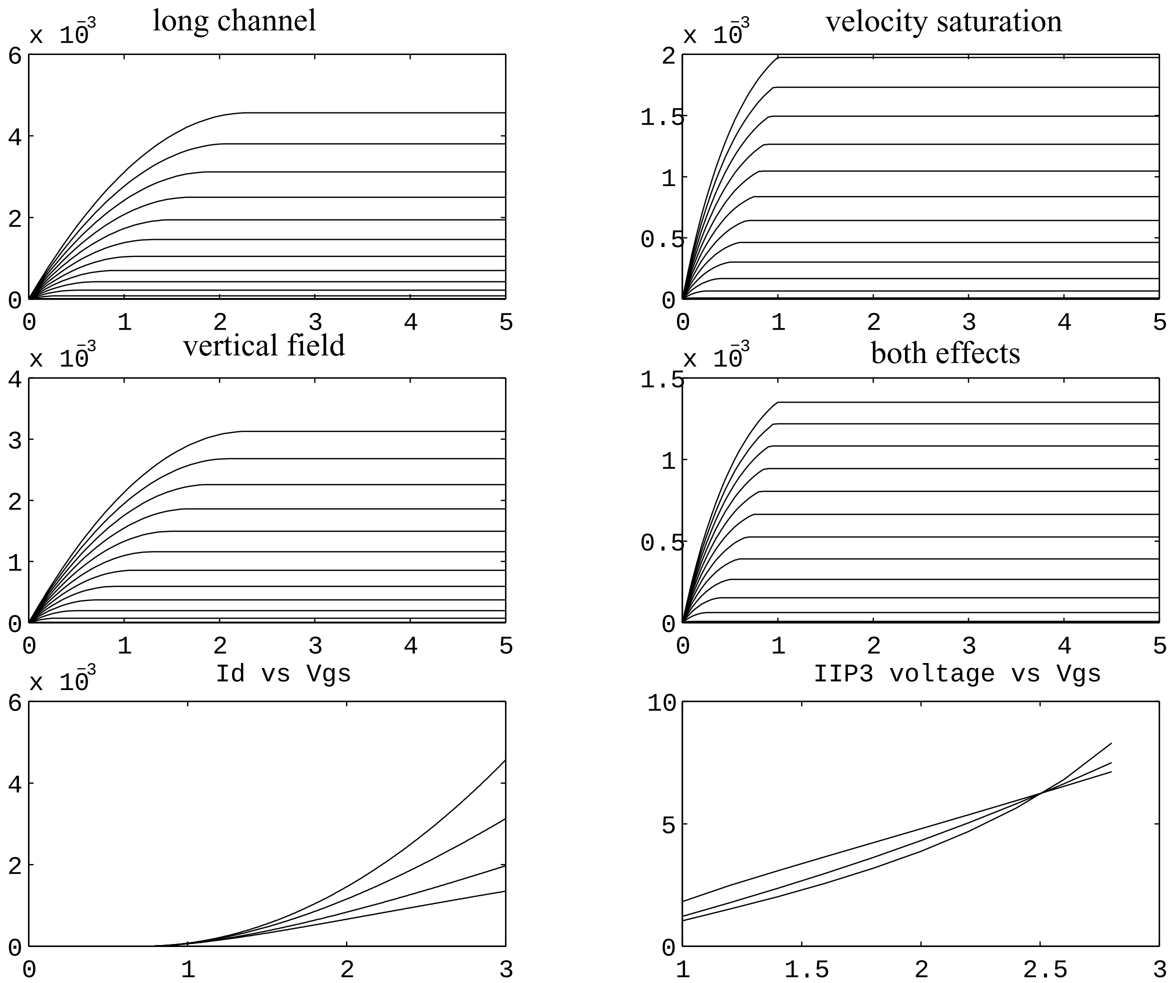
Solutions exercise 2 (components)

1. Problem 3.4

Solved numerically in Matlab.



The two plots in the bottom are extras. They show how Id and IIP3 vs Vgs looks for the four cases. One case is omitted in the IIP3 plot, since in the long channel case IIP3 is infinite.

2. Problem 3.1

$$L_D \approx \frac{2}{3}x_j \qquad C_{ov} = L_D W C_{ox} \approx \frac{2}{3}x_j W C_{ox}$$

Assume the transistor to be in saturation:

$$C_{gd} = C_{ov} \qquad C_{gs} = \frac{2}{3}C_{gc} + C_{ov} = \frac{2}{3}C_{ox}W(L-2L_D) + L_D W C_{ox}$$

$$\frac{C_{gd}}{C_{gs}} = \frac{L_D W C_{ox}}{\frac{2}{3}C_{ox}W(L-2L_D) + L_D W C_{ox}} = \frac{L_D}{\frac{2}{3}L - \frac{1}{3}L_D} = \frac{3L_D}{2L - L_D} = \frac{2x_j}{2L - \frac{2}{3}x_j} = \frac{3x_j}{3L - x_j}$$

The curves can be plotted in Matlab. Some extreme values:

x_j	L	C_{gd}/C_{gs}
250nm	0.5um	60%
50nm	5um	1%
250nm	0.5um	5%
50nm	0.5um	10%

3. Problem 3.3

Eq. 3.28 & 3.29 => $$I_D = \frac{\mu C_{ox}}{2} \cdot \frac{W}{L}(V_{gs} - V_t)^2 \cdot \frac{LE_{sat}}{(V_{gs} - V_t) + LE_{sat}}$$

$$g_m = \frac{\partial I_D}{\partial V_{gs}} = \frac{\mu C_{ox}}{2} \cdot \frac{W}{L} \cdot LE_{sat} \cdot \frac{2(V_{gs} - V_t)[(V_{gs} - V_t) + LE_{sat}] - (V_{gs} - V_t)^2}{[(V_{gs} - V_t) + LE_{sat}]^2}$$

$$= \frac{\mu C_{ox}}{2} \cdot \frac{W}{L} \cdot LE_{sat} \cdot (V_{gs} - V_t) \cdot \frac{2[(V_{gs} - V_t) + LE_{sat}] - (V_{gs} - V_t)}{[(V_{gs} - V_t) + LE_{sat}]^2}$$

$$= \frac{\mu C_{ox}}{2} \cdot WE_{sat} \cdot (V_{gs} - V_t) \cdot \frac{(V_{gs} - V_t) + 2LE_{sat}}{[(V_{gs} - V_t) + LE_{sat}]^2}$$

$$C_{gs} \approx \frac{2}{3}WLC_{ox} \qquad \text{(in saturation)}$$

$$\omega_t \approx \frac{g_m}{C_{gs}} \approx \frac{3}{2}\frac{1}{WLC_{ox}}\frac{\mu C_{ox}}{2}WE_{sat} \cdot (V_{gs} - V_t) \cdot \frac{(V_{gs} - V_t) + 2LE_{sat}}{[(V_{gs} - V_t) + LE_{sat}]^2}$$

$$= \frac{3}{4}\frac{\mu E_{sat}}{L} \cdot (V_{gs} - V_t) \cdot \frac{(V_{gs} - V_t) + 2LE_{sat}}{[(V_{gs} - V_t) + LE_{sat}]^2}$$

To verify the result, check the limits LE_{sat} towards zero and infinity. They must be possible to simplify to the short-channel and long-channel equations.

4. Problem 3.5

$$g_m = \mu C_{ox}\frac{W}{L}(V_{gs}-V_t) \Rightarrow \{(V_{gs}-V_t) = 1V = \text{ constant}\} \Rightarrow g_m \sim \mu \sim T^{-\frac{3}{2}}$$

$$\frac{g_{m400\text{K}}}{g_{m300\text{K}}} = \left(\frac{400}{300}\right)^{-\frac{3}{2}} = 0.65$$

Result: g_m decreases by 35%

5. Problem 3.8

The short-channel effects will affect the matching if the length of the transistors are different.

6. Inductor design

Requirements: L=5nH, f_s > 4.8GHz
Process: t_{met}=3um, t_{ox}=4um
Goal: maximize Q

Since the substrate losses is neglected, go for maximum dimensions to minimize the series resistance. The dimensions are limited by the f_s requirement.

$$f_s = \frac{1}{2\pi\sqrt{L \cdot C_{ox}/2}} \Rightarrow C_{ox} = \frac{2}{4\pi^2 f_s^2 L} = 440\text{fF}$$

$$C_{ox} = \frac{Area}{t_{ox}} \cdot \varepsilon \Rightarrow Area = \frac{C_{ox} \cdot t_{ox}}{\varepsilon} = 51000(\mu\text{m})^2$$

A track width W of 16um is first tested. The maximum length then becomes 3.19mm if the maximum area is not to be exceeded. Test four turns and a spacing of 2um.

$$a = \frac{length}{\#turns \cdot 8} = 100\mu\text{m} \qquad L = \frac{37.5 \cdot \mu_0 \cdot \#turns^2 \cdot a^2}{22r - 14a} = 5.4\text{nH}$$

Test W=20um, length=2.55mm, #turns=5, and spacing=2um => L=5.4nH OK.
This is possible to build (innermost turn has enough radius). It has a lower resistance than the first solution thanks to its larger width and shorter length, so we choose this one.

Lets now calculate the Q:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 1.7\mu\text{m} \quad R_s = \frac{length \cdot \rho}{W\delta(1-e^{-t_{met}/\delta})} = \frac{2.55\text{mm} \cdot 2.7 \cdot 10^{-8}\Omega\text{m}}{20\mu\text{m} \cdot 1.7\mu\text{m} \cdot (1-e^{-3/1.7})} = 2.44\Omega$$

$$Q = \frac{\omega L}{R_s} = 30.9$$

The Q is very high, since the substrate losses are neglected.

7. Problem 2.1

$$C_{corr} = \varepsilon \cdot \frac{\pi(R+H)^2}{H} = \varepsilon \cdot \left(\frac{\pi R^2}{H} + \underbrace{2\pi R}_{\text{correction term}}\right) = C_{uncorr} + \varepsilon \cdot 2\pi R$$

$$\frac{C_{corr}}{C_{uncorr}} = \frac{\frac{\pi R^2}{H} + 2\pi R}{\frac{\pi R^2}{H}} = 1 + 2\frac{H}{R} = 1 + 4\frac{H}{D}$$

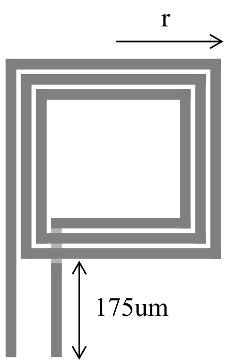
where H is oxide thickness, R is the radius and D is the diameter.

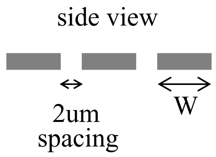
H/D	C_{corr}/C_{uncorr}	from table 2.5
0.005	1.02	1.023
0.01	1.04	1.042
0.025	1.1	1.094
0.05	1.2	1.167
0.1	1.4	1.286

The errors become substantial

8. Problem 2.2

a.





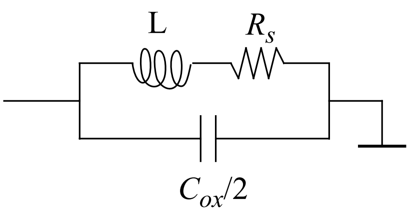
first test: r=120um
w=5um => a=85um
n=11

$$L = \frac{37.5 \cdot \mu_0 \cdot n^2 \cdot a^2}{22r - 14a} = 28nH$$

Too high. Try smaller r and n and fine tune:

r=86um
w=5um a=65 um
n=7 L=10nH

b. model:

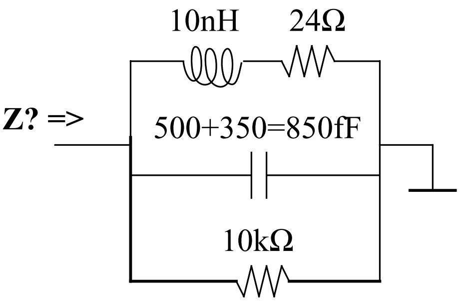


$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} = 2\mu\text{m}$$

$$R_s = \frac{l}{w\sigma\delta(1-e^{-t/\delta})} = \frac{8na + 350\mu\text{m}}{w\sigma\delta(1-e^{-t/\delta})} = 25\Omega$$

$$C_{ox} = \frac{wl\varepsilon_{ox}}{t_{ox}} = 700\text{fF}$$

c.



$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1.73\text{GHz}$$

$$Q_L = \frac{\omega_0 L}{R_s} = 4.35$$

$$R_{pL} = R_s(Q_L^2 + 1) = 500\Omega$$

$$Z|_{f_0} = R_{ptot} = \frac{R_{pL} \cdot 10\text{k}\Omega}{R_{pL} + 10\text{k}\Omega} = 475\Omega$$

With more metal layers, a higher Q and/or lower parasitic capacitance can be achieved.

9. Problem 2.5

Given:

$$i_D = I_s \cdot e^{v_i/v_T} \qquad v_T = 25\text{mV} \qquad \& \qquad v_j = 0.5\text{V} \Rightarrow i_D = 1\text{mA}$$

$$C_{j0} = 2\text{pF} \qquad n = \frac{1}{2} \qquad \phi = 0.8\text{V}$$

a. Calculate the incremental resistance at V_j = 0.5V

$$g_d = \frac{\partial i_D}{\partial v_i} = \frac{I_s}{v_T} \cdot e^{v_j/v_T} = \frac{i_D}{v_T}$$

at 0.5V: $$g_d = \frac{1\text{mA}}{25\text{mV}} = 0.04\text{S} \Rightarrow r_d = \frac{1}{g_d} = \frac{1}{0.04} = 25\Omega$$

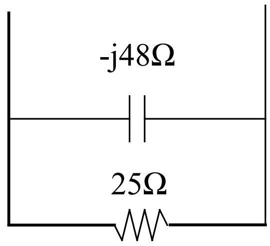
b. Calculate the capacitance at V_j = 0.5V

$$C_j = \frac{C_{j0}}{(1 - V_j/\phi)^n} = \frac{2\text{pF}}{\sqrt{1 - 0.5/0.8}} = 3.3\text{pF}$$

c. Calculate reactance at 1GHz (V_j=0.5V)

$$\frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 1 \cdot 10^9 \cdot 3.3 \cdot 10^{-12}} = -j48\Omega$$

The varactors appears mainly resistive at 1GHz
(see figure to the right)



10. Problem 2.9

$$R_0 = 10\ \text{k}\Omega \qquad R_{square} = 100\Omega$$

a. $W = W_0 \pm 0.2\mu\text{m} \qquad R = R_0 \pm 5\%$

$$R = R_0 \cdot \frac{W_0}{W} \Rightarrow 0.95 < \frac{R}{R_0} = \frac{W_0}{W} > 1.05 \Rightarrow \frac{W_0}{W_0 + 0.2\mu\text{m}} > 0.95,\ \frac{W_0}{W_0 - 0.2\mu\text{m}} < 1.05$$

$$\Rightarrow W_0 \geq 4.2\mu\text{m}$$

$$L_0 = W_0 \cdot \frac{R_0}{R_{square}} = 4.2\mu\text{m} \cdot \frac{10000}{100} = 420\mu\text{m}$$

b. $$C_{parasitic} = \frac{W_0 L_0 \varepsilon_{ox}}{t_{ox}} = 61\,\text{fF}$$

$$f_{limit} = \frac{1}{2\pi \cdot R_0 \cdot C_{parasitic}} = \frac{1}{2\pi \cdot 10 \cdot 10^3 \cdot 61 \cdot 10^{-15}} = 260\text{MHz}$$

The higher the accuracy required, the wider the resistor and the lower the frequency limit.