

LUNDS TEKNISKA HÖGSKOLA
Inst. for Elektro- och Informationsteknik

Tentamen 2017-06-01
SIGNALBEHANDLING I MULTIMEDIA, ETI265
Tid: 08.00-13.00
Sal: Vic 2 - Hela

Hjälpmedel: Miniräknare, formelsamling i signalbehandling och en valfri bok i matematik.
[Allowed items on exam: calculator, DSP and mathematical tables of formulas]

Observandum: För att underlätta rättningen: [In order to simplify the correction:]
-Lös endast **en** uppgift per blad. [Only solve one problem per paper sheet.]
-Skriv kod+personlig identifierare på **samtliga** blad.
[Please write your code+personal identifier on every paper sheet.]
Påståenden måste motiveras via resonemang och/eller ekvationer.
[Statements must be motivated by reasoning and/or equations.]
Poäng från inlämningsuppgifterna adderas till tentamensresultatet.
[The points from the tasks will be added to the examination score.]
Max Tot. poäng (tentamen + båda inl.uppg) = 5.0 + 0.5 + 0.5 = 6.0
[Max Tot. score (exam + 2 tasks) = 5.0 + 0.5 + 0.5 = 6.0]
Betygsgränser för kursen: 3 ($\geq 3.0p$), 4 ($\geq 4.0p$), 5 ($\geq 5.0p$).
[Grading; 3 ($\geq 3.0p$), 4 ($\geq 4.0p$), 5 ($\geq 5.0p$).]

1. Följande tids-diskreta signaler är givna;
[The following discrete time signals are given]

$$x_1(n) = [-2 \underset{\uparrow}{1} \ 0 \ -1 \ -2], \quad x_2(n) = [-\underset{\uparrow}{1} \ 2 \ -2 \ 1 \ 1]$$

Bestäm följande;
[Determine the following;]

- a) Den linjära faltningen av sekvenserna, dvs $y(n) = x_1(n) * x_2(n)$. (0.1p)
[The linear convolution between the sequences, i.e. $y(n) = x_1(n) * x_2(n)$.]
- b) Den cirkulära faltningen modulo 4 av sekvenserna, dvs $y(n) = x_1(n) \circledast_4 x_2(n)$. (0.1p)
[The circular convolution modulus 4 between the sequences, i.e. $y(n) = x_1(n) \circledast_4 x_2(n)$.]
- c) Den linjära korrelationen av sekvenserna, dvs $r_{x_1x_2}(n) = x_1(n) * x_2(-n)$ (0.1p)
[The linear correlation between the sequences, i.e. $y(n) = x_1(n) * x_2(-n)$.]
- d) Den cirkulära korrelationen av sekvenserna modulo 5, dvs
 $r_{x_1x_2}(n) = x_1(n) \circledast_5 x_2(-n)$ (0.2p)
[The circular correlation modulus 5 between the sequences, i.e. $r_{x_1x_2}(n) = x_1(n) \circledast_5 x_2(-n)$.]

2. Signaler samplas, sampelomvandlas och rekonstrueras enligt deluppgifter nedan. Bestäm vilka signaler som erhålls.

[*Signals are sampled, decimated or interpolated, and reconstructed as given below. Determine the resulting signals.*]

- a) Signalen $\cos(2\pi 250t)$ samplas med $F_s = 1000$ Hz, nedsamplas (dvs decimeras) med en faktor 2, samt rekonstrueras idealt (med $F_s = 1000$ Hz). (0.2p)

[*The signal $\cos(2\pi 250t)$ is sampled using $F_s = 1000$ Hz, downsampled (i.e. decimated) by the factor 2, and reconstructed ideally (using $F_s = 1000$ Hz).*]

- b) Signalen $\cos(2\pi 14100t)$ samplas med $F_s = 400$ Hz, uppsamplas (dvs interpoleras) med en faktor 3, samt rekonstrueras idealt med en ny samplefrekvens, $F_s = 500$ Hz. (0.3p)

[*The signal $\cos(2\pi 14100t)$ is sampled using $F_s = 400$ Hz, up-sampled (i.e. interpolated) by the factor 3, and then reconstructed ideally using a new sample frequency, $F_s = 500$ Hz.*]

3. På nästa sida visas 6 st frekvensresponser samt 6 st pol-/nollställe-diagram. Matcha de olika figurerna till respektive LTI-system (S1-S6) givet nedan. Det ingår i uppgiften att avgöra vilka storheter vi har på axlarna. Motivera ditt svar! (1.0p)

[*On next page there are given 6 magnitude responses and 6 pole/zero diagrams. Combine the diagrams with the corresponding LTI-systems (S1-S6) provided below. The task includes to decide the x- and y-axis variables. Motivate your answer.*]

S1: $y(n) = 0.77y(n-1) + x(n) + x(n-1)$

S2: $H(z) = \frac{1-z^{-1}}{1+0.77z^{-1}}$

S3: $H(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5}$

S4: $y(n) = \sum_{k=0}^7 x(n-k)$

S5: $H(z) = 3 - 3z^{-1}$

S6: $y(n) = x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4) + x(n-5)$

4. Fibonacci-serien för varje heltalsindex, n, ges av summan av de två föregående talen, enligt,

[*The Fibonacci series, for each integer number, n, is given by the sum of the two previous numbers, according to,*]

$$y(n) = \{ \underset{\uparrow}{1}, 1, 2, 3, 5, 8, 13, \dots \}, \quad n = 0, 1, 2, \dots$$

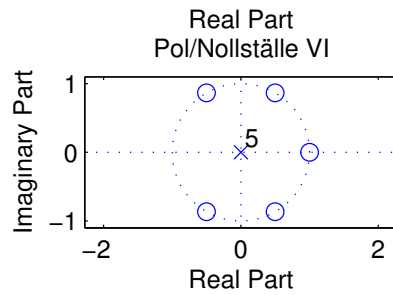
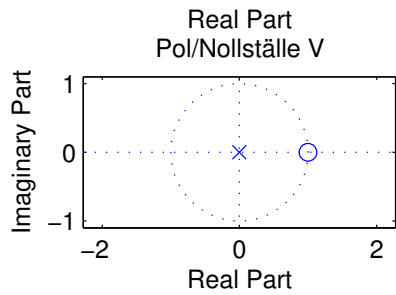
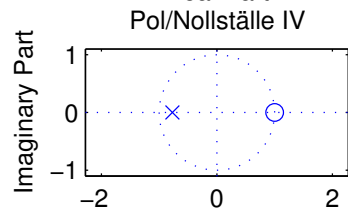
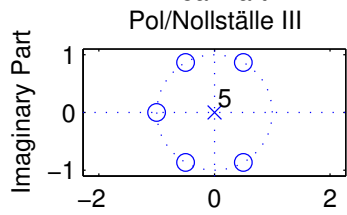
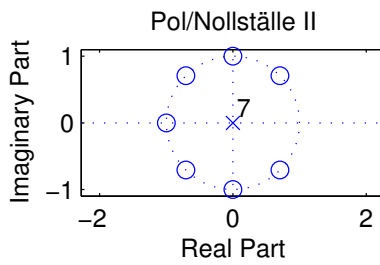
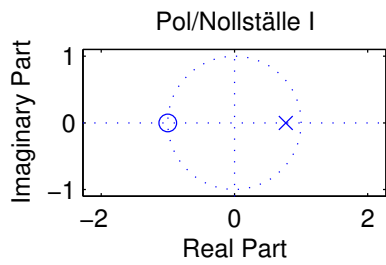
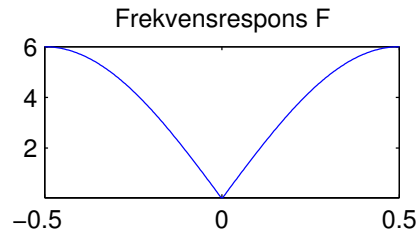
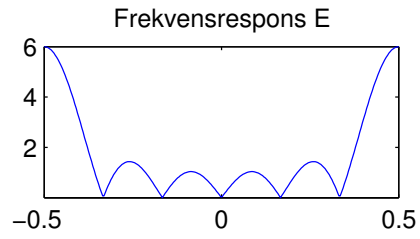
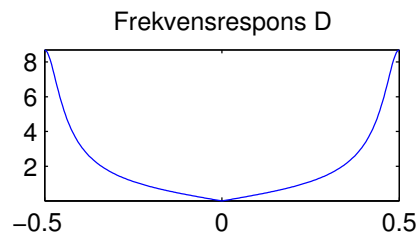
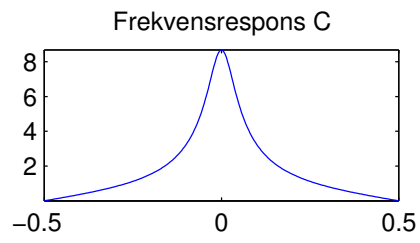
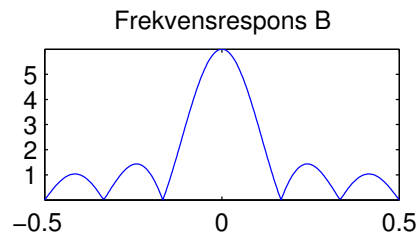
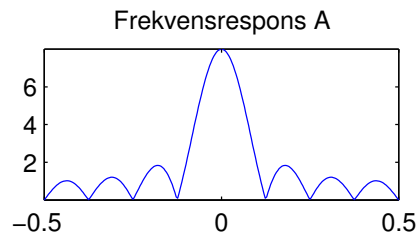
Detta ger differens-ekvationen,

[*This leads to the following difference equation,*]

$$y(n) = y(n-1) + y(n-2), \quad n \geq 2$$

med begynnelsevärden, $y(0) = 1, y(1) = 1$. Bestäm ett slutet uttryck för Fibonacci-serien, dvs lös ovanstående differens-ekvation (1.0p)

[*with initial conditions, $y(0) = 1, y(1) = 1$. Determine a closed form solution for the Fibonacci series, i.e. solve the above difference equation.*]



5. Ett LTI-system är beskrivet av nedanstående differensekvation,
[An LTI system is given by the following difference equation,]

$$y(n) = 0.5y(n-1) + bx(n)$$

- a) Bestäm parametern b så att $|H(\omega)| = 1$ vid vinkelfrekvensen $\omega = 0$. (0.3)
[Determine the parameter b such that $|H(\omega)| = 1$ at the angular frequency $\omega = 0$.]
- b) Bestäm "half-power point" (dvs vinkelfrekvensen, ω , för vilken $|H(\omega)|^2$ är lika med hälften av dess toppvärde). (0.7)
[Determine the "half-power point" (i.e. the angular frequency ω where $|H(\omega)|^2$ equals half its top value)]
6. Bestäm impulssvaret, $g(n)$, till ett LTI filter så att det uppfyller följande egenskap: Om insignalen är summan av impulssvaret och stegsvaret från ett LTI-system (vilket som helst) så skall utsignalen vara impulssvaret från samma system! Motivera ditt svar! (1.0)
[Determine the impulse response, $g(n)$, to an LTI filter such that it fulfills the following property: If the input signal is the sum of the impulse response and the step response from ANY linear and time invariant system, then the output signal should be the impulse response from the same system. Motivate your answer!]

Lycka Till!

Please remember to answer the Course-Evaluation-Questionnaire, CEQ!

SVAR OCH LÖSNINGAR Tentamen, ETI265, 2017-06-01

SVAR 1. a)

$$y(n) = x_1(n) * x_2(n) = [2 \quad -\underset{\uparrow}{5} \quad 6 \quad -3 \quad -1 \quad -1 \quad 3 \quad -3 \quad -2]$$

b)

$$y(n) = x_1(n) \otimes_4 x_2(n) = [1 \quad -\underset{\uparrow}{3} \quad 5 \quad -3] = [-\underset{\uparrow}{3} \quad 5 \quad -3 \quad 1]$$

c)

$$r_{x_1x_2}(n) = x_1(n) * x_2(-n) = [-2 \quad -1 \quad 5 \quad -7 \quad 1 \quad -\underset{\uparrow}{1} \quad 2 \quad -3 \quad 2]$$

d)

$$y(n) = x_1(n) \otimes_5 x_2(-n) = [-\underset{\uparrow}{3} \quad 1 \quad 2 \quad -5 \quad 1]$$

SVAR 2a. Före sampling har vi frekvenserna ± 250 Hz. Efter sampling har vi de normaliserade frekvenserna

$$f = \pm \frac{250}{1000} \pm k = \pm \frac{1}{4} \pm k$$

varv/sampel. Efter decimering med faktor 2 har vi de normaliserade frekvenserna,

$$f = \pm \frac{1}{4} * 2 \pm k = \pm \frac{1}{2} \pm k$$

Efter ideal rekonstruktion har vi frekvenserna $F = \pm f * F_s = \pm 500$ Hz, dvs $y(t) = \cos(2\pi 500t)$.

SVAR 2b. Före sampling har vi frekvenserna ± 14100 Hz. Efter sampling har vi de normaliserade frekvenserna

$$f = \pm \frac{14100}{400} \pm k = \pm (35 + \frac{1}{4}) \pm k = \pm \frac{1}{4} \pm k$$

varv/sampel. Efter interpolering med faktor 3 har vi de normaliserade frekvenserna,

$$f = \frac{\pm \frac{1}{4} \pm k}{3} = \pm \frac{1}{12}, \pm \frac{3}{12}, \pm \frac{5}{12} \pm k$$

Efter ideal rekonstruktion har vi frekvenserna $F = \pm f * F_s = \pm f * 500 = \pm \frac{125}{3}, 125, \frac{625}{3}$ Hz, dvs

$$y(t) = \cos(2\pi \frac{125}{3}t) + \cos(2\pi 125t) + \cos(2\pi \frac{625}{3}t)$$

SVAR 3. A-S4-II
 B-S6-III
 C-S1-I
 D-S2-IV
 E-S3-VI
 F-S5-V

SVAR 4. The Fibonacci series is given by

$$y(n) = y(n - 1) + y(n - 2), \quad n \geq 2$$

with initial conditions, $y(0) = 1, y(1) = 1$.

Use the single sided Z-transform to transform the difference equation: \Rightarrow

$$\begin{aligned} Y^+(z) &= z^{-1}Y^+(z) + y(-1) \cdot z^0 + z^{-2} \cdot Y^+(z) + y(-1)z^{-1} + y(-2) \cdot z^0 = \\ &= z^{-1}Y^+(z) + y(-1) + z^{-2} \cdot Y^+(z) + y(-1)z^{-1} + y(-2) \end{aligned}$$

Bring all $Y^+(z)$ terms to the left \Rightarrow

$$Y^+(z) = \frac{y(-1) + y(-2) + y(-1)z^{-1}}{1 - z^{-1} - z^{-2}}$$

We calculate backwards in the difference equation to get the values of $y(-1)$ and $y(-2)$, i.e.

$$y(n - 2) = y(n) - y(n - 1)$$

$$y(-1) = y(1) - y(0) = 0 \quad (n = 1)$$

$$y(-2) = y(0) - y(-1) = 1 \quad (n = 0)$$

\Rightarrow

$$Y^+(z) = \frac{1}{1 - z^{-1} - z^{-2}} = \frac{z^2}{z^2 - z - 1}$$

We need to find the inverse Z-transform in order to get a closed form expression of $y(n)$. We use partial fraction expansion.

The roots to the polynomial $z^2 - z - 1 = 0$ are given by

$$p_1 = \frac{1 + \sqrt{5}}{2} \quad p_2 = \frac{1 - \sqrt{5}}{2}$$

\Rightarrow

$$Y^+(z) = \frac{A_1}{1 - \frac{1+\sqrt{5}}{2}z^{-1}} + \frac{A_2}{1 - \frac{1-\sqrt{5}}{2}z^{-1}}$$

where

$$A_1 = \left(1 - \frac{1 + \sqrt{5}}{2}z^{-1}\right) Y^+(z) \Big|_{z=\frac{1+\sqrt{5}}{2}} = \frac{1}{1 - \frac{1-\sqrt{5}}{1+\sqrt{5}}} = \frac{1 + \sqrt{5}}{2\sqrt{5}}$$

$$A_2 = \left(1 - \frac{1 - \sqrt{5}}{2}z^{-1}\right) Y^+(z) \Big|_{z=\frac{1-\sqrt{5}}{2}} = \frac{1}{1 - \frac{1+\sqrt{5}}{1-\sqrt{5}}} = -\frac{1 - \sqrt{5}}{2\sqrt{5}}$$

This gives the answer

$$y(n) = \underline{\underline{\left(\frac{1 + \sqrt{5}}{2\sqrt{5}} \cdot \left(\frac{1 + \sqrt{5}}{2} \right)^n - \frac{1 - \sqrt{5}}{2\sqrt{5}} \cdot \left(\frac{1 - \sqrt{5}}{2} \right)^n \right) u(n)}}$$

SVAR 5. Givet ett LTI-system

$$y(n) = 0.5y(n - 1) + bx(n)$$

a) Bestäm b så att $|H(\omega)| = 1$ vid frekvensen $\omega = 0$.

svar: Z-transformera differensekvationen ger,

$$\begin{aligned} Y(z) &= 0.5z^{-1}Y(z) + bX(z) \\ Y(z)(1 - 0.5z^{-1}) &= bX(z) \\ Y(z) &= \frac{b}{(1 - 0.5z^{-1})}X(z) \\ \Rightarrow H(z) &= \frac{b}{(1 - 0.5z^{-1})} \end{aligned}$$

Ur $H(z)$ fås Fouriertransformen genom

$$\begin{aligned} H(\omega) &= H(z)|_{z=e^{j\omega}} = \frac{b}{(1 - 0.5e^{-j\omega})} \quad (1) \\ \Rightarrow |H(0)| &= \left| \frac{b}{(1 - 0.5)} \right| \equiv 1 \Rightarrow \underline{\underline{b = \pm \frac{1}{2}}} \end{aligned}$$

b) Bestäm ω för vilken $|H(\omega)|^2$ är lika med hälften av dess toppvärd.

svar: Toppvärdet fås då Ekv (1) har sitt största värde, dvs när nämnaren har sitt minsta värde. Detta sker då nämnaren blir 0.5 och toppvärdet blir 1 (anv. $b = 0.5$), dvs

$$\begin{aligned} |H(\omega)|^2 &= \left(\frac{1}{2} \frac{1}{\sqrt{\underbrace{(1 - 0.5\cos(\omega))^2}_{real} + \underbrace{(0.5\sin(\omega))^2}_{imag}}} \right)^2 \equiv \frac{1}{2} \\ \Rightarrow \frac{1}{4(1.25 - \cos(\omega))} &= \frac{1}{2} \end{aligned}$$

Multiplitera båda sidor med 4, samt invertera båda sidor,

$$\begin{aligned} 1.25 - \cos(\omega) &= \frac{1}{2} \\ \Rightarrow \cos(\omega) &= 1.25 - 0.5 \\ \Rightarrow \omega &= \underline{\underline{\text{acos}(1.25 - 0.5) \approx 0.7227}} \end{aligned}$$

SVAR 6. We have the following scenario;

$$\delta(n)+u(n) \rightarrow \{\text{ANY system}\} \rightarrow h(n)+s(n) \rightarrow \{\text{syst. } g(n)\} \rightarrow y(n) = [h(n) + s(n)]*g(n)$$

where $s(n)$ is the step response. In the Z-domain this becomes,

$$1+\frac{1}{1-z^{-1}} \rightarrow \{\text{ANY system}\} \rightarrow H(z)+S(z) \rightarrow \{\text{syst. } G(z)\} \rightarrow Y(z) = [H(z) + S(z)] G(z)$$

We need to chose $G(z)$ such that the output $Y(z) = H(z)$, and since $S(z) = H(z)\frac{1}{1-z^{-1}}$, we have;

$$Y(z) = \left[H(z) + H(z)\frac{1}{1-z^{-1}} \right] G(z) = H(z) \left[1 + \frac{1}{1-z^{-1}} \right] G(z) = H(z) \left[\frac{2-z^{-1}}{1-z^{-1}} \right] G(z)$$

So if we choose $G(z)$ as the inverse of the term within the right hand brackets as

$$G(z) = \frac{1-z^{-1}}{2-z^{-1}} = \frac{0.5-0.5z^{-1}}{1-0.5z^{-1}}$$

we get the desired output. The inverse Z-transform of $G(z)$ is,

$$g(n) = 0.5\left(\frac{1}{2}\right)^n u(n) - 0.5\left(\frac{1}{2}\right)^{n-1} u(n-1) = \underline{\underline{0.5\delta(n) - \left(\frac{1}{2}\right)^{n+1} u(n-1)}}$$