EXAM QUESTION EXAMPLES

ETIN10, CHANNEL MODELING FOR WIRELESS COMMUNICATIONS, 2017

Question 1

This question is regarding the concepts of large-scale and small-scale fading:

a) Please give a brief physical explanation as to why large-scale and small-scale fading occur in wireless systems.

b) Give an example of a distribution that is typically used to model the following (give one example for each):

- Small-scale fading in a line-of-sight scenario.
- Small-scale fading in a non-line-of-sight scenario.
- Large-scale fading.

Question 2

Consider two different wireless systems **a** and **b**. For **a**, the signal bandwidth of the system is much smaller than the coherence bandwidth of the channel. Conversely, **b** employs a signal bandwidth that is much larger than the coherence bandwidth of the channel. Which system (**a** or **b**) is best suited for employing frequency diversity techniques? Motivate Your answer.

Question 3

The power delay profile of a channel is modeled as

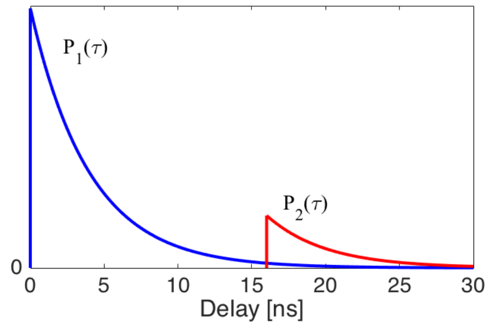
$$P(\tau) = P_1(\tau) + P_2(\tau),$$

where

$$P_1(\tau) = \begin{cases} a_1 \mathrm{e}^{-\tau/\gamma}, \text{ for } \tau \geq 0 \\ 0, \text{ for } \tau < 0 \end{cases}$$

$$P_2(\tau) = \begin{cases} a_2 \mathrm{e}^{-(\tau-\tau_2)/\gamma}, \text{ for } \tau \geq \tau_2 \\ 0, \text{ for } \tau < \tau_2 \end{cases}$$

Here, $a_1 = 1 \cdot 10^{-4}$, $a_2 = 2 \cdot 10^{-5}$, $\gamma = 4$ ns and $\tau_2 = 16$ ns. $P_1(\tau)$ and $P_2(\tau)$ are shown in the figure below.



a) Calculate the *time-integrated power* given by

$$P_m = \int_{-\infty}^{\infty} P(\tau) d\tau$$

b) Calculate the *mean delay* given by

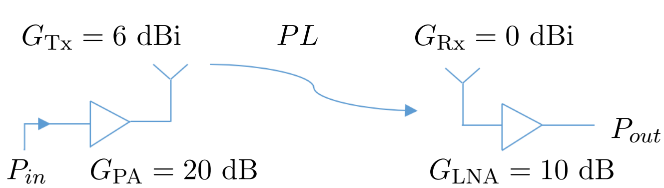
$$T_m = \frac{\int_{-\infty}^{\infty} P(\tau)\tau d\tau}{P_m}$$

c) Calculate the *rms delay spread* given by

$$S_\tau = \sqrt{\frac{\int_{-\infty}^{\infty} P(\tau)\tau^2 d\tau}{Pm} - T_m^2}$$

Question 4

A cellular system is operating at a freqeuncy of 1800 MHz. The base station is equipped with a power amplifier (PA) with a gain of $G_{PA} = 20$ dB. The PA is directly connected to the BS antenna, which has a gain of $G_{\mathrm{Tx}} = 6$ dBi. The typical user-equipment is equipped with an omnidirectional antenna with a gain of $G_{\mathrm{Rx}} = 0$ dBi, that is directly connected to a low-noise amplifier (LNA) with a gain of $G_{\mathrm{LNA}} = 10$ dB. The system noise figure is assumed to be 0 dB, and the receiver sensitivity is $S_{\mathrm{UE}} = -120$ dBm. The system needs to be able to operate up to distances of 5 km.



a) Assume that the path loss, PL, is modelled as free space path loss plus a lognormal large-scale fading term with $\sigma = 6$ dB and $\mu = 0$ dB. Based on these parameters for the large-scale fading, a fading margin of 10 dB is chosen with respect to the receiver sensitivity.

Calculate the minimum required input power P_{in} to the PA at the BS in order to fulfil this fading margin. Give the answer in units of Watts.

b) Repeat this calculation with the same settings, but for a path loss that is modeled by the log-distance power law with $PL(d_0) = 43$ dB, where $d_0 = 1$ m, and the path loss exponent is $n = 3.4$.

d) Now, assume the same settings as in **b)**, but the standard deviation of the large-scale fading is 8 dB instead. Calculate a new fading margin with respect to the receiver sensitivity, so that the probability of having a received power smaller than the receiver sensitivity is not changed.

Answers

Answer to Question 1. See text book.

Answer to Question 2. Since system **a** has a signal bandwidth that is much smaller than the coherence bandwidth, it will experience frequency flat fading. This means that, for each channel realization, all the frequency components will experience highly correlated fading. System **b** on the other hand, will experience frequency selective fading. Since the signal bandwidth is much larger than the coherence bandwidth of the channel, there will be many frequency components that experience uncorrelated fading. Hence, frequency diversity techniques are better suited for use in system **b**.

Answer to Question 3. See the appended solution at the end of this document.

Answer to Question 4. **a)** Based on the link-budget of this system, assuming a system noise-figure of 0 dB[1], the output power is given by

$$P_{out} = P_{in} + G_{Tx} + G_{PA} - PL + G_{Rx} + G_{LNA}$$

Based on the chosen fading margin relative to the receiver sensitivity, M_F, we know that the minimum required output power is given by

$$P_{out,min} = S_{UE} + M_F = -120 + 10 = -110 \text{ dBm}.$$

Based on this we can now calculate the required minimum input power as

$$P_{in,min} = P_{out,min} - G_{Tx} - G_{PA} + PL - G_{Rx} - G_{LNA}$$

In **a**, the path loss as a function of distance is given by

$$PL_a(d) = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right).$$

Now, we can evaluate (4) at a distance of 5 km and then use this value in (3). This gives us

$$P_{in,min} = -34.5 \text{ dBm} = -34.5 \text{ dBW}.$$

Answer: The minimum required input power is 0.36 μW.

[1] In practice, this type of assumption is typically not reasonable.

b) We now replace the path loss in the above calculation with the log-distance power law, i.e.

$$PL_b(d) = P(d_0) + 10n\log_{10}\left(\frac{d}{d_0}\right).$$

Evauluating this at a distance of 5 km gives us

$$P_{in,min} = 22.765 \text{ dBm} = -7.235 \text{ dBW}.$$

Answer: The minimum required input power is 0.19 W.

c) We want to have the same outage probability for the old case and the new case. Hence, the tail probabilities for these cases should be equal. We can safely assume that the large scale fading is modelled by a normal distribution, and hence we can use the Q-function for the tail probabilities. Let M_{F1} be the old fading margin and M_{F2} be the new fading margin. Then

$$Q\left(\frac{M_{F1}}{\sigma_{F1}}\right) = Q\left(\frac{M_{F2}}{\sigma_{F2}}\right),$$

and,

$$Q\left(\frac{10}{6}\right) = Q\left(\frac{M_{F2}}{8}\right),$$

which gives

$$M_{F2} = \frac{10}{6} \cdot 8 = \frac{40}{3}.$$

Answer: The new fading margin is 13.33 dB.[2]

[2]Note that this also gives us the tail probability that was used to choose the fading margin. It turns out that this was chosen based on a value of the tail probability close to 5%.

Example. 3

① time - integrated power.

$$P_m = \int_{-\infty}^{\infty} a_1 e^{-\tau/\gamma} + a_2 e^{-(\tau-\tau_0)/\gamma} d\tau \qquad \tau - \tau_0 = \tau_3 \; ; \; \tau_3 > 0$$

$$= \int_0^{\infty} a_1 e^{-\tau/\gamma} + a_2 e^{-\tau_3/\gamma} d\tau \qquad -\frac{1}{\gamma} = b.$$

$$= -a_1 \gamma \cdot e^{-\tau/\gamma} \Big|_0^{\infty} - a_2 \gamma \cdot e^{-\tau_3/\gamma} \Big|_0^{\infty}$$

$$= a_1 \gamma + a_2 \gamma$$

$$= (1 \cdot 10^{-4} + 2 \cdot 10^{-5}) \cdot 4 \cdot 10^{-9}$$

$$= 4.8 \times 10^{-13}$$

② mean delay

$$T_m = \left(\int_{-\infty}^{\infty} \tau \cdot P(\tau) \, d\tau \right) \Big/ P_m$$

$$= \left(\int_0^{\infty} \tau (\cdot a_1 e^{-\tau/\gamma} + a_2 e^{-\tau_3/\gamma}) \, d\tau \right) \Big/ P_m$$

$$= \left[\underbrace{\int_0^{\infty} \tau \cdot a_1 e^{-\tau/\gamma} d\tau}_{①} + \underbrace{\int_0^{\infty} \tau \cdot a_2 e^{-\tau_3/\gamma} d\tau_3}_{②} \right] \Big/ P_m.$$

$$① = a_1 \int_0^{\infty} \tau \cdot e^{-\tau/\gamma} d\tau$$

$$= a_1 \left[-\tau \cdot \gamma - \gamma^2 \right] e^{-\tau/\gamma} \Big|_0^{\infty}$$

$$= \underbrace{-a_1 \tau \cdot \gamma \cdot e^{-\tau/\gamma} \Big|_0^{\infty}}_{=0} - a_1 \gamma^2 e^{-\tau/\gamma} \Big|_0^{\infty}$$

$$= a_1 \gamma^2$$

Example. 3

① time - integrated power.

$$P_m = \int_{-\infty}^{\infty} a_1 e^{-\tau/\gamma} + a_2 e^{-(\tau-\tau_2)/\gamma} d\tau \qquad \tau - \tau_2 = \tau_3 \; ; \; \tau_3 > 0$$

$$= \int_0^{\infty} a_1 e^{-\tau/\gamma} + a_2 e^{-\tau_3/\gamma} d\tau \qquad -\frac{1}{\gamma} = b.$$

$$= -a_1 \gamma \cdot e^{-\tau/\gamma} \Big|_0^{\infty} - a_2 \gamma \cdot e^{-\tau_3/\gamma} \Big|_0^{\infty}$$

$$= a_1 \gamma + a_2 \gamma$$

$$= (1 \cdot 10^{-4} + 2 \cdot 10^{-5}) \cdot 4 \cdot 10^{-9}$$

$$= 4.8 \times 10^{-13}$$

② mean delay

$$T_m = \left(\int_{-\infty}^{\infty} \tau \cdot P(\tau)\, d\tau \right) \Big/ P_m$$

$$= \left(\int_0^{\infty} \tau (a_1 e^{-\tau/\gamma} + a_2 e^{-\tau_3/\gamma}) d\tau \right) \Big/ P_m$$

$$= \left[\underbrace{\int_0^{\infty} \tau \cdot a_1 e^{-\tau/\gamma} d\tau}_{①} + \underbrace{\int_0^{\infty} \tau \cdot a_2 e^{-\tau_3/\gamma} d\tau_3}_{②} \right] \Big/ P_m.$$

$$① = a_1 \int_0^{\infty} \tau \cdot e^{-\tau/\gamma} d\tau$$

$$= a_1 \left[-\tau \cdot \gamma - \gamma^2 \right] e^{-\tau/\gamma} \Big|_0^{\infty}$$

$$= \underbrace{-a_1 \tau \cdot \gamma \cdot e^{-\tau/\gamma} \Big|_0^{\infty}}_{=0} - a_1 \gamma^2 e^{-\tau/\gamma} \Big|_0^{\infty}$$

$$= a_1 \gamma^2$$

② . $a_2 \int_0^\infty \tau \cdot e^{-\tau_3/\gamma} d\tau_3$.

$$= a_2 \int_0^\infty (\tau_3 + \tau_2) \cdot e^{-\tau_3/\gamma} d\tau_3$$

$$= \underbrace{a_2 \int_0^\infty \tau_3 e^{-\tau_3/\gamma} d\tau_3}_{\text{same as ①}} + a_2 \tau_2 \int_0^\infty e^{-\tau_3/\gamma} d\tau_3$$

$$= a_2 \gamma^2 - a_2 \cdot \tau_2 \cdot \gamma \cdot e^{-\frac{\tau_3}{\gamma}} \Big|_0^\infty$$

$$= a_2 \gamma^2 + a_2 \tau_2 \cdot \gamma$$

So we have:

$$T_m = (a_1 \gamma^2 + a_2 \gamma^2 + a_2 \tau_2 \gamma) / P_m$$

$$= (a_1 \gamma + a_2 \gamma + a_2 \tau_2) / (a_1 + a_2)$$

$$= 6.6667 \times 10^{-9}$$

③ . rms delay spread

$$S_\tau = \sqrt{\frac{\int_{-\infty}^\infty P(\tau) \tau^2 d\tau}{P_m} - T_m^2}$$

Deal with $\int_{-\infty}^\infty P(\tau) \tau^2 d\tau$ first,

$$\int_{-\infty}^\infty (a_1 e^{-\tau/\gamma} + a_2 e^{-\tau_3/\gamma}) \cdot \tau^2 d\tau$$

$$= \underbrace{\int_0^\infty a_1 e^{-\tau/\gamma} \cdot \tau^2 d\tau}_{①} + \underbrace{\int_0^\infty a_2 \cdot \tau^2 \cdot e^{-\tau_3/\gamma} d\tau}_{②}$$

① : $\int_0^{\infty} a_1 \cdot \tau^2 e^{-\tau/r} d\tau$

$= a_1 \cdot (-\tau^2 \cdot r - 2\tau \cdot r^2 - 2r^3) e^{-\tau/r} \Big|_0^{\infty}$

$= \underbrace{-a_1(\tau^2 \cdot r + 2\tau \cdot r^2) e^{-\tau/r} \Big|_0^{\infty}}_{0} - 2a_1 \cdot r^3 \cdot e^{-\tau/r} \Big|_0^{\infty}$

$= 2a_1 r^3$

② $\int_0^{\infty} a_2 \tau^2 e^{-\tau_3/r} d\tau_3$

$= a_2 \int_0^{\infty} (\tau_3 + \tau_2)^2 e^{-\tau_3/r} d\tau_3$

$= a_2 \int_0^{\infty} (\tau_3^2 + \tau_2^2 + 2\tau_2\tau_3) e^{-\tau_3/r} d\tau_3$

$= a_2 \int_0^{\infty} \tau_3^2 e^{-\tau_3/r} d\tau_3 + a_2 \int_0^{\infty} \tau_2^2 \cdot e^{-\tau_3/r} d\tau_3 + 2\tau_2 \cdot a_2 \int_0^{\infty} \tau_3 e^{-\tau_3/r} \cdot d\tau_3$

$= 2a_2 r^3 + a_2 \tau_2^2 \cdot r + 2\tau_2 \cdot a_2 r^2$

So $S_\tau = \sqrt{\dfrac{2(a_1+a_2)r^3 + a_2\tau_2^2 r + 2\tau_2 a_2 r^2}{(a_1+a_2) r} - T_m^2}$

$= 7.1802 \times 10^{-9}$