Selected answers*
Problem set 2

Wireless Communications, 2nd Ed.

5.5 Large-scale log-normal fading

a) Outage happens when the propagation loss is 8 dB higher than the deterministic 127 dB (at 135 dB). With $\sigma_F = 7\ \mathrm{dB}$ log-normal fading, the **outage probability** becomes

$$P_{\mathrm{out}} = Q\left(\frac{8}{7}\right) = 12.7\% \tag{1}$$

b) Possible solutions for improving outage:

i. *Increase TX power*: **YES**, this will work, since it increases the fading margin. Spectrum regulations may, however, prevent us from doing so.

ii. *Decrease the deterministic path loss*: **YES**, this may be possible. It can be done by making a BS taller, for instance.

iii. *Change the antennas*: **YES**, this will also work. If the signal can be focused better the total loss will become smaller. The focusing of energy will, however, make the system more sensitive to the direction in which the TX/RX is in. Spectrum regulations, limiting EIRP may also prevent us from doing this.

iv. *Lower the σ_F*: **NO**, this will not work in general. We would need to re-design the radio propagation environment, which is not exactly realistic.

v. *Build a better RX*: **YES**, this will work, in principle. The system can tolerate a lower received power if a better receiver is designed. It may, however, not be a practical solution, since better receivers tend to be more expensive from several points of view.

5.6 Small-scale Rayleigh fading

(a) Since a Rayleigh-distributed amplitude r corresponds to an exponentially distributed power Kr^2 the **outage probability** in terms of RX sensitivity (power) level $C_{\min}$ and mean received power $\bar{C}$ is

$$P_{\mathrm{out}} = 1 - e^{-C_{\min}/\bar{C}} \tag{1}$$

(b) Using the definition of fading margin, $M = \bar{C}/C_{\min}$, we get

$$P_{\mathrm{out}} = 1 - e^{-1/M} \tag{2}$$

which gives us the required **fading margin**, in dB, as

$$M = -10\log_{10}\left(-\ln\left(1 - P_{\mathrm{out}}\right)\right)\ \mathrm{dB}. \tag{3}$$

* Note: Solutions provided here are less detailed than the ones expected during the exam. Many steps are excluded.

[5.7] Approximation of (Rayleigh) fading margin

a) With the approximation $P_{\text{out}} \approx r_{\text{min}}^2/2\sigma^2$ we get an **approximate fading margin**

$$\tilde{M} = -10\log_{10} P_{\text{out}} \text{ dB}. \tag{1}$$

b) The error between the exact and approximate fading margin is

$$M - \tilde{M} = -10\log_{10}\left(-\ln\left(1 - P_{\text{out}}\right)\right) + 10\log_{10} P_{\text{out}} \text{ dB} \tag{2}$$

and the **largest error**, -0.11 dB, occurs at the edge of the interval, namely where $P_{\text{out}} = 5\%$. Given how large uncertainties we normally have when designing radio systems, this error is insignificant.

5.9 Level crossing rate and average duration of fades

The level crossing rate of Rayleigh fading under isotropic scattering conditions (Jakes' fading), expressed in terms of the received amplitude r threshold,

$$N_R(r) = \sqrt{2\pi}\nu_{\max}\frac{r}{\sqrt{2\Omega_0}}e^{-\frac{r^2}{2\Omega_0}} \quad . \tag{1}$$

where $\sqrt{2\Omega_0}$ is the RMS value of the received amplitude. Hence, $r^2/2\Omega_0 = C/\bar{C} = 1/M$ and the **level crossing rate** can be written

$$N_R(M) = \sqrt{2\pi}\nu_{\max}\frac{1}{\sqrt{M}}e^{-1/M} \quad . \tag{2}$$

When expressed in terms of amplitude, the product $N_R(r)ADF(r)$ equals the cdf of the Rayleigh distribution. Now, when expressed in terms of power instead of amplitude, the distribution is exponential instead. Hence, the **average duration of fades** becomes

$$A\,DF(M) = \frac{1 - e^{-1/M}}{N_R(M)} = \frac{\sqrt{M}}{\sqrt{2\pi}\nu_{\max}}\left(e^{1/M} - 1\right) \tag{3}$$

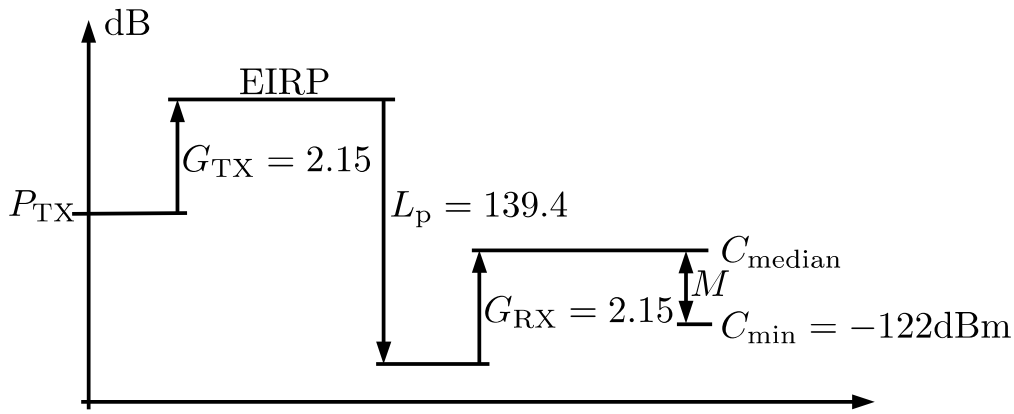
5.10 Large- and small-scale fading

The propagation loss is given by

$$L_{\text{p}} = 10\log_{10}\frac{d^4}{h_{BS}^2 h_{MS}^2} + 10\log_{10}\left(\frac{f_{MHz}^2}{1,600}\right) = 139.4 \text{ dB}. \tag{1}$$

Check: Are we inside the radio horizon? $d_h \approx 4100\left(\sqrt{20} + \sqrt{1.5}\right) = 23 \times 10^3$ ... YES!

a) With the parameters given in the problem statement, the **link budget** becomes:



b) The fading margin required for the small scale fading, relative to the *average* received power, with $P_{\mathrm{out}} = 5\%$ is

$$M_{\mathrm{small}} = -10\log_{10}\left(-\ln\left(1 - P_{\mathrm{out}}\right)\right) = 12.9 \text{ dB}. \tag{2}$$

Since the propagation loss in the problem gives a *median* value, we need to compensate for the difference between median and average power of the Rayleigh fading signal. This correction is 1.58 dB. Hence, the **fading margin related to the median power** is

$$M_{\mathrm{small,median}} = 12.9 - 1.58 = 11.3 \text{ dB}. \tag{3}$$

c) To obtain 95% boundary coverage, we need a fading margin M_{large} against $\sigma_F = 5$ dB log-normal fulfilling

$$Q\left(\frac{M_{\mathrm{large}}}{5}\right) = 5\% \tag{4}$$

which gives

$$M_{\mathrm{large}} = 5 \times 1.645 = 8.2 \text{ dB}. \tag{5}$$

d) The fading margin to be inserted in the link budget, given that we add the respective fading margins, is

$$M = M_{\mathrm{small,median}} + M_{\mathrm{large}} = 19.5 \text{ dB}. \tag{6}$$

With this value inserted in the link budget, the required **transmit power** becomes

$$P_{TX} = -122 + 19.5 - 2.15 + 139.4 - 2.15 = 32.6 \text{ dBm}. \tag{7}$$

6.7 CDMA signals

With a maximum excess delay of $1.3\ \mu s$ and a chip duration of $0.26\ \mu s$, the multipath components fall in $1.3/0.26 = 5$ delay bins. This means that we experience leakage of energy between chips and the channel is therefore **wide-band**.

If the maximum excess delay is $100\ ns$, then the channel is much shorter than a chip and we do not experience any significant leakage of energy between chips. The channel is therefore considered as **narrow-band** in this case.

7.2 Okumura and Okumura-Hata channel models

With the given parameters we get the follwing results:

a) Okumura's model (by calculating free-space loss and compensating according to figures 7.12 and 7.13)

$$L_{Oku} = 133 \text{ dB} \tag{1}$$

b) Okumura-Hata model

$$L_{Oku-Hata} = 123.4 + 34.4 \log_{10} 2 + 0 = 133.7 \text{ dB} \tag{2}$$

c) The two models agree well, which is quite natural since the second is a parameterization of the first. The range of validity for the Okumura-Hata model is, however, a bit smaller.

9.1 User influence on coverage

Let's assume that the MS antenna is a patch antenna, since helix antennas are very rare on terminals these days. Figure 9.1 then shows that the median user decreases the radiation efficiency by about 3.5 dB. Since the propagation exponent is 3.2, a 3.5 dB reduction in the link budget gives

$$10 \log_{10} \left(\frac{d}{37} \right)^{3.2} = -3.5 \text{ dB} \tag{1}$$

and a **new coverage distance** of $d = 29$ km.