop bounds 4 and 3.5, respectively. Iteration bound is the larger of the two, that is, 4.

(c) Transposition is done as usual. Re-inspection of the iteration bound on the transposed filter gives the same result, that is, the bound is characteristic for an algorithm, not a specific implementation.

9. (a) There are 6 loops in this architecture. The iteration bound is the maximum of the loop bounds, that is,

$$T_\infty = \max\{\frac{3T_{\mathrm{add}} + 2T_{\mathrm{mult}}}{1}, \frac{T_{\mathrm{add}} + T_{\mathrm{mult}}}{1}, \frac{5T_{\mathrm{add}} + 2T_{\mathrm{mult}}}{2},$$
$$\frac{5T_{\mathrm{add}} + T_{\mathrm{mult}}}{3}, \frac{3T_{\mathrm{add}} + T_{\mathrm{mult}}}{2}\} = 3T_{\mathrm{add}} + 2T_{\mathrm{mult}}$$

(b) The critical path is $4T_{\mathrm{add}} + 2T_{\mathrm{mult}}$.

10. The critical path is either **d-f-a-b** or **d-e-c-b**, which both take 5 t.u. There are 4 loops in this graph, that is, **a-b**, **b-c**, **c-d-e**, and **a-b-c-d-f**. Their loop bounds are 1, 3, 4/3, and 7/4, respectively, that is, $T_\infty = 3$.

11. The iteration bound is 7/2. Good luck with the program.