Digital modulation
Contents

• Brief overview of a wireless communication link
• Radio signals and complex notation (again)
• Modulation basics
• Important modulation formats
STRUCTURE OF A WIRELESS COMMUNICATION LINK
A simple structure

Speech

Data

A/D → Speech encoder → Encrypt. → Chann. encoding → Modulation

(Read Chapter 10 for more details)
RADIO SIGNALS AND COMPLEX NOTATION
(from Lecture 3)
Simple model of a radio signal

• A transmitted radio signal can be written

\[ s(t) = A \cos(2\pi ft + \phi) \]

- Amplitude
- Frequency
- Phase

• By letting the transmitted information change the amplitude, the frequency, or the phase, we get the three basic types of digital modulation techniques:
  
  - **ASK** (Amplitude Shift Keying)
  - **FSK** (Frequency Shift Keying)
  - **PSK** (Phase Shift Keying)

Constant envelope
Example: Amplitude, phase and frequency modulation

\[ s(t) = A(t) \cos\left(2\pi f_c t + \phi(t)\right) \]

<table>
<thead>
<tr>
<th></th>
<th>(A(t))</th>
<th>(\phi(t))</th>
<th>Comment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>4ASK</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td>- Amplitude carries information&lt;br&gt;- Phase constant (arbitrary)</td>
</tr>
<tr>
<td>4PSK</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
<td>- Amplitude constant (arbitrary)&lt;br&gt;- Phase carries information</td>
</tr>
<tr>
<td>4FSK</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
<td>- Amplitude constant (arbitrary)&lt;br&gt;- Phase slope (frequency) carries information</td>
</tr>
</tbody>
</table>
The IQ modulator

I-channel (in-phase)

\[ s_I(t) \]

\[ \times \]

\[ \cos(2\pi f_c t) \]

\[ \bigwedge \]

\[ f_c \]

\[ -90^\circ \]

\[ -\sin(2\pi f_c t) \]

Q-channel (quadrature)

\[ s_Q(t) \]

\[ \times \]

Transmitted radio signal

\[ s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t) \]

Take a step into the complex domain:

Complex envelope \[ \tilde{s}(t) = s_I(t) + js_Q(t) \]

Carrier factor \[ e^{j2\pi f_c t} \]

\[ s(t) = \text{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\} \]
Interpreting the complex notation

Complex envelope (phasor)

Transmitted radio signal

\[ s(t) = \text{Re} \left\{ \tilde{s}(t)e^{j2\pi f_c t} \right\} \]
\[ = \text{Re} \left\{ A(t)e^{j\phi(t)}e^{j2\pi f_c t} \right\} \]
\[ = \text{Re} \left\{ A(t)e^{j(2\pi f_c t + \phi(t))} \right\} \]
\[ = A(t)\cos(2\pi f_c t + \phi(t)) \]

Polar coordinates:

\[ \tilde{s}(t) = s_I(t) + js_Q(t) = A(t)e^{j\phi(t)} \]

By manipulating the amplitude \( A(t) \) and the phase \( \phi(t) \) of the complex envelope (phasor), we can create any type of modulation/radio signal.
MODULATION BASICS
Pulse amplitude modulation (PAM)  
The modulation process

![Diagram showing the modulation process with complex numbers, PAM, and radio signal]

**Bits**  
$b_m$ → **Mapping** → $c_m$ → **PAM** → $s_{LP}(t)$ → $\text{Re}\{\}$ → **Radio signal**

**Complex domain**

**Complex numbers**

**PAM:**  
$$s_{LP}(t) = \sum_{m=-\infty}^{\infty} c_m g(t - m T_s)$$

**“Standard” basis pulse criteria**

$$\int_{-\infty}^{\infty} |g(t)|^2 \, dt = 1 \text{ or } = T_s \quad \text{(energy norm.)}$$

$$\int_{-\infty}^{\infty} g(t) g^*(t - m T_s) \, dt = 0 \text{ for } m \neq 0 \quad \text{(orthogonality)}$$

**Many possible pulses**

"Standard" basis pulse criteria

$$\int_{-\infty}^{\infty} |g(t)|^2 \, dt = 1 \text{ or } = T_s \quad \text{(energy norm.)}$$

$$\int_{-\infty}^{\infty} g(t) g^*(t - m T_s) \, dt = 0 \text{ for } m \neq 0 \quad \text{(orthogonality)}$$
Pulse amplitude modulation (PAM)
Basis pulses and spectrum

Assuming that the complex numbers $c_m$ representing the data are independent, then the **power spectral density** of the base band PAM signal becomes:

$$S_{LP}(f) \sim \left| \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} \, dt \right|^2$$

which translates into a radio signal (band pass) with

$$S_{BP}(f) = \frac{1}{2} \left( S_{LP}(f - f_c) + S_{LP}(-f - f_c) \right)$$
Pulse amplitude modulation (PAM)

Basis pulses and spectrum

Illustration of power spectral density of the (complex) base-band signal, $S_{LP}(f)$, and the (real) radio signal, $S_{BP}(f)$.

Can be asymmetric, since it is a complex signal.

Symmetry (real radio signal)

What we need are basis pulses $g(t)$ with nice properties like:

- Narrow spectrum (low side-lobes)
- Relatively short in time (low delay)
Pulse amplitude modulation (PAM)

Basis pulses

**TIME DOMAINT**

- Rectangular [in time]

**FREQ. DOMAIN**

- (Root-) Raised-cosine [in freq.]
Pulse amplitude modulation (PAM) Interpretation as IQ-modulator

For real valued basis functions $g(t)$ we can view PAM as:

$$s_1(t) = \Re{s_{LP}(t)}$$

$$s_Q(t) = \Im{s_{LP}(t)}$$

(Both the rectangular and the (root-) raised-cosine pulses are real valued.)
Multi-PAM
Modulation with multiple pulses

Bits

\[ b_m \]

\[ \text{Mapping} \]

\[ c_m \]

\[ \text{multi-PAM} \]

\[ s_{LP}(t) \]

\[ \text{Re}\{ \} \]

Radio signal

\[ \exp(j2\pi f_c t) \]

Complex domain

\[
\text{multi-PAM: } s_{LP}(t) = \sum_{m=-\infty}^{\infty} g_{c_m}(t - mT_s)
\]

“Standard” basis pulse criteria

\[
\int |g_{c_m}(t)|^2 dt = 1 \text{ or } =T_s \quad \text{(energy norm.)}
\]

\[
\int g_{c_m}(t) g_{c_m}^*(t - kT_s) dt = 0 \text{ for } k \neq 0 \quad \text{(orthogonality)}
\]

\[
\int g_{c_m}(t) g_{c_n}^*(t) dt = 0 \text{ for } c_m \neq c_n \quad \text{(orthogonality)}
\]

Several different pulses

3 April 2017
Multi-PAM
Modulation with multiple pulses

Frequency-shift keying (FSK) with $M$ (even) different transmission frequencies can be interpreted as multi-PAM if the basis functions are chosen as:

$$g_k(t) = e^{-j\pi k \Delta f t} \text{ for } 0 \leq t \leq T_s$$

and for $k = +/- 1, +/- 3, ..., +/- M/2$

Bits: 00 01 10 11
Continuous-phase FSK (CPFSK)

The modulation process

Bits $b_m$ → Mapping $c_m$ → CPFSK $s_{LP}(t)$ → Re{ }

Complex domain

Radio signal

CPFSK: $s_{LP}(t) = A \exp\left(j \Phi_{CPFSK}(t)\right)$

where the amplitude $A$ is constant and the phase is

$\Phi_{CPFSK}(t) = 2\pi h_{mod} \sum_{m=-\infty}^{\infty} c_m \int_{-\infty}^{t} \tilde{g}(u-mT) \, du$

where $h_{mod}$ is the modulation index.
Continuous-phase FSK (CPFSK)
The Gaussian phase basis pulse

In addition to the rectangular phase basis pulse, the Gaussian is the most common.

\[ BT_s = 0.5 \]
IMPORTANT MODULATION FORMATS
Binary phase-shift keying (BPSK)
Rectangular pulses

Base-band

Radio signal
Binary phase-shift keying (BPSK)
Rectangular pulses

Complex representation

Signal constellation diagram

\[ \sqrt{E_B} \]

\[ -\sqrt{E_B} \]
Binary phase-shift keying (BPSK)
Rectangular pulses

Power spectral density for BPSK

<table>
<thead>
<tr>
<th>Contained percentage of total energy</th>
<th>Spectral efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>0.59 Bit/s/Hz</td>
</tr>
<tr>
<td>99%</td>
<td>0.05 Bit/s/Hz</td>
</tr>
</tbody>
</table>
Binary phase-shift keying (BPSK)
Raised-cosine pulses (roll-off 0.5)

Base-band

Radio signal
Binary phase-shift keying (BPSK)
Raised-cosine pulses (roll-off 0.5)
Binary phase-shift keying (BPSK)
Raised-cosine pulses (roll-off 0.5)

Power spectral density for BAM

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<tbody>
<tr>
<td>90%</td>
<td>1.02 Bit/s/Hz</td>
</tr>
<tr>
<td>99%</td>
<td>0.79 Bit/s/Hz</td>
</tr>
</tbody>
</table>

Much higher spectral efficiency than BPSK (with rectangular pulses).
Quaternary PSK (QPSK or 4-PSK)
Rectangular pulses

Complex representation

Radio signal
Quaternary PSK (QPSK or 4-PSK)

Rectangular pulses

Power spectral density for QPSK

Twice the spectrum efficiency of BPSK (with rect. pulses). TWO bits/pulse instead of one.

<table>
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<th>Contained percentage of total energy</th>
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<tbody>
<tr>
<td>90%</td>
<td>1.18 Bit/s/Hz</td>
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<tr>
<td>99%</td>
<td>0.10 Bit/s/Hz</td>
</tr>
</tbody>
</table>
Quadrature ampl.-modulation (QAM)
Root raised-cos pulses (roll-off 0.5)

Complex representation

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</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>2.04Bit/s/Hz</td>
</tr>
<tr>
<td>99%</td>
<td>1.58Bit/s/Hz</td>
</tr>
</tbody>
</table>

Much higher spectral efficiency than QPSK (with rectangular pulses).
Signals with high amplitude variations leads to less efficient amplifiers.

Complex representation of QPSK

It is a problem that the signal passes through the origin, where the amplitude is ZERO. (Infinite amplitude variation.)

Can we solve this problem in a simple way?
Amplitude variations
A solution

Let’s rotate the signal constellation diagram for each transmitted symbol!
Amplitude variations
A solution

Looking at the complex representation ...

QPSK without rotation

QPSK with rotation

A “hole” is created in the center. No close to zero amplitudes.
Still uses the same rectangular pulses as QPSK - the power spectral density and the spectral efficiency are the same.

This modulation type is used in several standards for mobile communications (due to it’s low amplitude variations).
Offset QPSK (OQPSK)
Rectangular pulses

There is one bit-time offset between the in-phase and the quadrature part of the signal (a delay on the Q channel). This makes the transitions between pulses take place at different times!
Offset QPSK
Rectangular pulses

This method also creates a hole in the center, giving less amplitude variations.
Offset QAM (OQAM)
Raised-cosine pulses

This method also creates a hole in the center, but has larger amplitude variations than OQPSK.
Continuous-phase modulation

**Basic idea:**
- Keep **amplitude constant**
- Change phase continuously

In this particular example we change the phase in a piecewise linear fashion by +/- $\pi/2$, depending on the data transmitted.

This type of modulation can be interpreted both as phase and frequency modulation. It is called **MSK** (minimum shift keying) or **FFSK** (fast frequency shift keying).
Minimum shift keying (MSK)

Simple MSK implementation

01001

Rectangular pulse filter

Voltage controlled oscillator (VCO)

MSK signal
Minimum shift keying (MSK)

Power spectral density of MSK

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<td>90 %</td>
<td>1.29 Bit / s / Hz</td>
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<tr>
<td>99 %</td>
<td>0.85 Bit / s / Hz</td>
</tr>
</tbody>
</table>
Gaussian filtered MSK (GMSK)

Further improvement of the phase: Remove 'corners'

(Simplified figure)

MSK
(Rectangular pulse filter)

Gaussian filtered MSK - GMSK
(Gaussian pulse filter)
Gaussian filtered MSK (GMSK)

Simple GMSK implementation

When implemented this “simple” way, it is usually called Gaussian filtered frequency shift keying (GFSK).

GFSK is used in e.g. Bluetooth.
This is a more precise implementation of GMSK, which is used in e.g. GSM.
Gaussian filtered MSK (GMSK)

Power spectral density of GMSK.

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<tbody>
<tr>
<td>90 %</td>
<td>1,45 Bit / s / Hz</td>
</tr>
<tr>
<td>99 %</td>
<td>0,97 Bit / s / Hz</td>
</tr>
</tbody>
</table>
How do we use all these spectral efficiencies?

Example: Assume that we want to use MSK to transmit 50 kbit/sec, and want to know the required transmission bandwidth.

Take a look at the spectral efficiency table:

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<td>0.85 Bit / s / Hz</td>
</tr>
</tbody>
</table>

The 90% and 99% bandwidths become:

\[
B_{90\%} = \frac{50000}{1.29} = 38.8 \text{ kHz}
\]

\[
B_{99\%} = \frac{50000}{0.85} = 58.8 \text{ kHz}
\]
### Summary

BPSK with root-raised cosine pulses

<table>
<thead>
<tr>
<th>Modulation method</th>
<th>spectral efficiency for 90% of total energy (Bit / s / Hz)</th>
<th>spectral efficiency for 99% of total energy (Bit / s / Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>0.59</td>
<td>0.05</td>
</tr>
<tr>
<td>BAM ($\alpha=0.5$)</td>
<td>1.02</td>
<td>0.79</td>
</tr>
<tr>
<td>QPSK, OQPSK,</td>
<td>1.18</td>
<td>0.10</td>
</tr>
<tr>
<td>MSK</td>
<td>1.29</td>
<td>0.85</td>
</tr>
<tr>
<td>GMSK ($B_g \ T = 0.5$)</td>
<td>1.45</td>
<td>0.97</td>
</tr>
<tr>
<td>QAM ($\alpha = 0.5$)</td>
<td>2.04</td>
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TABLE 11.1 in textbook.