

ETEN05 Electromagnetic Wave Propagation, 2015

Workshop 4: Birefringent media

1 Aims

In this workshop we study wave propagation in birefringent media. In particular, we see how the refractive index changes when the propagation direction is not along the axes of symmetry in the material. Special attention is paid to the difference between chiral media and gyrotropic media.

There is also a demonstration lab, where you will see a real microwave equipment, and how to manipulate the polarization. Feel free to also continue your work with the handin if you have not finished.

2 Task

The script `propconst.m` from workshop 2 has been rewritten in order to be a function taking two arguments and giving three outputs:

```
function [n, E, H] = propconst2(theta, phi)
```

The full source code can be viewed in `propconst2.m`, which can be downloaded from the course web site. The input angles `theta` and `phi` give the direction of propagation in a standard spherical coordinate system (both are given in radians). Look in the script to see how rotation matrices can be used to shift the arbitrary direction given by θ and φ to a local z -direction.

The outputs correspond to the refractive indices and polarizations of the *two* waves propagating in the (positive) direction indicated by `theta` and `phi`, look at the end of `propconst2.m` to see how these are found.

Using this function, another script `plotsurfaces.m` has been written (can also be downloaded from the course web site) that first computes the refractive indices $n_{1,2}$ for each angle θ and φ , and then plots them as radial functions of the angles, *i.e.*,

$$x = n_{1,2} \sin \theta \cos \varphi, \quad y = n_{1,2} \sin \theta \sin \varphi, \quad z = n_{1,2} \cos \theta$$

are plotted as functions of θ and/or φ . The surfaces are colored corresponding to the polarization state $p = -\hat{\mathbf{k}} \cdot (\mathbf{E}_0 \times \mathbf{E}_0^*) / |\mathbf{E}_0|^2$, where $p = -1$ corresponds to

left hand circular polarization (LCP, blue), $p = 0$ corresponds to linear polarization (LP, green), and $p = +1$ corresponds to right hand circular polarization (RCP, red). You can make plots in the principal planes $x = 0$, $y = 0$, and $z = 0$ by choosing the viewing angle, see the end of the script.

When multiplied by the wave number in vacuum k_0 , these surfaces are known as \mathbf{k} -surfaces, since for a fixed propagation direction (θ, φ) they give the corresponding wave vectors $\mathbf{k}_{1,2} = n_{1,2}k_0\hat{\mathbf{k}}$, where $\hat{\mathbf{k}}$ is a unit vector pointing in the direction given by θ and φ .

Try this visualization for the following material models: a biaxial medium

$$\boldsymbol{\epsilon} = \epsilon_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \quad \boldsymbol{\mu} = \mu_0 \mathbf{I}, \quad \boldsymbol{\xi} = \boldsymbol{\zeta} = \mathbf{0} \quad (2.1)$$

a chiral medium

$$\boldsymbol{\epsilon} = \epsilon_0 \mathbf{I}, \quad \boldsymbol{\mu} = \mu_0 \mathbf{I}, \quad \boldsymbol{\xi} = 0.1j/c_0 \mathbf{I}, \quad \boldsymbol{\zeta} = -0.1j/c_0 \mathbf{I} \quad (2.2)$$

and a gyroelectric medium

$$\boldsymbol{\epsilon} = \epsilon_0 \begin{pmatrix} 1 & 0.1j & 0 \\ -0.1j & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{\mu} = \mu_0 \mathbf{I}, \quad \boldsymbol{\xi} = \boldsymbol{\zeta} = \mathbf{0} \quad (2.3)$$

Your figures should look like the ones in Figures 1 and 2. Note that you can rotate 3D-figures manually in matlab to get a better feeling for the geometry. Change some of the parameters and see how the surfaces change. In particular, try changing the signs of the imaginary entries in (2.2) and (2.3). Try to explain the difference between the chiral and gyrotropic cases.

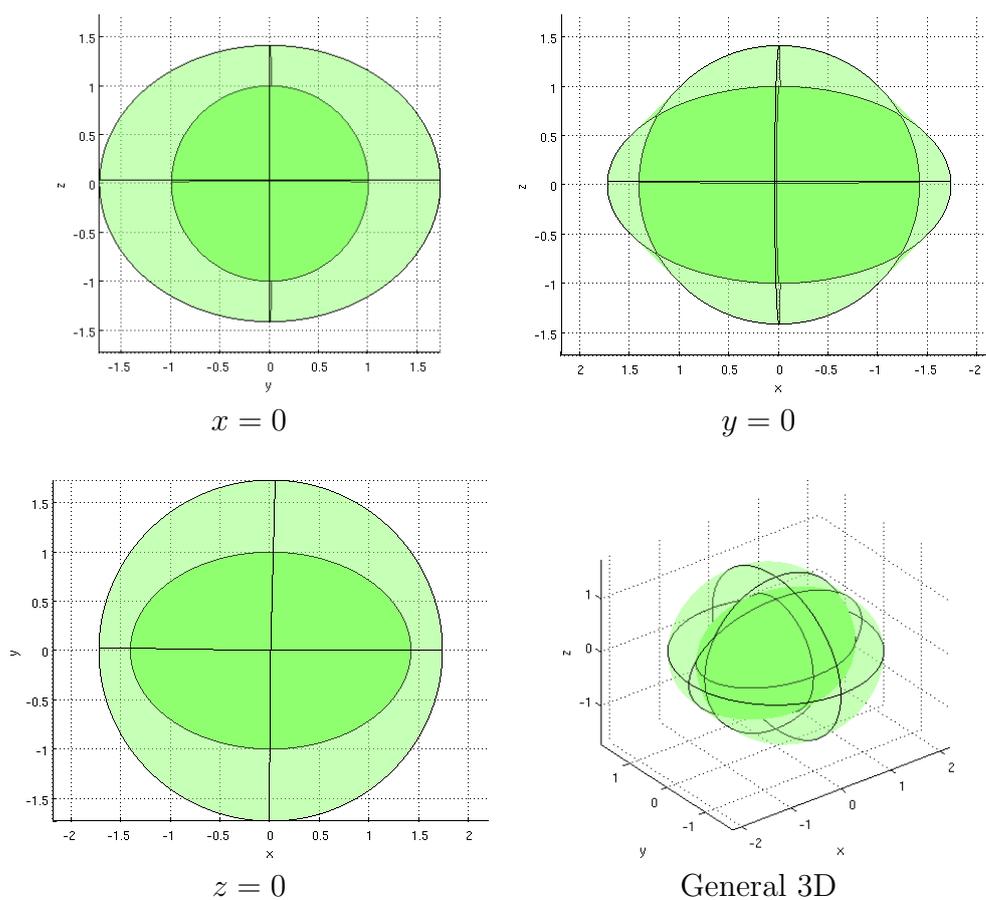


Figure 1: Example of surfaces corresponding to the two refractive indices for different propagation directions in the medium given in (2.1). The surfaces are two intersecting ellipsoids.

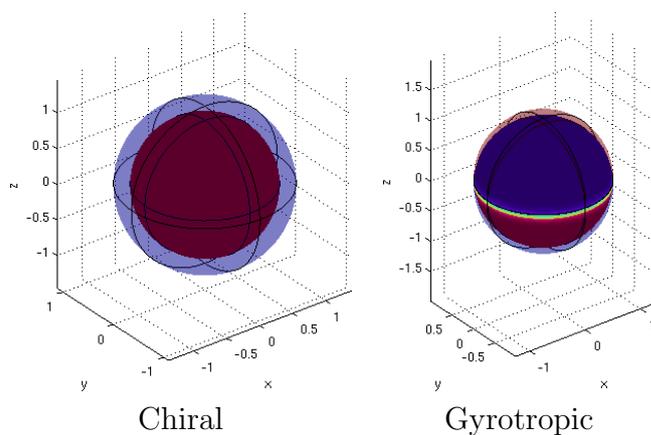


Figure 2: Example of surfaces corresponding to the two refractive indices for different propagation directions in the media given in (2.2) and (2.3).