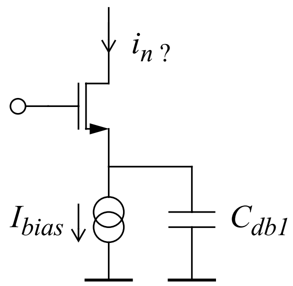
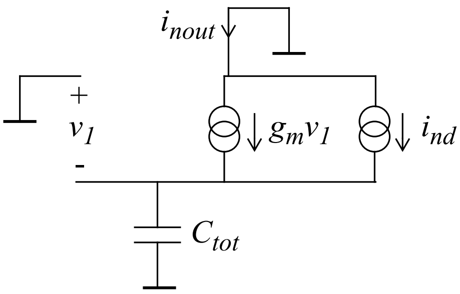
Solutions exercise 5 (LNA)

1. Problem 11.1
Assume c_{db}=c_{gs} ,W_1=W_2 (and L_1=L_2)

The output noise due to the cascode device M_2 is to be calculated in two different cases a and b. We draw a simplified schematic to find this noise in the general case:



Small signal equivalent:



$$g_m \cdot v_1 + i_{nd} + v_1 \cdot sC_{tot} = 0 \Rightarrow v_1 = -\frac{i_{nd}}{g_m + sC_{tot}}$$

$$i_{nout} = i_{nd} + g_m \cdot v_1 = i_{nd} \cdot \left(1 - \frac{g_m}{g_m + sC_{tot}}\right) = i_{nd} \cdot \frac{sC_{tot}}{g_m + sC_{tot}}$$

$$\overline{i_{nout}^2} = \overline{i_{nd}^2} \cdot \left(\frac{sC_{tot}}{g_m + sC_{tot}}\right)^2 = \overline{i_{nd}^2} \cdot \left(\frac{j\omega}{g_m/C_{tot} + j\omega}\right)^2$$

Just the magnitude matters:

$$\overline{i_{nout}^2} = \overline{i_{nd}^2} \cdot \frac{\omega^2}{(g_m/C_{tot})^2 + \omega^2}$$

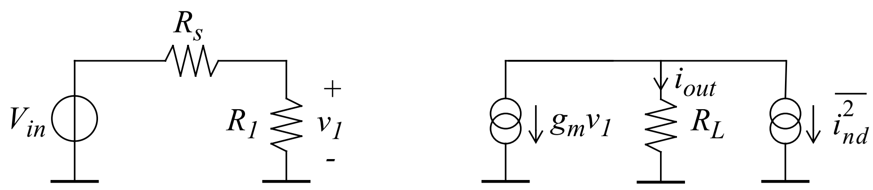
a. $C_{tot} = C_{db1} + C_{sb2} + C_{gs2} = 3 \cdot C_{gs2} \Rightarrow \overline{i_{nout}^2} = \overline{i_{nd}^2} \cdot \dfrac{\omega^2}{(\omega_T/3)^2 + \omega^2} = \overline{i_{nd}^2} \cdot \dfrac{9\omega^2}{\omega_T^2 + 9\omega^2}$

b. $C_{tot} = C_{db1} + C_{gs2} = 2 \cdot C_{gs2} \Rightarrow \overline{i_{nout}^2} = \overline{i_{nd}^2} \cdot \dfrac{\omega^2}{(\omega_T/2)^2 + \omega^2} = \overline{i_{nd}^2} \cdot \dfrac{4\omega^2}{\omega_T^2 + 4\omega^2}$

2. Problem 11.5

Disregard C_{gs} , since when it has influence the matching is bad anyway (due to non-resistive input impedance)

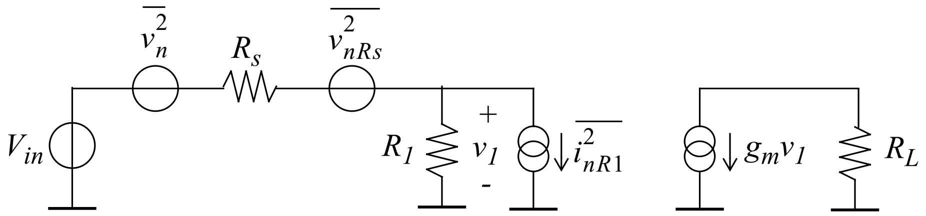
a.



$$G = \frac{i_{out}}{v_{in}} = \frac{g_m}{2}$$

Move i_{nd} and noise due to R_L (i_{nL}) from the output to the input:

$$\overline{v_n^2} = (\overline{i_{nd}^2} + \overline{i_{nL}^2}) \cdot \frac{1}{G^2}$$



Use trick from the book and compare the short-circuit noise current through R_1:

$$F = \frac{\overline{i_{nR1}^2} + \overline{v_{nRs}^2} \cdot 1/R_s^2 + \overline{v_n^2} \cdot 1/R_s^2}{\overline{v_{nRs}^2} \cdot 1/R_s^2} = \frac{R_s^2 \cdot \overline{i_{nR1}^2} + \overline{v_{nRs}^2} + \overline{v_n^2}}{\overline{v_{nRs}^2}} = 1 + R_s^2 \cdot \frac{\overline{i_{nR1}^2}}{\overline{v_{nRs}^2}} + \frac{\overline{v_n^2}}{\overline{v_{nRs}^2}}$$

$$= 1 + \frac{R_s}{R_1} + \frac{(4kT\gamma g_m + 4kT \cdot 1/R_L) \cdot 4/g_m^2}{4kTR_s} = 1 + \frac{R_s}{R_1} + \frac{4\gamma}{g_m R_s} + \frac{4}{g_m^2 R_L R_s}$$

Impedance match $\Rightarrow R_1 = R_s \Rightarrow F = 2 + \dfrac{4\gamma}{g_m R_s} + \dfrac{4}{g_m^2 R_L R_s}$

b. The gate-induced noise appears in parallel with i_{nR1}, that is replace i_{nR1} in a. with:

$$\frac{4kT}{R_1} + 4kT\delta\frac{(\omega C_{gs})^2}{5g_{d0}}$$

The difference can be significant if the second term is comparable to the first:

$$\delta\frac{(\omega C_{gs})^2}{5g_{d0}} \approx \frac{(\omega C_{gs})^2}{g_m} = \omega^2\left(\frac{C_{gs}}{g_m}\right)^2 \cdot g_m \approx \left(\frac{\omega}{\omega_T}\right)^2 \cdot g_m \approx \frac{1}{R_1} \Rightarrow \omega = \omega_T \cdot \frac{1}{\sqrt{g_m R_1}}$$

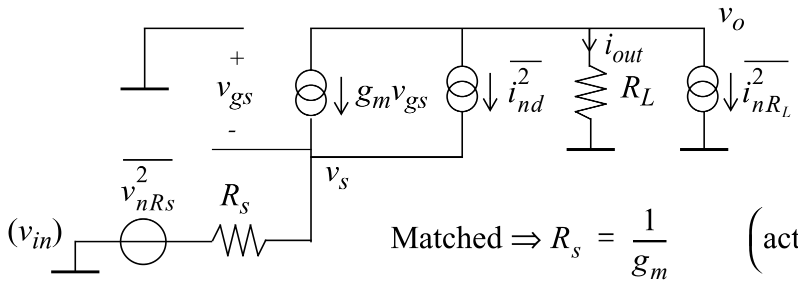
In a practical design $g_m R_1$ is probably not more than 10, that is gate-induced noise has no practical influence for frequencies below approximately $f_T/3$ in this LNA.

$$\left(F \approx 2 + \frac{4\gamma}{g_m R_s} + \frac{4}{g_m^2 R_L R_s} + \left(\frac{\omega}{\omega_T}\right)^2 \cdot g_m R_s\right)$$

Problem 11.7

a.

Small-signal equivalent



Matched $\Rightarrow R_s = \dfrac{1}{g_m}$ $\left(\text{actually } \dfrac{1}{g_m + g_{mb}}\right)$

All noise sources are transformed to the output and then compared in order to find F.

$$\begin{cases} (v_{nR_s} - v_s)\cdot\dfrac{1}{R_s} - g_m v_s + i_{nd} = 0 \\ g_m v_s - i_{nd} - i_{nR_L} - \dfrac{v_o}{R_L} = 0 \end{cases} \Rightarrow \begin{pmatrix} v_s\left(\dfrac{1}{R_s} + g_m\right) = v_{nR_s}\cdot\dfrac{1}{R_s} + i_{nd} \\ v_s\cdot g_m - v_o\cdot\dfrac{1}{R_L} = i_{nd} + i_{nR_L} \end{pmatrix}$$

$$g_m = \frac{1}{R_s} \Rightarrow \begin{pmatrix} v_s\cdot 2g_m = v_{nR_s}\cdot g_m + i_{nd} & (1) \\ v_s\cdot g_m - v_o\cdot\dfrac{1}{R_L} = i_{nd} + i_{nR_L} & (2) \end{pmatrix}$$

$$(1) - 2^*(2):\quad 2v_o\cdot\frac{1}{R_L} = v_{nR_s}\cdot g_m - i_{nd} - 2i_{nR_L} \Rightarrow i_{out} = \frac{1}{2}\cdot g_m v_{nR_s} - \frac{1}{2}i_{nd} - i_{nR_L}$$

The sign of the noise currents can be disregarded:

$$\overline{i_{n,\,out}^2} = \frac{1}{4}\cdot g_m^2\cdot\overline{v_{nR_s}^2} + \frac{1}{4}\overline{i_{nd}^2} + \overline{i_{nR_L}^2} = \left(\frac{1}{4}\cdot\overline{i_{nR_s}^2} + \frac{1}{4}\overline{i_{nd}^2} + \overline{i_{nR_L}^2}\right)$$

$$F = \frac{g_m^2 kTR_s + kT\gamma g_m + 4kT/R_L}{g_m^2 kTR_s} = 1 + \frac{\gamma}{g_m R_s} + \frac{4}{g_m R_L\cdot g_m R_s} = 1 + \gamma + \frac{4}{g_m R_L}$$

$\gamma = \dfrac{2}{3} \Rightarrow \text{NF} > 2.2\text{ dB}$ $\qquad\gamma = 2 \Rightarrow \text{NF} > 4.8\text{ dB}$

b.

$$F = \frac{\frac{1}{4}\cdot\overline{i_{nR_s}^2} + \frac{1}{4}\overline{i_{ng}^2} + \frac{1}{4}\overline{i_{nd}^2} + \overline{i_{nR_L}^2}}{\frac{1}{4}\cdot\overline{i_{nR_s}^2}} = 1 + \gamma + \frac{4}{g_m R_s} + \frac{\overline{i_{nd}^2}}{\overline{i_{nR_s}^2}}$$

$$= 1 + \gamma + \frac{4}{g_m R_L} + \frac{\delta(\omega C_{gs})^2}{5g_m}\cdot R_s \approx 1 + \gamma + \frac{4}{g_m R_L} + \frac{\delta}{5}\cdot\left(\frac{\omega}{\omega_T}\right)^2\cdot g_m R_s$$

$$\approx 1 + \gamma + \frac{4}{g_m R_L} + \left(\frac{\omega}{\omega_T}\right)^2$$

(the gate-induced noise appears in parallel with the noise due to Rs, and has influence only at very high frequencies, like in problem 11.5)

4. Design problem

Test *Q*=3:

$$Q = \frac{1}{2\pi f_0 2R_s C_{gs}} \Rightarrow C_{gs} = \frac{1}{2\pi f_0 2R_s Q} = 330\ \text{fF}$$

$$A_v = 2Qg_m R_L \Rightarrow g_m = \frac{A_v}{2QR_L} = \frac{31.6}{2 \cdot 3 \cdot 300} = 17.6\ \text{mS}$$

$$R_{in} = g_m \cdot \frac{L_s}{C_{gs}} \Rightarrow L_s = \frac{R_{in} C_{gs}}{g_m} = 0.94\ \text{nH}$$

$$L_g = \frac{1}{\omega_0^2 \cdot C_{gs}} - L_s = 29\ \text{nH}$$

$$C_{gs} = \frac{2}{3} \cdot WLC_{ox},\quad L = L_{min} = 0.4\ \mu\text{m} \Rightarrow W = \frac{3}{2} \cdot \frac{C_{gs}}{L_{min} C_{ox}} = 270\ \mu\text{m}$$

$$g_m = \mu C_{ox} \cdot \frac{W}{L} \cdot (V_{gs} - V_T) \Rightarrow V_{gs} - V_T = V_{od} = \frac{g_m}{\mu C_{ox} \cdot W/L} = 0.24\ \text{V}$$

$$I_d = \frac{1}{2} \mu C_{ox} \cdot \frac{W}{L} \cdot V_{od}^2 = 2.1\ \text{mA}\ \ \text{(per side)}$$

$$F = \frac{1 + 4/5 \cdot (3^2 + 1) + 2/4}{50 \cdot 3^2 \cdot 17.6 \cdot 10^{-3}} = 3.2\ \text{dB}$$

Was not quite good enough, but more current can be used:

$$I_d = 4\ \text{mA}\ \ \text{(per side)} \Rightarrow V_{od} = \sqrt{\frac{2I_d}{\mu C_{ox} W/L}} = 0.33\ \text{V}\ \ \text{(same W/L as before)}$$

$$g_m = \mu C_{ox} \cdot \frac{W}{L} \cdot (V_{gs} - V_T) = 24.5\ \text{mS}$$

$$A_v = 2Qg_m R_L = 44.1 = 32.9\ \text{dB}\ \ \text{(some additional gain)}$$

$$L_s = \frac{R_{in} C_{gs}}{g_m} = 0.67\ \text{nH}\ \ \text{(lower than bond wire, build differentially)}$$

$$L_g = \frac{1}{\omega_0^2 \cdot C_{gs}} - L_s = 29.3\ \text{nH}\ \text{(off-chip to minimize noise due to inductor losses)}$$

$$F = 2.5\ \text{dB}\ \text{(some margin to accomodate for cascode device noise)}$$

$$V_{G1} = V_T + V_{od} = 0.85\ \text{V}$$

$$V_{G2} = V_T + 2V_{od} + V_{margin} = 1.5\ \text{V}\ \text{(the margin also includes input voltage swing)}$$

(The cascode device is assumed to be identical to M_1)

Furthermore the LC output circuits must be designed so the output nodes resonate at f_0.