Crash course part 2

Frequency compensation

Agenda

- Frequency dependance
- Feedback amplifiers
- Frequency dependance of the Transistor
- Frequency Compensation
- Phantom Zero

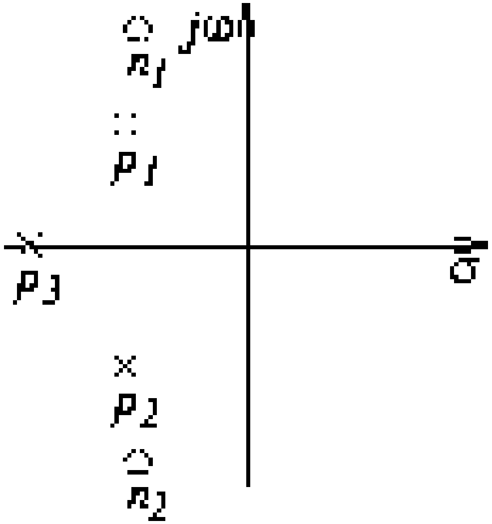
- Examples

poles and zeros

- In general a transfer function can be written as:

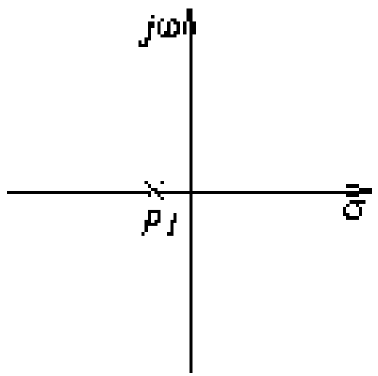
$$H(s) = k \frac{\left(1-\frac{s}{n_1}\right)\left(1-\frac{s}{n_2}\right)\cdots\left(1-\frac{s}{n_m}\right)}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\cdots\left(1-\frac{s}{p_n}\right)}$$

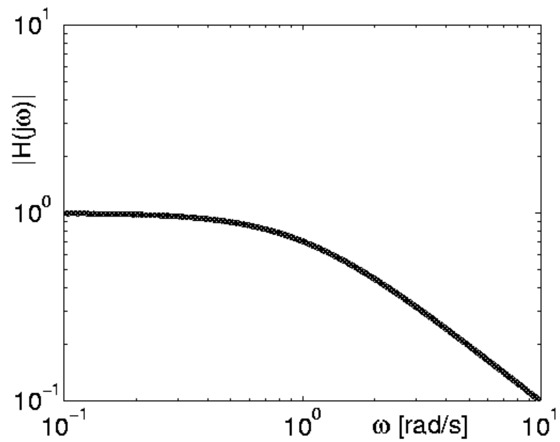
- $n_1, n_2, \ldots, n_m$ are zeros
- $p_1, p_2, \ldots, p_n$ are poles
- poles and zeros are shown in the s-plane
- poles are drawn as ×
- zeros as ○

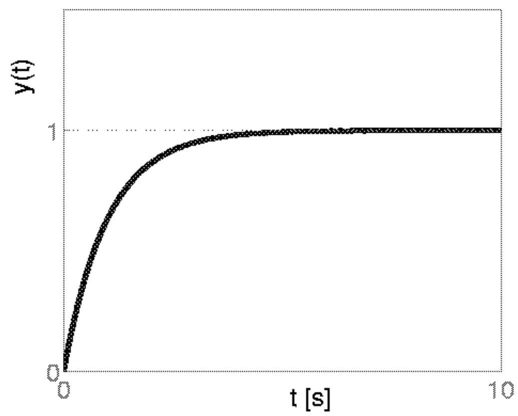


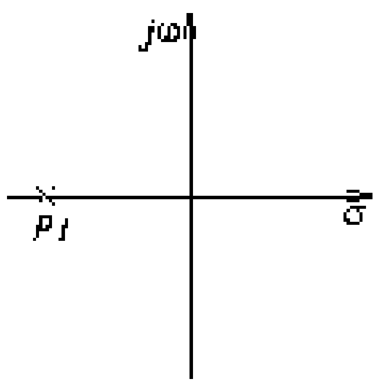
System with one pole

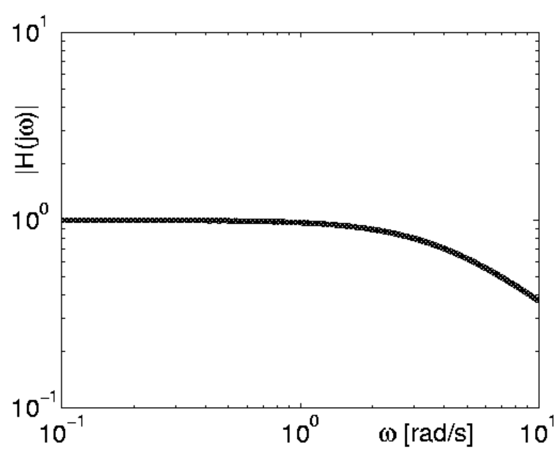
Pole placement Magnitude Step response

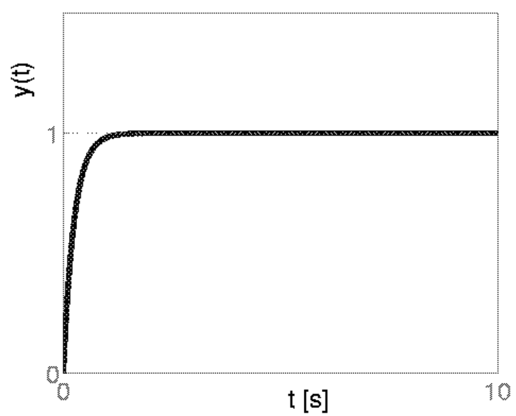






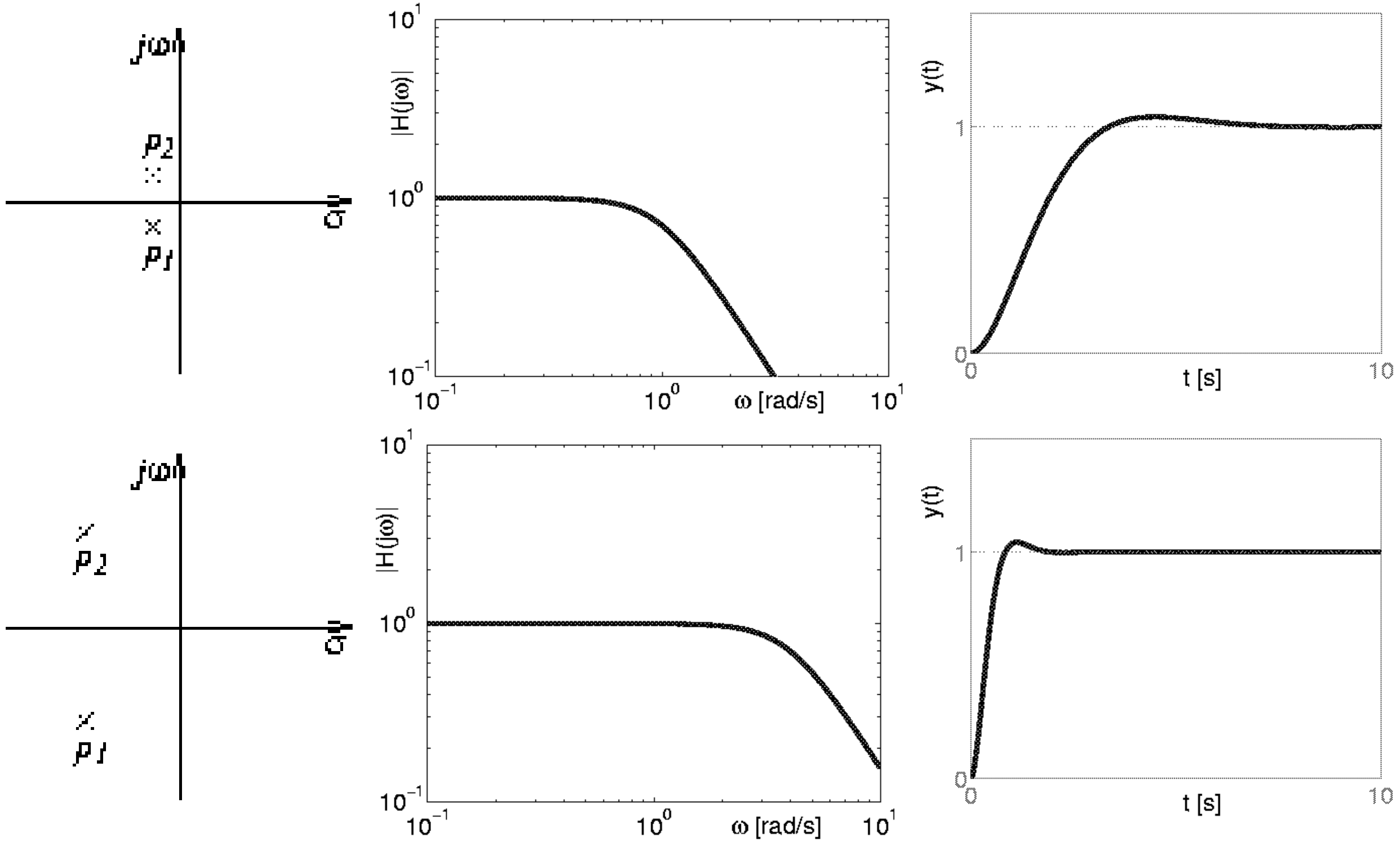






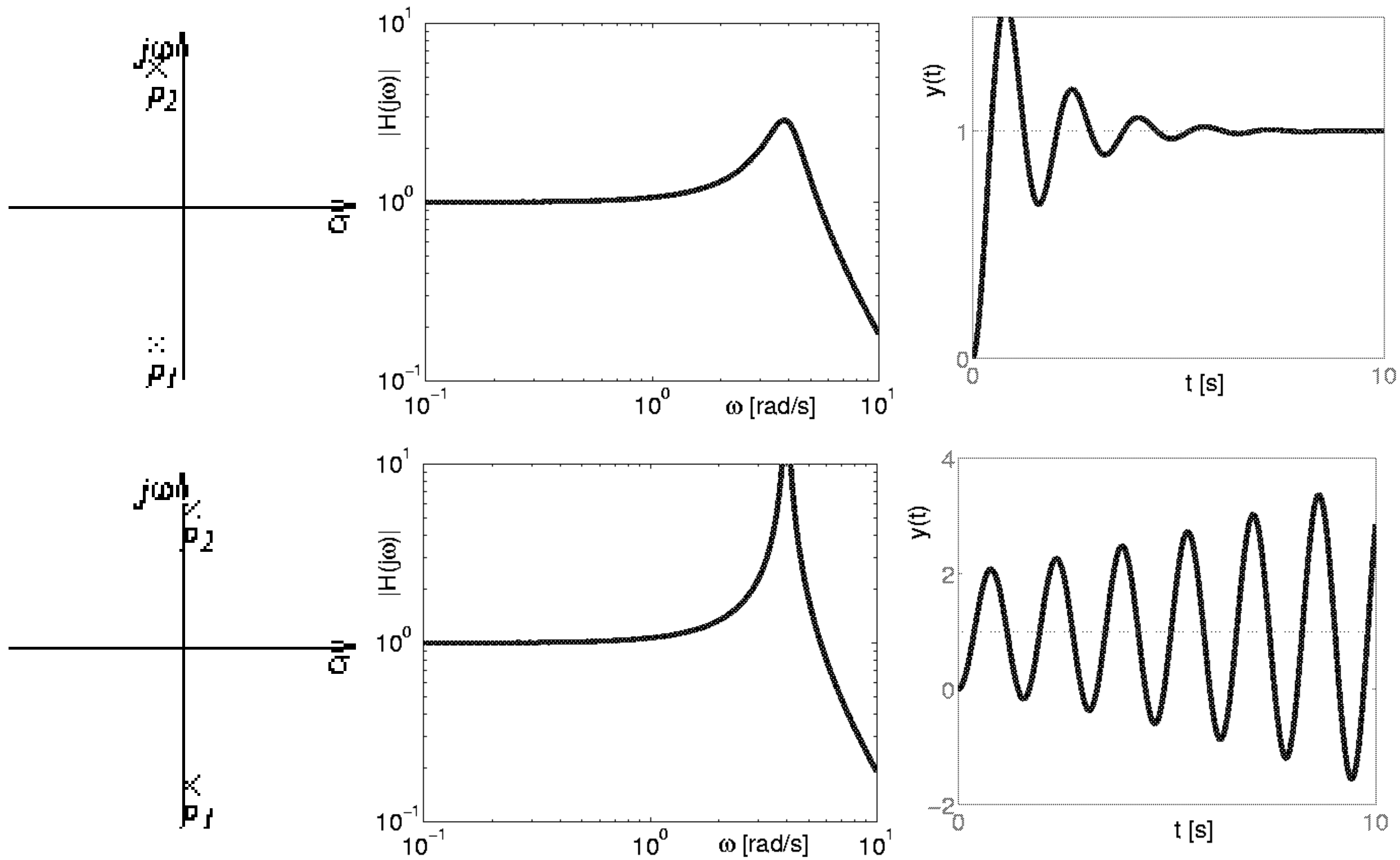
System with complex pole pair

Pole placement Magnitude Step response



Complex poles close to jω-axis

Pole placement Magnitude Step response



Discrepancy factor, frequency dependance

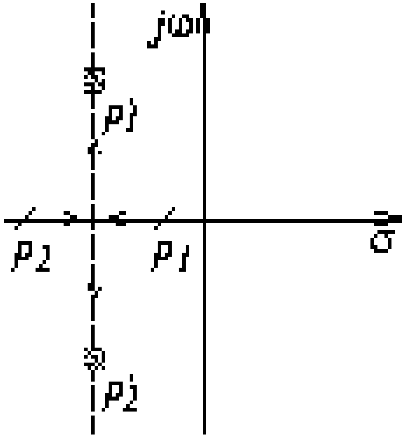
- Discrepancy factor can be written as:

$$\frac{-A\beta(s)}{1-A\beta(s)} = \frac{-A\beta(0)\dfrac{N(s)}{P(s)}}{1-A\beta(0)\dfrac{N(s)}{P(s)}} = \frac{-A\beta(0)N(s)}{P(s)-A\beta(0)N(s)}$$

- Interpretation of the above:
  - zeros in $A\beta(s)$ is zeros in A_t
  - Roots of equation $P(s) - A\beta(0)N(s) = 0$ are poles in A_t
  - The system poles depends on the loop gain $A\beta(0)$
- $P(s) - A\beta(0)N(s)$ is called the caracteristic polynome

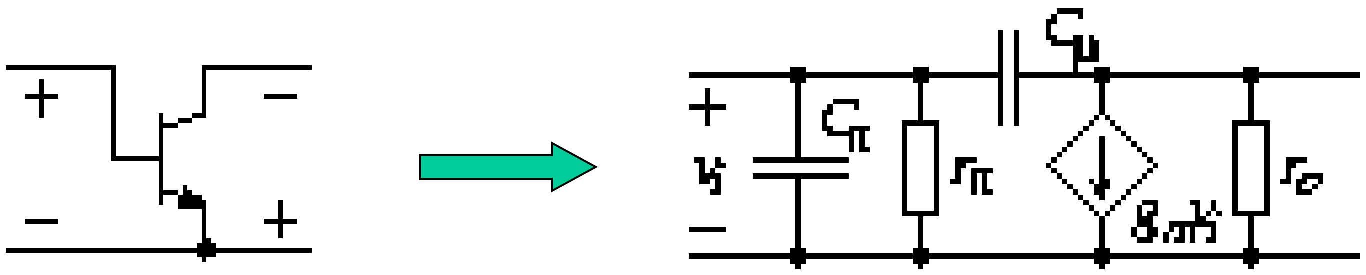
Root locus

- Root locus is a graphical representation of how the system poles moves with increasing $A\beta(0)$
  - Arrows in root locus shows increasing $A\beta(0)$
- Loop poles are $p_1, p_2, \ldots, p_n$
- System poles are $p'_1, p'_2, \ldots, p'_n$



High frequecy model for BJT

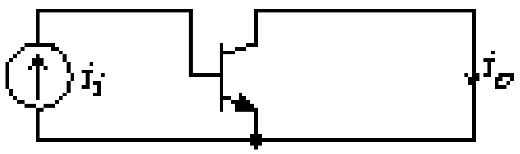
- Capacitances are included in the hybrid-π-model
- C_π capacitance between base and emitter
- C_μ capacitance between base and collector



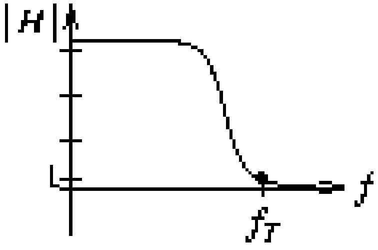
- C_μ is much less than C_π and is assumed to have no importance in our calculations.
- C_μ (and r_o) are neglected if not stated otherwise!

Transit frequency f_T

- At very high frequencies the CE-stage has no gain
- f_T is the frequency when $|H(s)|=|i_o/i_i|=1$ for a current driven CE-stage with a short circuit at the output



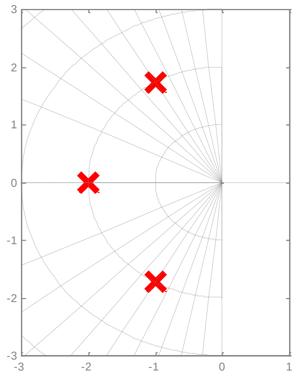
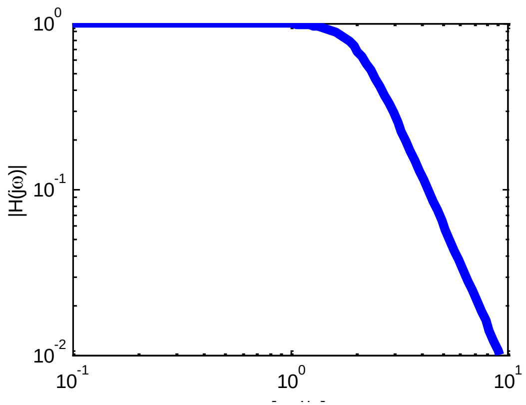
$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

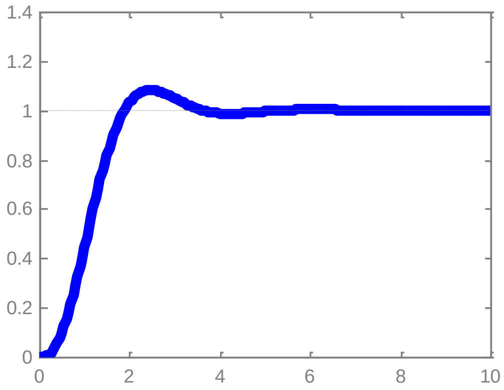


- f_T varies heavily between different types of transistors
- 1N26 f_T ~ 1kHz but today there are transistors having f_T > 50GHz

Maximal flat magnitude

- Often you want to have constant gain for all signal frequencies
- This is equal to having the system poles in Butterworth position
- They are located on a half circle equally spaced in left half plane



Loop-gain-Poles-product (LP-product)

- The characteristic polynominal for a system having two loop poles and no loop zeroes is:

$$P(s) - A\beta(0)N(s) = s^2 - (p_1 + p_2)s + [1 - A\beta(0)]p_1p_2$$

- The same system with poles in Butterworth position is:

$$P(s) - A\beta(0)N(s) = s^2 - (p_a + p_b)s + \omega_0^2$$

- Identifying:
- $\omega_0^2 = |[1 - A\beta(0)]p_1p_2|$
- ω_0 is the maximum **possible** bandwidth for the system.

LP-product definition

- The LP-product for a system with two loop poles:

$$LP_2 = \left| \left(1 - A\beta(0)\right) p_1 p_2 \right|$$

- In general the LP-product is:

$$LP = \left| \left(1 - A\beta(0)\right) \prod_{\forall n} p_n \right|$$

- With the poles in Butterworth position the band width is, ω_0:

$$\omega_0 = \sqrt[n]{LP} = \sqrt[n]{\left| \left(1 - A\beta(0)\right) \prod_{\forall n} p_n \right|}$$

Poles in Butterworth

- Poles in Butterworth is found from:

$$s^n + a_1 s^{n-1} + \cdots + a_n = 0 \text{ där } |a_n| = \omega_0^n$$

- Location for the poles are (2:nd and 3:rd order)

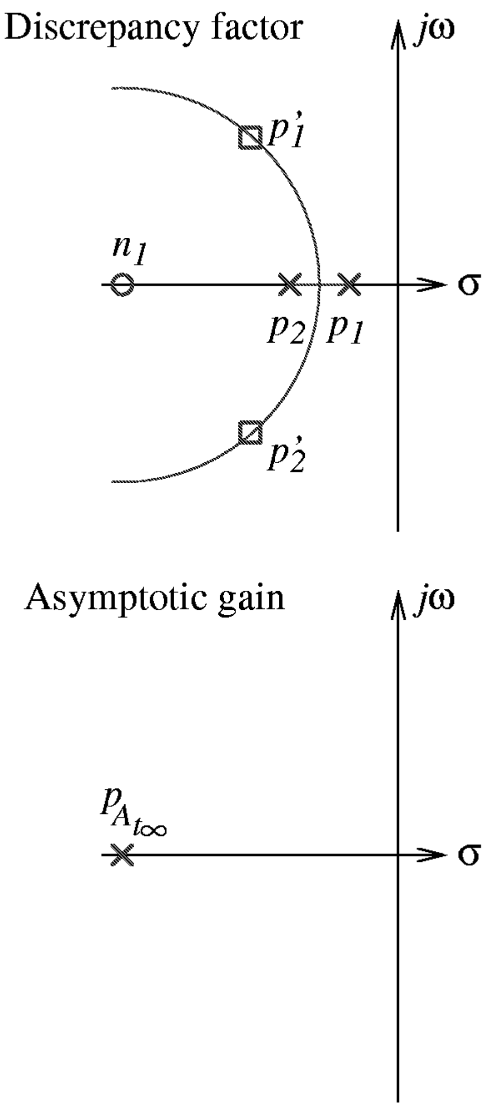
  - 2:nd order $-\omega_0 \frac{1}{\sqrt{2}}\left(1 \pm j\right)$
  - 3:rd order $-\omega_0, \quad -\omega_0 \frac{1}{2}\left(1 \pm j\sqrt{3}\right)$

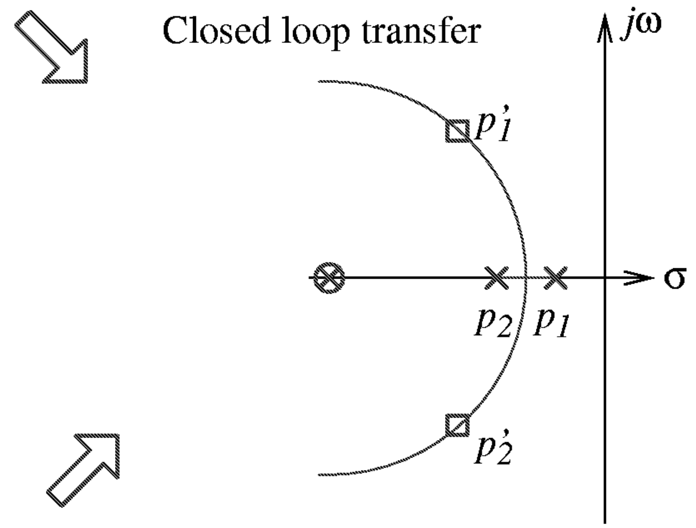
Flow chart for band width estimation

- Caculate the LP-product
- Estimate the band width
- Choose the location of the system poles (Butterworth) for estimated band width
- Calculate the sum of the loop poles and the sum of the system poles
- Check that the loop poles are dominant
- If a non-dominant pole exists remove the most negative one and re-calculate
- Finished

Phantom zero, grafical interpretation

$A_{t\infty}$ is made frequency independant





Where to put the phantom zero (2:nd order)

- Identification of s-terms gives
  - $p_1 + p_2 + \frac{\omega_0^2}{n} = p_1' + p_2' \quad , |A\beta(0)| >> 1$
- Place the zero at $$n = \frac{-\omega_0^2}{-(p_1' + p_2') + p_1 + p_2}$$
- System poles sum

Characteristics	$p'_1 + p'_2$
Butterworth	$-\sqrt{2}\omega_0$
Bessel	$-1.73\omega_0$
Real double polee	$-2\omega_0$

← MFM

Implementation of phantom zero

- β should increase for higher frequencies

$$\beta(s) = \beta(0)\left(1 - \frac{s}{n}\right)$$

- Three places to put a phantom zero in β
  - In the feedback net
  - At the amplifiers output
  - At the amplifiers input