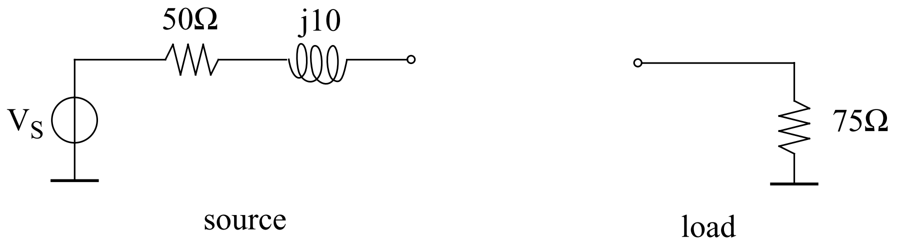
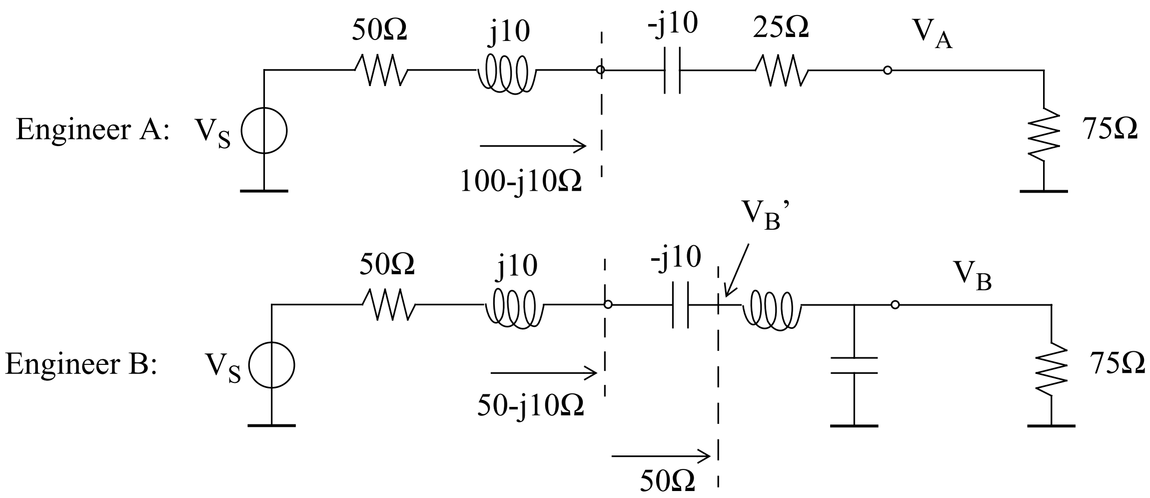
Solutions exercise 3 (RLC-circuits & amplifiers)

1. Problem 4.1

The source and the load in the figure are to be matched for maximum power transfer.



Two engineers come up with two different solutions:



$$\frac{P_A}{P_B} = \frac{V_A^2}{V_B^2} = \frac{(V_A/V_S)^2}{(V_B'/V_S)^2 \cdot (V_B/V_B')^2} = \frac{(1/2)^2}{(1/2)^2 \cdot 75/50} = \frac{50}{75} = \frac{2}{3}$$

I solution A the load seen by the source is 100-j10Ω, instead of the optimal 50-j10Ω of solution B. Furthermore, signal power is wasted in the extra 25Ω resistor. One third of the power is lost in A, compared to the optimal solution in B. (The series connected capacitor and inductor of B can be combined in just one component).

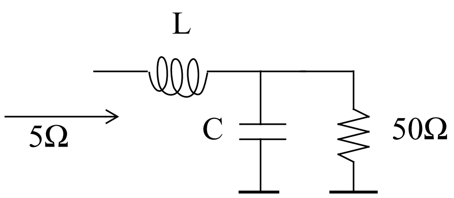
2. Problem 4.2 with extension

Given:

$$R_L = 50\Omega \qquad P = 1W \Rightarrow \hat{V}_L = \sqrt{2 \cdot P \cdot R_L} = 10\text{V}$$

$$f = 1\text{GHz} \qquad \hat{V}_{prim} = \frac{6.3}{2}V \Rightarrow R_{prim} = 50\Omega \cdot \left(\frac{6.3}{2 \cdot 10}\right)^2 = 5.0\Omega$$

a. L-match



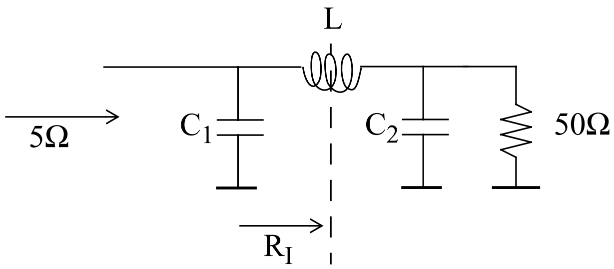
$$R_{prim} = \frac{50\Omega}{Q^2 + 1} \Rightarrow Q = 3$$

$$C = \frac{Q}{\omega R_L} = 9.5\text{pF}$$

$$L = \frac{1}{C\omega^2} = 2.65\text{nH}$$

$$\hat{I}_L = \frac{\hat{V}_{prim}}{R_{prim}} = 0.63\text{A}$$

b. Pi-match



$$Q = 10$$

$$R_I \approx \frac{(\sqrt{R_{prim}} + \sqrt{R_L})^2}{Q^2} = 0.866\Omega$$

$$L = \frac{Q \cdot R_I}{\omega} = 1.4\text{nH}$$

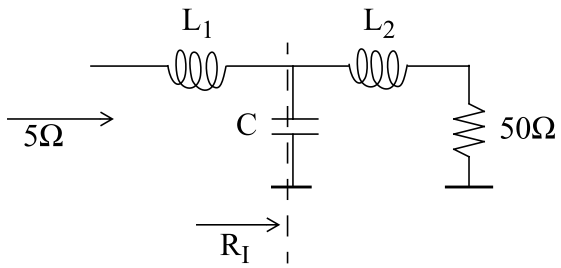
$$Q_{left} = \sqrt{\frac{R_{prim}}{R_I} - 1} = \sqrt{\frac{5.0}{0.866} - 1} = 2.18 \qquad Q_{right} = \sqrt{\frac{R_L}{R_I} - 1} = \sqrt{\frac{50}{0.866} - 1} = 7.53$$

$Q = Q_{left} + Q_{right} = 9.7 \approx 10 \Rightarrow$ close enough

$$C_1 = \frac{Q_{left}}{\omega R_{prim}} = 70\text{pF} \qquad C_2 = \frac{Q_{right}}{\omega R_L} = 24\text{pF}$$

$$\hat{I}_L = Q_{right} \cdot \frac{\hat{V}_L}{R_L} = 7.5 \cdot \frac{10}{50} = 1.5\text{A}$$

c. T-match



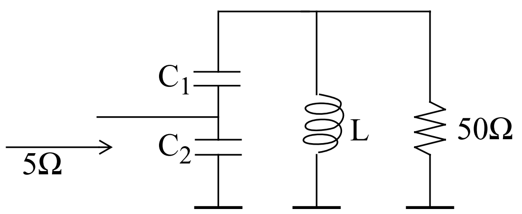
$Q = 10$

$$R_I = 300\Omega \Rightarrow Q = Q_{left} + Q_{right} = 7.68 + 2.24 = 9.92 \Rightarrow \text{close enough}$$

$$C = \frac{Q}{\omega R_I} = 52\text{pF} \qquad L_1 = \frac{Q_{left} \cdot R_{prim}}{\omega} = 6.1\text{nH} \qquad L_2 = \frac{Q_{right} \cdot R_L}{\omega} = 18\text{nH}$$

$$\hat{I}_{L_1} = \frac{\hat{V}_{prim}}{R_{prim}} = 0.63\text{A} \qquad \hat{I}_{L_2} = \frac{\hat{V}_L}{R_L} = 0.20\text{A}$$

d. Capacitive tap



$Q = 10$

$$L = \frac{R_L}{\omega \cdot Q} = 0.80\text{nH}$$

$$Q_2^2 = \frac{R_{prim}}{R_L} \cdot (Q^2 + 1) - 1 = 9.1$$

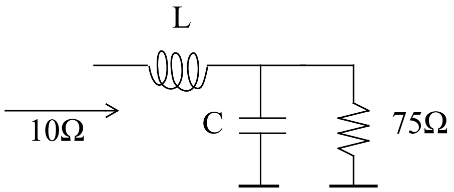
$$C_2 = \frac{Q_2}{\omega \cdot R_L} = 9.6\text{pF} \qquad C_1 = C_2 \cdot \frac{Q_2^2 + 1}{Q \cdot Q_2 - Q_2^2} = 4.2\text{pF}$$

$$\hat{I}_L = \frac{\hat{V}_L}{\omega \cdot L} = 2.0\text{A}$$

e. Which solutions are integratable?

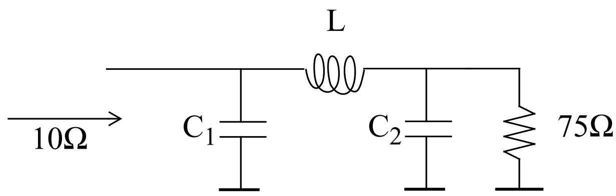
All solutions have component values that are compatible with integration, but the large current levels may be a problem. Also the low intermediate resistance in (b) is a problem, since the series resistances in the components must be mych smaller. Best suited for integration is (a).

3. Problem 4.5



$$f_0 = 150\text{MHz}$$
$$Q = \sqrt{\frac{75}{10} - 1} = 2.55$$
$$L = \frac{Q \cdot R_{prim}}{\omega_0} = 27\text{nH}$$
$$C = \frac{Q}{\omega_0 \cdot R_L} = 36\text{pF}$$

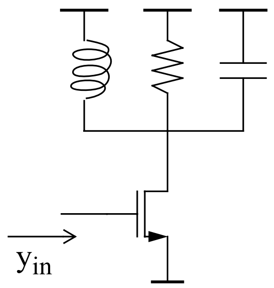
4. Problem 4.6



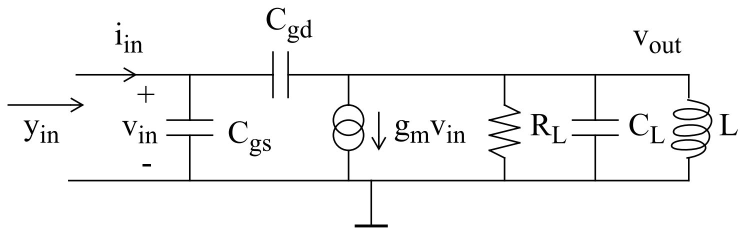
$$f_0 = 150\text{MHz}$$
$$BW = 15\text{MHz} \Rightarrow Q = \frac{BW}{f_0} = 10$$

$$R_I = 1.3\Omega \Rightarrow Q = Q_{left} + Q_{right} = 2.6 + 7.5 = 10.1 \Rightarrow \text{close enough}$$
$$L = \frac{Q \cdot R_I}{\omega_0} = 13.8\text{nH} \qquad C_1 = \frac{Q_{left}}{\omega_0 R_{prim}} = 276\text{pF} \qquad C_2 = \frac{Q_{right}}{\omega_0 R_L} = 106\text{pF}$$

5. Problem 8.5



Small signal equivalent:



The negative real part of y_{in} occurs due to C_{gd} and the load being inductive at some frequencies. To simplify the calculations, assume $C_{gd} \ll C_L$, so that the influence of C_{gd} on the voltage gain can be disregarded:

$$v_{out} = -v_{in} \cdot g_m \cdot Z_L \Rightarrow A_v = -g_m \cdot Z_L$$
$$\text{Miller} \Rightarrow i_{in} = v_{in}(1 - A_v) \cdot sC_{gd} + sC_{gs} = v_{in} \cdot (1 + g_m Z_L) \cdot sC_{gd} + sC_{gs}$$
$$y_{in} = i_{in}/v_{in} = sC_{gd} \cdot (1 + g_m Z_L) + sC_{gs} = s(C_{gs} + C_{gd}) + sC_{gd} \cdot g_m Z_L$$

The real part is due to the term $sC_{gd} \cdot g_m Z_L = j\omega C_{gd} \cdot g_m Z_L$

$$Z_L = \frac{1}{\frac{1}{R} + j\omega C_L + \frac{1}{j\omega L}} = \frac{1}{\frac{1}{R} + j\left(\omega C_L - \frac{1}{\omega L}\right)} = \frac{\frac{1}{R} - j\left(\omega C_L - \frac{1}{\omega L}\right)}{\left(\frac{1}{R}\right)^2 + \left(\omega C_L - \frac{1}{\omega L}\right)^2}$$

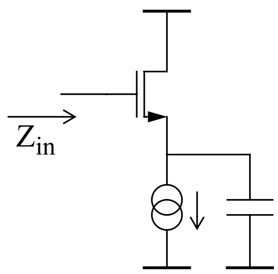
$$\mathrm{Re}(y_{in}) = \mathrm{Re}(j\omega C_{gd} \cdot g_m Z_L) = g_m \omega C_{gd} \cdot \frac{\omega C_L - \frac{1}{\omega L}}{\left(\frac{1}{R}\right)^2 + \left(\omega C_L - \frac{1}{\omega L}\right)^2} < 0 \Rightarrow \omega C_L < \frac{1}{\omega L}$$

$$\Rightarrow \omega < \frac{1}{\sqrt{L \cdot C_L}} = 2\pi \cdot f_0$$

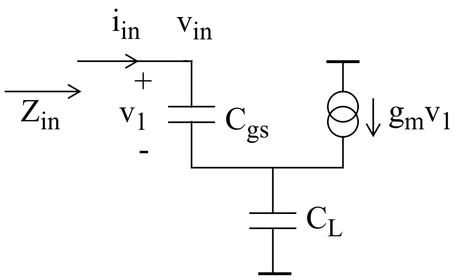
The real part is negative below the resonance frequency.
Two solutions:
1. Minimize C_{gd} since y_{in} is proportional to C_{gd}. For example by cascoding or neutralization.
2. Make sure that $\mathrm{Re}(y_{gen}) > -\mathrm{Re}(y_{in})$, that is drive with low impedance.

6. Problem 8.7

a.



Small signal equivalent:



$$\begin{cases} g_m(v_{in} - v_{out}) + (v_{in} - v_{out})sC_{gs} - v_{out}sC_L = 0 \Rightarrow v_{in}(g_m + sC_{gs}) = v_{out}(g_m + sC_{gs} + sC_L) \\ i_{in} = (v_{in} - v_{out})sC_{gs} \end{cases}$$

$$i_{in} = \left(v_{in} - v_{in} \cdot \frac{g_m + sC_{gs}}{g_m + sC_{gs} + sC_L}\right)sC_{gs} = v_{in} \cdot \left(1 - 1 + \frac{sC_L}{g_m + sC_{gs} + sC_L}\right)sC_{gs}$$

$$\Rightarrow Z_{in} = \frac{v_{in}}{i_{in}} = \frac{g_m + s(C_{gs} + C_L)}{s^2 C_L C_{gs}} = -\frac{g_m + j\omega(C_{gs} + C_L)}{\omega^2 C_{gs} C_L} \Rightarrow \mathrm{Re}(Z_{in}) = -\frac{g_m}{\omega^2 C_{gs} C_L}$$

b. As can be seen from (a) all C_L gives a negative $\mathrm{Re}(Z_{in})$!

c. One way is to put a resistor R_p from the input to signal ground. If $1/R_p > -1/\mathrm{Re}(Y_{in})$, $\mathrm{Re}(Z_{in})$ will be positive.

$$Y_{in} = \frac{1}{Z_{in}} = -\frac{\omega^2 C_{gs} C_L}{g_m + j\omega(C_{gs} + C_L)} = -\frac{\omega^2 C_{gs} C_L (g_m - j\omega(C_{gs} + C_L))}{g_m^2 + \omega^2 (C_{gs} + C_L)^2}$$

$$\mathrm{Re}(Y_{in}) = -\frac{g_m\omega^2 C_{gs}C_L}{g_m^2 + \omega^2(C_{gs}+C_L)^2}$$

$$R_p < -\frac{1}{\mathrm{Re}(Y_{in})} = \frac{g_m^2 + \omega^2(C_{gs}+C_L)^2}{g_m\omega^2 C_{gs}C_L} = \frac{g_m}{\omega^2 C_{gs}C_L} + \frac{1}{g_m}\cdot\frac{(C_{gs}+C_L)^2}{C_{gs}C_L}$$

To get a positive real part at all frequencies: $R_p < \frac{1}{g_m}\cdot\frac{(C_{gs}+C_L)^2}{C_{gs}C_L}$

7. Design problem

Max input signal = 0.15V(peak) => Set V_{gs}-V_t = 0.2V to avoid input clipping.
Frequency = 1GHz, Bandwidth = 200MHz => Q=5

Test if 20nH inductor, which has a Q of 5 can be used:

$$R_{pL} = \omega LQ = 630\Omega$$
$$A_v = g_m R_{pL} = 10 \Rightarrow g_m = 16\text{mS}$$
$$g_m = \frac{2I_d}{V_{gs}-V_t} \Rightarrow I_d = 1.6\text{mA}$$
$$\frac{W}{L} = \frac{I_d}{\mu C_{ox}(V_{gs}-V_t)^2} = 362,\ L = 0.4\mu\text{m} \Rightarrow W = 145\mu\text{m}$$
$$C_{db} = C_{jn}\cdot\frac{W}{2}\cdot 0.6 = 0.93\cdot 72.5\cdot 0.6\text{fF} = 40\text{fF}$$ (fingered transistor layout)
$$C_{dg} = C_{gd0}\cdot W = 0.21\cdot 145\text{fF} = 30\text{fF}$$
$$C_{dtot} = C_{db} + C_{dg} = 70\text{fF}$$

Total capacitance needed to resonate with L at 1GHz: $C_{tot} = \frac{1}{\omega_0^2\cdot L} = 1.27\text{pF}$

$C = C_{tot} - C_{dtot} - C_L = 1.1\text{pF}$ (In practice also correction for inductor parasitics needed)

The choice of inductance was fine (everything worked out to reasonable values).

It remains to choose V_{G1} and V_{G2}:
$V_{G1} = V_t + 0.2 = 0.72V$
$V_{G2} = V_t + 2V_{odmax} + 0.2V$ margin = 0.52 + 2 (0.2 + 0.15) + 0.2 = 1.42 V
Maximum output signal swing: v_{dd} - (1.42 - Vt) = 2.1V
The amplifier can handle 2.1V(peak). It would have been sufficient with $A_v V_{inmax}$=1.5V(peak).
This is OK, and the design is thereby successfully finished.