Solutions to Exam in Integrated Radio Electronics

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1.

a) $L = \dfrac{37{,}5 \cdot \mu_0 n^2 a^2}{22 r - 14 a}$

$n=1 \Rightarrow r = a = 50\,\mu m \Rightarrow L = \dfrac{37{,}5 \cdot 4\pi \cdot 10^{-7} \cdot 1^2 \cdot (50\,\mu m)^2}{8 \cdot 50\,\mu m} = 0{,}29\ nH$

$n=2 \Rightarrow r = 60\,\mu m,\ a = 55\,\mu m \Rightarrow L = \dfrac{37{,}5 \cdot 4\pi \cdot 10^{-7} \cdot 2^2 \cdot (55\,\mu m)^2}{22 \cdot 60\,\mu m - 14\ 55\,\mu m} = 1{,}0\ nH$

$n=4 \Rightarrow r = 80\,\mu m,\ a = 65\,\mu m \Rightarrow L = \dfrac{37{,}5 \cdot 4\pi \cdot 10^{-7} \cdot 4^2 \cdot (65\,\mu m)^2}{22 \cdot 80\,\mu m - 14\ 65\,\mu m} = 3{,}7\ nH$

$n=8 \Rightarrow r = 120\,\mu m,\ a = 85\,\mu m \Rightarrow L = \dfrac{37{,}5 \cdot 4\pi \cdot 10^{-7} \cdot 8^2 (85\,\mu m)^2}{22\ 120\,\mu m - 14 \cdot 85\,\mu m} = 15\ nH$

b) $f_s = \dfrac{1}{2\pi \sqrt{L\, C_{ox}/2}}$, $C_{ox} = W \cdot l \cdot \dfrac{\varepsilon_{ox}}{t_{ox}}$, $l = 8 \cdot n \cdot a$

$n=1 \Rightarrow l = 8 \cdot 1 \cdot 50\,\mu m = 400\,\mu m$, $C_{ox} = 8\,\mu m \cdot 400\,\mu m \cdot \dfrac{8{,}85 \cdot 10^{-12} \cdot 3{,}9}{5\,\mu m} = 22\ fF \Rightarrow$

$f_s = \dfrac{1}{2\pi \sqrt{0{,}29\,nH \cdot 22\,fF/2}} = 89\ GHz$

$n=2 \Rightarrow l = 8 \cdot 2 \cdot 55\,\mu m = 880\,\mu m$, $C_{ox} = 8\,\mu m \cdot 880\,\mu m\ \dfrac{8{,}85 \cdot 10^{-12} \cdot 3{,}9}{5\,\mu m} = 48\ fF \Rightarrow$

$f_s = \dfrac{1}{2\pi \sqrt{1{,}0\,nH \cdot 48\,fF/2}} = 32\ GHz$

$n=4 \Rightarrow l = 8 \cdot 4 \cdot 65\,\mu m = 2080\,\mu m$, $C_{ox} = 8\,\mu m \cdot 2080\,\mu m \cdot \dfrac{8{,}85 \cdot 10^{-12} \cdot 3{,}9}{5\,\mu m} = 115\ fF \Rightarrow$

$f_s = \dfrac{1}{2\pi \sqrt{3{,}7\,nH \cdot 115\,fF/2}} = 11\ GHz$

$n=8 \Rightarrow l = 8 \cdot 8 \cdot 85\,\mu m = 5440\,\mu m$, $C_{ox} = 8\,\mu m\ 5440\,\mu m \cdot \dfrac{8{,}85 \cdot 10^{-12} \cdot 3{,}9}{5\,\mu m} = 301\ fF \Rightarrow$

$f_s = \dfrac{1}{2\pi \sqrt{15\,nH \cdot 301\,fF/2}} = 3{,}3\ GHz$

c) $Q = \dfrac{\omega L}{R_s}$, $R_s \simeq \dfrac{l\, \rho}{W\, \delta\, (1 - e^{-t/\delta})}$, $\delta = \sqrt{\dfrac{2\rho}{\omega \mu_0}} = \sqrt{\dfrac{2 \cdot 27 \cdot 10^{-9}}{2\pi \cdot 2 \cdot 10^9 \cdot 4\pi\, 10^{-7}}} = 1{,}85\,\mu m$

$R_s = l \cdot \dfrac{27 \cdot 10^{-9}}{8\, 10^{-6} \cdot 1{,}85 \cdot 10^{-6} \cdot (1 - e^{-2/1{,}85})} = l \cdot 2{,}76\ k\Omega$

$n=1 \Rightarrow R_s = 400\,\mu m \cdot 2{,}76\ k\Omega/m = 1{,}1\,\Omega$, $Q = \dfrac{2\pi \cdot 2 \cdot 10^9 \cdot 0{,}29 \cdot 10^{-9}}{1{,}1} = 3{,}3$

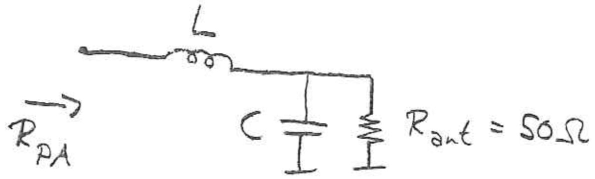
$n=2 \Rightarrow R_s = 880\,\mu m \cdot 2{,}76\ k\Omega/m = 2{,}4\,\Omega$, $Q = \dfrac{2\pi \cdot 2 \cdot 10^9 \cdot 1{,}0 \cdot 10^{-9}}{2{,}4} = 5{,}2$

$n=4 \Rightarrow R_s = 2080\,\mu m \cdot 2{,}76\ k\Omega/m = 5{,}7\,\Omega$, $Q = \dfrac{2\pi \cdot 2 \cdot 10^9 \cdot 3{,}7 \cdot 10^{-9}}{5{,}7} = 8{,}2$

$n=8 \Rightarrow R_s = 5440\,\mu m \cdot 2{,}76\ k\Omega/m = 15\,\Omega$, $Q = \dfrac{2\pi \cdot 2 \cdot 10^9 \cdot 15 \cdot 10^{-9}}{15} = 13$

2

a)



$$Q = \sqrt{\frac{R_{ant}}{R_{PA}} - 1} = \sqrt{\frac{50}{5{,}0} - 1} = 3{,}0$$

$$Q = \omega_0 R_{ant} C \Rightarrow C = \frac{Q}{\omega_0 R_{ant}} = \frac{3}{2\pi \cdot 2 \cdot 10^9 \cdot 50} = 4{,}8 \text{ pF}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi \cdot 2 \cdot 10^9)^2 \cdot 4{,}8 \cdot 10^{-12}} = 1{,}3 \text{ nH}$$

b) $\underline{R_{ant} = 30\Omega}$

$$Q = \omega_0 R_{ant} C = 1{,}8$$

$$R_{PA} = \frac{R_{ant}}{Q^2+1} = \frac{30}{1{,}8^2+1} = 7{,}1\ \Omega$$

$\underline{R_{ant} = 100\Omega}$

$$Q = \omega_0 R_{ant} C = 6$$

$$R_{PA} = \frac{R_{ant}}{Q^2+1} = \frac{100}{6^2+1} = 2{,}7\ \Omega$$

c) $\underline{R_{ant} = 30\ \Omega}$

$V_{PA} = 4V_{pp} \Rightarrow I_{PA} = \frac{4V_{pp}}{7{,}1\Omega} = 560 \text{ mA}_{pp} < 800 \text{ mA} \Rightarrow$ voltage limited

$$P = \frac{V_{PA,pk}^2}{2R_{PA}} = \frac{(2V_p)^2}{2 \cdot 7{,}1} = 0{,}28\,\text{W}$$

$\underline{R_{ant} = 50\Omega}$

$V_{PA} = 4V_{pp} \Rightarrow I_{PA} = \frac{4V_{pp}}{5\Omega} = 800 \text{ mA}_{pp} \Rightarrow$ limited in voltage and current simultaneously. (optimum power)

$$P = \frac{V_{PA,pk}^2}{2R_{PA}} = \frac{2^2}{2 \cdot 5} = 0{,}40\ \text{W}$$

$\underline{R_{ant} = 100\Omega}$

$V_{PA} = 4V_{pp} \Rightarrow I_{PA} = \frac{4V_{pp}}{2{,}7\Omega} = 1{,}5\text{A} > 800 \text{mA} \Rightarrow$ current limited

$(\Rightarrow V_{PA} = 800 \text{mA}_{pp} \cdot 2{,}7\Omega = 2{,}16\ V_{pp})$

$$P = I_{PA,pk}^2 \cdot R_{PA}/2 = 0{,}4^2 \cdot \frac{2{,}7}{2} = 0{,}22\,\text{W}$$

3

a) $R_L = \omega_0 L Q = 2\pi \cdot 1{,}8\cdot 10^9 \cdot 8\cdot 10^{-9}\cdot 12 = 1{,}09\ k\Omega$ (per side)

$P_{sig} = 2\cdot \frac{V_{sig}^2}{2R_L} = \frac{V_{sig}^2}{1{,}09k\Omega}$

$\mathcal{L}(\Delta f) = 10\cdot \log\left[\frac{2FkT}{P_{sig}}\cdot\left(\frac{f_0}{2Q\cdot\Delta f}\right)^2\right] = -138\ dBc/Hz$ @ 3 MHz $\Delta f \Rightarrow$

$P_{sig} = \frac{2FkT}{10^{-13,8}}\cdot\left(\frac{f_0}{2Q\cdot\Delta f}\right)^2 = \frac{2\cdot 3\cdot 4{,}2\cdot 10^{-21}}{10^{-13,8}}\cdot\left(\frac{1800}{2\cdot 12\cdot 3}\right)^2 = 1{,}0\ mW \Rightarrow$

$V_{sig} = \sqrt{1{,}0mW\cdot 1{,}09k\Omega} = 1{,}04\ V$

0,5V margin $\Rightarrow V_{dd} = 1{,}04+0{,}5 = 1{,}54V \Rightarrow$ choose $V_{dd} = 1{,}6V$

$V_{sig} = I_{dc}\cdot\frac{2}{\pi}\cdot R_L \Rightarrow I_{dc} = \frac{1{,}04V\cdot\pi}{2\cdot 1{,}09k\Omega} = 1{,}5\ mA$

b) Varactor

$f_{high} = 1{,}8GHz + 7\% = 1{,}93\ GHz$ $\quad f_{low} = 1{,}8GHz - 7\% = 1{,}67\ GHz$

$C_{tot\,max} = \frac{1}{(2\pi)^2 L f_{low}^2} = 1{,}14\ pF/side$ $\quad C_{tot\,min} = \frac{1}{(2\pi)^2 L f_{high}^2} = 0{,}85\ pF/side$

$C_{var\,max} - C_{var\,min} = C_{tot\,max} - C_{tot\,min} = 1{,}14pF - 0{,}85pF = 0{,}29\ pF$

$\frac{C_{var\,max}}{C_{var\,min}} = 2 \Rightarrow C_{var\,max}\left(1-\frac{1}{2}\right) = 0{,}29pF \Rightarrow C_{var,max} = 0{,}58\ pF$

$C_{var\,max} = WLC_{ox} + 2WC_{gdo} \Rightarrow W = \frac{C_{var\,max}}{LC_{ox}+2C_{gdo}} = \frac{580}{0{,}4\cdot 4{,}6 + 2\cdot 0{,}21}\mu m = 256\ \mu m$

Switches

$C_{rest} = C_{tot\,max} - C_{var\,max} = 1{,}14 - 0{,}58 = 0{,}56\ pF$

$C_{ind,par} = \frac{1}{(2\pi)^2 L f_s^2} = \frac{1}{(2\pi)^2\cdot 8nH\cdot(4GHz)^2} = 0{,}20\ pF$ (parasitics of inductor)

$C_{transistor,par} = C_{rest} - C_{ind,par} = 0{,}36\ pF$

$C_{transistor,par} = C_{gs} + C_{db} + 4C_{gd} = \frac{2}{3}WLC_{ox} + WC_{gdo} + \frac{W}{2}\cdot 1{,}1\cdot c_{jn} + 4WC_{gdo} =$

$= W(\mu m)\cdot\left(\frac{2}{3}\cdot 0{,}4\cdot 4{,}6 + 0{,}21 + 0{,}55\cdot 0{,}93 + 4\cdot 0{,}21\right) fF = W\cdot 2{,}8\ fF/\mu m \Rightarrow$

$W = 360fF / 2{,}8fF/\mu m = 130\ \mu m$

Tail transistor: $W = L\cdot\frac{2I_d}{\mu C_{ox} V_{od}^2} = 0{,}4\cdot\frac{2\cdot 1{,}5m}{110\mu\cdot 0{,}25^2}\mu m = 175\ \mu m$

$V_{bias} = V_{T_0} + V_{od} = 0{,}70\ V$

c) $g_{m,startup} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \sqrt{2\cdot 110\cdot 10^{-6}\cdot\frac{130}{0{,}4}\cdot\frac{1{,}5m}{2}} = 7{,}3\ mS$

$A_{l,startup} = g_{m,startup}\cdot R_L = 7{,}3mS\cdot 1{,}09k\Omega = 8 > 3$ OK

Results: $V_{dd} = 1{,}6V$, $I_{dc} = 1{,}5mA$, $W_{var} = 256\mu m$, $W_{sw} = 130\mu m$, $W_{tail} = 175\mu m$, $A_{l,startup} = 8$, $V_{bias} = 0{,}70V$

4.

a)

$$Q_{in} = \frac{1}{2\omega_o R_s C_{gs}} \Rightarrow C_{gs} = \frac{1}{2\omega_o R_s Q_{in}} = \frac{1}{2\cdot 2\pi\cdot 1{,}575\cdot 10^9\cdot 50\cdot 1{,}5} = 670\ fF$$

$$C_{gs} = W C_{gdo} + \frac{2}{3} W L C_{ox} \Rightarrow W = \frac{C_{gs}}{C_{gdo}+\frac{2}{3}L C_{ox}} = \frac{670}{0{,}21+\frac{2}{3}\cdot 0{,}4\cdot 4{,}60} = 470\ \mu m$$

$$I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}V_{od}^2 \Rightarrow V_{od} = \sqrt{\frac{2I_d}{\mu C_{ox}\frac{W}{L}}} = \sqrt{\frac{2\cdot 1{,}5m}{110\cdot 10^{-6}\cdot 470/0{,}4}} = 152\ mV$$

$$g_m = \frac{2I_d}{V_{od}} = \frac{3\ mA}{152\ mV} = 19{,}7\ mS$$

$$R_{in} = g_m\frac{L_s}{C_{gs}} \Rightarrow L_s = \frac{R_{in}C_{gs}}{g_m} = \frac{50\cdot 670\cdot 10^{-15}}{19{,}7\cdot 10^{-3}} = 1{,}7\ nH$$

$$L_g = \frac{1}{\omega_o^2 C_{gs}} - L_s = 13{,}5\ nH$$

$$V_g = V_{T_0} + V_{od} = 652\ mV$$

$$F = 1 + \frac{(\delta/5)\cdot(Q^2+1)+\gamma/4}{R_s Q^2 g_m} = 1 + \frac{(2{,}8/5)\cdot 3{,}25 + 1{,}4/4}{50\cdot 2{,}25\cdot 19{,}7\cdot 10^{-3}} = 1{,}98 = 3{,}0\ dB$$

$$V_{G,cascode} = V_{S,cascode} + V_t + V_{od} = V_{od} + 0{,}3V + V_t + V_{od} = 1{,}1\ V$$

b)

$$C_{d,cascode} = C_{db,cascode} + C_{dg,cascode}$$

$$C_{db,cascode} = \frac{C_{dbo}}{(1+V_{dd}/V_j)^{m_j}} = \frac{\frac{W}{2}\cdot 1{,}1\cdot 0{,}93fF}{(1+2/0{,}69)^{0{,}31}} = 158\ fF$$

$$C_{dg,cascode} = W\cdot C_{gdo} = 470\cdot 0{,}21\ fF = 99\ fF$$

$$C_L = 150\ fF \Rightarrow C_{except\ coil} = 158+150+99 = 407\ fF$$

$$R_L = \omega_o L\cdot Q \sim L\cdot\frac{1}{\sqrt{L}} \sim \sqrt{L} \Rightarrow \text{higher } L \text{ is better}$$

choose max L that gives sufficient frequency:

$$C_{tot} = \frac{1}{\omega_o^2 L}, \quad C_{tot} = C_{except\ coil} + C_{coil} = 407\ fF + \frac{1}{\omega_s^2 L} \Rightarrow$$

$$\frac{1}{\omega_o^2 L} = \frac{1}{\omega_s^2 L} + 407\ fF \Rightarrow \frac{1}{\omega_o^2 L} = \frac{L}{(20\ Hz/H\cdot 2\pi)^2} + 407\ f \Rightarrow$$

$$L^2 + 407f\cdot(20\cdot 2\pi)^2 L - \frac{(20\cdot 2\pi)^2}{(2\pi\cdot 1{,}575\cdot 10^9)^2} = 0 \Rightarrow L = -3{,}2nH \pm\sqrt{(3{,}2n)^2 + 1{,}61\cdot 10^{-16}} =$$

$$= 9{,}9\ nH \Rightarrow Q_L = \frac{20}{\sqrt{9{,}9}} = 6{,}36, \quad f_s = \frac{20\ GHz}{9{,}9} = 2{,}02\ GHz$$

($\underline{\text{check}}$: $C_{coil} = \frac{1}{\omega_s^2 L} = 627\ fF \Rightarrow C_{tot} = 407+627 = 1{,}03\ pF \Rightarrow f_o = \frac{1}{2\pi\sqrt{LC}} = 1{,}575\ GHz$ $\underline{\text{ok}}$)

$$A_v = 2Q_{in}g_m R_L = 2\cdot 1{,}5\cdot 19{,}7\cdot 10^{-3}\cdot \underbrace{6{,}36}_{Q_L}\cdot \underbrace{2\pi\cdot 1{,}575\cdot 10^9}_{\omega_o}\cdot \underbrace{9{,}9\cdot 10^{-9}}_{L} =$$

$$= 37 = 31\ dB$$

5.

a) 0,2V in [illegible] (ε) ⇒ choose $V_{od,in} = 0,22$

$$g_{m,in} = \frac{2I_d}{V_{od}} = \frac{1mA}{0,22V} = 4,5mS$$

$$A_v = \frac{2}{\pi} \cdot g_m R_L \Rightarrow R_L = \frac{A_v}{\frac{2}{\pi} \cdot g_m} > \frac{1,78}{\frac{2}{\pi} \cdot 4,5m} = 0,62\ k\Omega$$

$$V_{R_L} = I_{R_L} \cdot R_L = 0,5m \cdot 0,62k = 0,31\ V$$

$$V_{in} = 0,2V \Rightarrow V_{out} = 0,2 \cdot 1,78 = 0,36\ V$$

neglect that $V_{out} > V_{R_L}$. Due to non-linearities. Assume pure IF sinusoid.

$$I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}V_{od}^2 \Rightarrow W = L \cdot \frac{2I_d}{\mu C_{ox} V_{od}^2} = 0,4\mu m \cdot \frac{1m}{110\mu \cdot 0,22^2} = 75\ \mu m$$

$$V_{G,in} = V_{t0} + V_{od} = 0,72\ V$$

<u>switches</u>

$$\sqrt{2}\, V_{od,sw} = \sin(0,15 \cdot 90^\circ) \cdot 1V_{pk} = (V_{LO} = 1V_{pk,diff}) \Rightarrow V_{od,sw} = 0,16\ V$$

$$W_{sw} = L \cdot \frac{2I_d}{\mu C_{ox} V_{od,sw}^2} = 0,4\mu m\ \frac{0,5m}{110\mu \cdot 0,16^2} = 71\ \mu m$$

<u>choose LO-bias voltage</u>

largest input voltage $\Rightarrow V_{od,in,max} = 0,2\ V + 0,22\ V = 0,42\ V$

at zero-crossing $\quad V_{od,sw,max} = V_{od,in,max}\sqrt{\frac{W_{in}}{2W_{sw}}} = 0,31\ V$

$\Rightarrow V_{LO,bias} > 0,42 + 0,31 + V_T = 1,23\ V$, choose 1,25 V

$$V_{LO,pk,high} = 1,25 + 0,5 = 1,75\ V$$
$$V_{IF,pk,low} = V_{dd} - 2V_{out} = 2 - 2 \cdot 0,36 = 1,28\ V$$

diff $= 0,47V < 0,50 = V_T$ ⇒ OK, small margin..

⇒ no switch transistor or input transistor will enter triode either at LO peak or zero-crossing ⇒ <u>OK</u>.

b) $$C_{in} = 2 \cdot C_{gs,in} + C_{routing} = 2 \cdot W_{in}\left(\frac{2}{3}LC_{ox} + C_{gdo}\right) + 50fF =$$
$$= 2 \cdot 75\left(\frac{2}{3} \cdot 0,4 \cdot 4,6 + 0,21\right) + 50 = 266\ fF > 150\ fF$$

$$\omega_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{9,9n \cdot (627+523)f}} = 1,492GHz$$

Frequency shift $= 1,575\ GHz - 1,492\ GHz = 83\ MHz = 5\%$

Result: The center frequency has moved 5% downwards.