Solutions to

Written Exam in

Integrated Radio Electronics (ETI 170)

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1 a, b, c

$$Q = \frac{\omega L}{R_s} \quad , \quad f_s = \frac{1}{2\pi\sqrt{L C_p/2}} \quad , \quad R_p = R_s (Q^2+1)$$

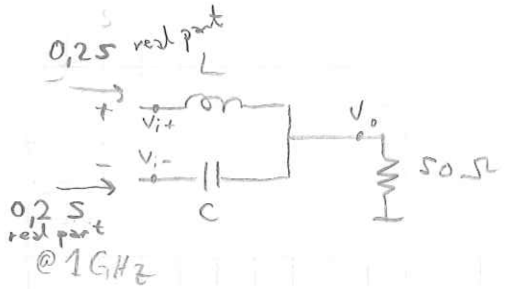
	Q	f_s	R_p
A	10,5	16 GHz	165 Ω
B	12,6	13,5 GHz	396 Ω
C	9,0	11,25 GHz	564 Ω

d. For large tuning range inductor A is best, since it has smallest L and highest f_s.

e For low power inductor C is best, since it has highest R_p

f For low phase noise without considering power, inductor A is best as it has the highest Q/L ratio.

2.



a) $(V_{i+} - V_o)\frac{1}{j\omega L} + (V_{i-} - V_o)\, j\omega C - V_o \cdot \frac{1}{R} = 0 \Rightarrow$

$V_{i+} \cdot \frac{1}{j\omega L} + V_{i-} \cdot j\omega C = V_o\left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right) \Rightarrow$

$V_{i+} \cdot \frac{1}{\omega L} - V_{i-} \cdot \omega C = jV_o\left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)$, vid resonans: $\omega C = \frac{1}{\omega L} \Rightarrow$

$\omega C(V_{i+} - V_{i-}) = jV_o \cdot \frac{1}{R} \Rightarrow$

$V_o = j\omega RC(V_{i-} - V_{i+})$, $V_{i+} = -V_{i-} \Rightarrow V_o = 2j\omega RC\, V_{i-} = -2j\omega RC\, V_{i+}$

$i_+ = (V_{i+} - V_o)\frac{1}{j\omega L} = V_{i+}(1 + 2j\omega RC) \cdot (-j\omega C) \Rightarrow$

$Y_+ = \frac{i_+}{V_{i+}} = -j\omega C + 2\omega^2 RC^2 = \frac{1}{j\omega L} + 2R\frac{C}{L}$

$i_- = (V_{i-} - V_o) \cdot j\omega C = V_{i-}(1 - 2j\omega RC) \cdot j\omega C \Rightarrow$

$Y_- = \frac{i_-}{V_{i-}} = j\omega C + 2\omega^2 RC^2 = j\omega C + 2R\frac{C}{L}$

$2 \cdot 50 \cdot \frac{C}{L} = \frac{1}{5} \Rightarrow \frac{C}{L} = \frac{1}{500}$

$\frac{1}{\sqrt{LC}} = 2\pi \cdot 1\,GHz \Rightarrow LC = 2{,}53 \cdot 10^{-20}$

$LC \cdot \frac{C}{L} = C^2 = \frac{1}{500} \cdot 2{,}53 \cdot 10^{-20} \Rightarrow$

$\underline{C = 7\ pF}$, $\underline{L = 3{,}6\ nH}$

b) $Y_+ = -j\,0{,}044 + 0{,}2\ S$

short capacitor:

$Y_s = j\omega C = j\,0{,}044 \Rightarrow$

$C = \frac{0{,}044}{\omega} = 7\ pF$

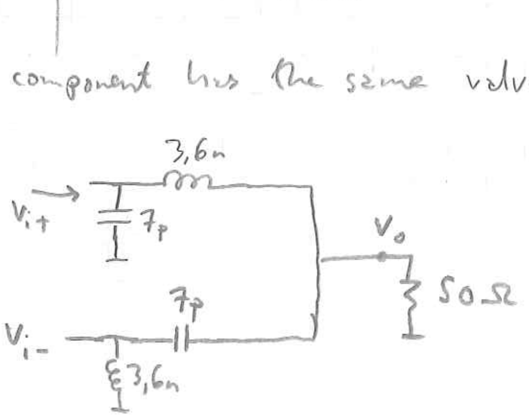
$Y_- = +j\,0{,}044 + 0{,}2\ S$

shunt inductor:

$Y_s = -\frac{j}{\omega L} = -j\,0{,}044 \Rightarrow$

$L = \frac{1}{\omega \cdot 0{,}044} = 3{,}6\ nH$

That is the shunt component has the same values as the series ones.



3. In active region. ($V_{GS} > V_T$, $V_{DS} > V_{GS} - V_T$)

$$C_{gs} = \frac{2}{3} C_{ox} \cdot WL = \frac{2}{3} \cdot 14 \cdot 100 \cdot 0{,}2 \text{ fF} = 187 \text{ fF}$$

$$C_{gd} = W \cdot C_{ol}/w = 100 \cdot 0{,}07 \text{ fF} = 7 \text{ fF}$$

$$r_g = \frac{1}{3} \cdot R_{sp} \cdot \frac{W}{L} \cdot \frac{1}{(n_{fingers})^2} = \frac{1}{3} \cdot 7{,}5 \cdot \frac{100}{0{,}2} \cdot \frac{1}{20^2} = 3{,}1\ \Omega$$

$$I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} \left(V_{GS} - V_t\right)^2 = \frac{1}{2} \cdot 400\mu \cdot \frac{100}{0{,}2} \cdot (0{,}5 - 0{,}3)^2 = 4{,}0 \text{ mA}$$

$$g_m = \frac{2I_D}{V_{ov}} = \frac{8{,}0 \text{ mA}}{0{,}2 \text{ V}} = 40 \text{ mS}$$

$$f_T = \frac{g_m}{C_{gs}} \cdot \frac{1}{2\pi} = \frac{40\text{m}}{187 \cdot 10^{-15} \cdot 2\pi} = 34 \text{ GHz}$$

$$f_{max} = \frac{1}{2\pi} \cdot \frac{1}{2} \sqrt{\frac{2\pi f_T}{r_g C_{gd}}} = 250 \text{ GHz}$$

$$g_{ds} = \frac{I_D}{L} \cdot \left| \frac{dx_{d,m}}{dV_{DS}} \right| = \frac{4 \text{ mA}}{0{,}2\mu\text{m}} \cdot 0{,}08\ \mu\text{m/V} = 1{,}6 \text{ mS}$$

$$A_v = \frac{g_m}{g_{ds}} = \frac{40}{1{,}6} = 25$$

$$C_{db0} = \frac{W}{2} \cdot D_{g-g} \cdot 1{,}0 \text{ fF} = 50 \cdot 0{,}4 \cdot 1{,}0 \text{ fF} = 20 \text{ fF}$$

$$\overline{i_{nd}^2} = 4kT\gamma g_m \Delta f = 1{,}67 \cdot 10^{-20} \cdot 1{,}5 \cdot 40 \cdot 10^{-3} \cdot 1 \cdot 10^6 \text{ A}^2 = 1{,}0 \cdot 10^{-15} \text{ A}^2$$

$$g_g = \frac{\omega^2 C_{gs}^2}{5 g_{d0}} = \frac{(2\pi \cdot 3 \cdot 10^9 \cdot 187 \cdot 10^{-15})^2}{5 \cdot 40 \cdot 10^{-3}} = 6{,}2 \cdot 10^{-5}$$

$$\overline{i_{ng}^2} = 4kT\delta g_g \Delta f = 1{,}67 \cdot 10^{-20} \cdot 3 \cdot 6{,}2 \cdot 10^{-5} \cdot 1 \cdot 10^6 \text{ A}^2 = 3{,}1 \cdot 10^{-18} \text{ A}^2$$

4.

a) $d_{avg} = 80\,\mu m \qquad d_{out} = 92\,\mu m \qquad n = 2$

$$L \approx 9{,}375 \cdot \mu_0 \cdot n^2 \frac{d_{avg}^2}{11\, d_{out} - 7\, d_{avg}} = 0{,}67\ nH$$

$$Length = n \cdot 4 \cdot d_{avg} = 640\ \mu m$$

$$\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} = \sqrt{\frac{2}{2\pi \cdot 10 \cdot 10^9 \cdot 4\pi \cdot 10^{-7} \cdot 59{,}6 \cdot 10^6}} = 0{,}65\ \mu m$$

$$R_s \approx \frac{Length}{W \cdot \sigma \cdot \delta (1 - e^{-t/\delta})} = \frac{640\mu}{8 \cdot 10^{-6} \cdot 59{,}6 \cdot 10^6 \cdot 0{,}65\mu \cdot (1 - e^{-2/0{,}65})} = 2{,}16\,\Omega$$

$$Q = \frac{\omega L}{R_s} = \frac{10 \cdot 10^9 \cdot 2\pi \cdot 0{,}67 \cdot 10^{-9}}{2{,}16} = 19{,}5$$

$$C_{par} = \frac{Length \cdot W}{D} \cdot \varepsilon_0 \varepsilon_r = \frac{640\mu \cdot 8\mu}{3{,}5\mu} \cdot 8{,}85 \cdot 10^{-12} \cdot 4 = 52\ fF$$

$$f_s \approx \frac{1}{2\pi \sqrt{L C_{par}/2}} = 38\ GHz$$

b) $$2 \cdot R_1 = \left(\frac{300\mu \cdot 100\mu}{100\mu}\right)^{-1} \cdot 5\,\Omega cm = 166\,\Omega \Rightarrow R_1 = 83\,\Omega$$

$$C_{par,noshield} = C_{par} \cdot \frac{3{,}5}{4} = 46\ fF$$

$$f_{s,noshield} \approx \frac{1}{2\pi \sqrt{L C_{par,noshield}/2}} = 40{,}5\ GHz$$

Transform both resistors to parallel:

$$Q_1 = 1/\omega R_1 C_{par,noshield}/2 = 1/2\pi \cdot 10 \cdot 10^9 \cdot 83 \cdot 23 \cdot 10^{-15} = 8{,}3$$

$$R_{p1} = R_1 \cdot (Q_1^2 + 1) = 83 \cdot (8{,}3^2 + 1) = 5{,}8\ k\Omega$$

$$R_{p2} = R_s \cdot (Q_{shield}^2 + 1) = 2{,}16 \cdot (19{,}5^2 + 1) = 823\,\Omega$$

$$R_p = R_{p1} \,//\, R_{p2} = 720\,\Omega$$

$$Q_{noshield} = \frac{R_p}{\omega L} = \frac{720}{2\pi\, 10\, 10^9 \cdot 0{,}67 \cdot 10^{-9}} = 17{,}1$$

<u>Result</u>: Q reduced from 19,5 to 17,1 at 10 GHz, and f_s increased from 38 GHz to 40,5 by removing shield

5. First neglect inductor losses & r_g & find parameters so that F is low:

Max current = 10 mA ⇒ 5mA / side

ICP > -13 dBm ⇒ -16 dBm / side   50Ω ⇒ 50 mV

Choose Q = 2 & $V_{ov} = 0{,}25V$ ($V_{ov} > 2Q \cdot V_{in}$ OK)

$$g_m = \frac{2I_D}{V_{ov}} = \frac{10\,mA}{0{,}25} = 40\,mS$$

$$C_T = \frac{1}{2\omega_0 R_s Q} = \frac{1}{2 \cdot 2\pi \cdot 5 \cdot 10^9 \cdot 50 \cdot 2} = 160\,fF$$

$$L_s = \frac{R_{in} \cdot C_T}{g_m} = \frac{50 \cdot 160 \cdot 10^{-15}}{40 \cdot 10^{-3}} = 0{,}2\,nH$$

$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}V_{ov}^2 \Rightarrow W = L \cdot \frac{2I_D}{\mu C_{ox} V_{ov}^2} = 0{,}2\mu m \cdot \frac{10m}{400\mu \cdot 0{,}25^2} = 80\,\mu m$$

$$C_{gs} = \frac{2}{3}WL\,C_{ox} = \frac{2}{3} \cdot 80 \cdot 0{,}2 \cdot 14\,fF = 150\,fF$$

$$P = \frac{C_{gs}}{C_T} = \frac{150}{160} = 0{,}94$$

$$F = 1 + \frac{(\delta/5)(Q^2+1) \cdot P^2 + \gamma/4}{R_s Q^2 g_m} = 1 + \frac{0{,}8 \cdot 5 \cdot 0{,}94^2 + 0{,}5}{50 \cdot 4 \cdot 40 \cdot 10^{-3}} = 1{,}50 = 1{,}8\,dB$$

seems OK ⇒ Continue:

$$L_g = \frac{1}{\omega_0^2 C_T} - L_s = 6{,}33\,nH - 0{,}2\,nH = 6{,}1\,nH$$ Too large!

$$R_{L_g+L_s} = \frac{\omega_0(L_g+L_s)}{Q_L} = \frac{199}{14} = 14\,\Omega$$

Too large

$$F_{L_{g,s}} = 1{,}50 + \frac{14}{50} + \frac{0{,}4}{50} = 1{,}79 = 2{,}5\,dB$$ with cascode 2,7 dB, too much!

Reduce Q from 2 to 1,4 to reduce L_g. Keep V_{ov}

$g_m = 40\,mS$ $C_T = 0{,}23\,pF$ $L_s = 0{,}29\,nH$

$L = 0{,}2\,\mu m$ $W = 80\,\mu m$ $C_{gs} = 0{,}15\,pF \Rightarrow P = 0{,}65$

$F = 1{,}38 = 1{,}4\,dB$ $L_g = 4{,}1\,nH$ <5 nH OK $R_{L_g+L_s} = 9{,}9\,\Omega$

$F_{L_{g,s}} = 1{,}38 + \frac{9{,}9}{50} + \frac{0{,}4}{50} = 1{,}586 = 2{,}0\,dB$ $F_{tot} = 2{,}0 + 0{,}2 = 2{,}2\,dB < 2{,}5\,dB$ OK

cascode

$$A_v = 2Q\,g_m R_L \Rightarrow R_L = \frac{A_v}{2Q g_m} = \frac{10}{2 \cdot 1{,}4 \cdot 40 \cdot 10^{-3}} = 90\,\Omega$$

5 continued.

Try 5nH load inductor for load:

$R_p = \omega_0 L Q = 2\pi \cdot 5 \cdot 10^9 \cdot 5 \cdot 10^{-9} \cdot 14 = 2{,}2\ k\Omega$

Load with 94Ω in parallell to get 90Ω in total

$C_{tot} = \frac{1}{\omega_0^2 \cdot L} = 200\ fF$

$C_{drain,cascode} < W \cdot \frac{1}{2} \cdot 0{,}4 \cdot 1{,}0\ fF = 80 \cdot \frac{1}{2} \cdot 0{,}4 \cdot 1{,}0\ fF = 16\ fF$

Result: $L_{Load} = 5nH$, $R_{Load} = 94\Omega$, $C_{Load} = 203 - 16\ fF = 0{,}19\ pF$

$V_{G1} = V_t + V_{ov} = 0{,}3 + 0{,}25 = 0{,}55\ V$

$V_{G2} = V_t + 2V_{ov,max} + 0{,}1V = 0{,}3 + 2 \cdot (0{,}25 + V_{in,max} \cdot 2a) + 0{,}1 = 1{,}2\ V$

$V_{out,max} = V_{in,max} \cdot A_v = 0{,}05V \cdot 10 = 0{,}5\ V$

$V_{DD} = V_{G2} - V_t + V_{out,max} = 1{,}2 - 0{,}3 + 0{,}5 = 1{,}4V$ is required

(should be OK to exceed 1,2V slightly, thanks to cascode)

$r_g = \frac{1}{3} \frac{W}{L} \cdot R_{sp} \cdot \frac{1}{n_{finger}^2} \Rightarrow n_{finger} = \sqrt{\frac{1}{3} \frac{W}{L} \frac{R_{sp}}{r_g}} = \sqrt{\frac{1}{3} \cdot \frac{80}{0{,}2} \cdot \frac{7{,}5}{0{,}4}} = 50$

50 gates if contacted on one side

(if contacted on both sides, 25 is enough.)

Results: $W_{1,2} = 80\mu m$ $\quad L_{1,2} = 0{,}2\mu m$

$L_s = 0{,}3\ nH$ $\quad L_g = 4{,}1\ nH$ $\quad L_{Load} = 5\ nH$

$R_{Load} = 94\Omega$ $\quad C_{Load} = 0{,}19\ pF$

$V_{G1} = 0{,}55V$ $\quad V_{G2} = 1{,}2V$ $\quad V_{DD} = 1{,}4V$