ETEN05 Electromagnetic Wave Propagation, 2016

Workshop 1: Modeling of materials

1 Aims

In this workshop we study the mathematical modeling of materials, and how to implement the models numerically. Although we will not discuss the exercises in Orfanidis explicitly at the workshop, you are of course welcome to ask about them.

2 The room E:4116

There are 12 computers and 24 seats in the room. You log in with your usual STIL-account. The files necessary for the labs are available for download from the course home page. You can also find them on the special partition "S:", more specifically `S:\\Courses\eit\eten05`.

Don't trust the local computer to keep your files, if you want to keep anything after the lab you should save the files on your own account, save them on a USB-stick, or email the files to your self.

Ground rules for the lab room: no food or beverages, and make sure you leave the room in a *better* state than you found it, *i.e.*, return all chairs to their intended positions, no leftover papers etc.

3 Time and frequency domain

The purpose of this task is to investigate in practice how a material reacts to an applied electric field. The underlying model is that the polarization is given by a convolution in the time domain, and a multiplication in the frequency domain.

We consider two different material models, the Debye model

$$\chi(t) = \alpha \mathrm{e}^{-t/\tau} u(t) \tag{3.1}$$

$$\chi(\omega) = \frac{\alpha\tau}{1 + \mathrm{j}\omega\tau} \tag{3.2}$$

and the Lorentz model (where $\nu_0^2 = \omega_0^2 - \nu^2/4$)

$$\chi(t) = \frac{\omega_\mathrm{p}^2}{\nu_0}\mathrm{e}^{-\nu t/2}\sin(\nu_0 t)u(t) \tag{3.3}$$

$$\chi(\omega) = \frac{\omega_\mathrm{p}^2}{-\omega^2 + \omega_0^2 + \mathrm{j}\omega\nu} \tag{3.4}$$

Write a matlab script that computes and plots the polarization as a function of time and frequency when the material is subjected to

1. A rectangle function $E(t) = u(t) - u(t - t_0) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t < t_0 \\ 0 & t > t_0 \end{cases}$.
2. A damped sine wave $E(t) = \mathrm{e}^{-t/t_0}\sin(\omega_\mathrm{a} t)u(t)$.

Check that your program produces results corresponding to the demo program `materialmodels.py` (or the windows executable `materialmodels.exe`), which can be downloaded from the course web site. (On the lab computers you may need to start the executable from the S: partition due to security policy restrictions.) Start by trying this program to get a feeling for which parameters do what in the material models. Suitable start parameters for your models can be:

$$\alpha = \tau = \omega_\mathrm{p} = \nu = \omega_0 = 1.5, \quad t_0 = 15, \quad \omega_\mathrm{a} = 1.5$$

and the time interval $t \in [0, 30]$ and frequency interval $\omega \in [0, 3]$ (these are the startup parameters in `materialmodels.py`).

Note that there are many ways to solve this problem, for instance by explicit solutions, using convolutions, the FFT algorithm etc. If you get stuck, there is some help in an example script `example.m` at the course web site, or at the end of the demo program `materialmodels.py` (although it is written in python).

4 Numerical solution of the Landau-Lifshitz-Gilbert equation

4.1 Introduction

In Handin 1, you are asked to do some investigations on the Landau-Lifshitz-Gilbert equation

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma\mu_0 \boldsymbol{M} \times \boldsymbol{H} + \alpha\frac{\boldsymbol{M}}{|\boldsymbol{M}|} \times \frac{\partial \boldsymbol{M}}{\partial t} \tag{$*$}$$

which models the behavior of the magnetization $\boldsymbol{M}$ in a ferromagnetic material when subjected to an applied magnetic field $\boldsymbol{H}$. The model is nonlinear, and can usually not be given an explicit solution as in the Debye and Lorentz models above. In the handin, you will solve this by linearization, where the typical frequency behavior can be studied. Here, we will study the full nonlinear model.

Since ($*$) is an ordinary differential equation (ODE), it can be solved numerically using matlab's ODE-solvers. These usually solve equations on the form $y'(t) = f(t, y)$, and since ($*$) is on the form $y'(t) = f(t, y, y')$, we need to reformulate it. We do this by using the equation inside itself:

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma\mu_0 \boldsymbol{M} \times \boldsymbol{H} + \alpha \frac{\boldsymbol{M}}{|\boldsymbol{M}|} \times \underbrace{\left(-\gamma\mu_0 \boldsymbol{M} \times \boldsymbol{H} + \alpha \frac{\boldsymbol{M}}{|\boldsymbol{M}|} \times \frac{\partial \boldsymbol{M}}{\partial t} \right)}_{= \frac{\partial \boldsymbol{M}}{\partial t}}$$

$$= -\gamma\mu_0 \boldsymbol{M} \times \boldsymbol{H} - \alpha\gamma\mu_0 \frac{\boldsymbol{M}}{|\boldsymbol{M}|} \times (\boldsymbol{M} \times \boldsymbol{H}) + \alpha^2 \frac{\boldsymbol{M}}{|\boldsymbol{M}|} \times \left(\frac{\boldsymbol{M}}{|\boldsymbol{M}|} \times \frac{\partial \boldsymbol{M}}{\partial t} \right)$$

From the cross products in the right hand side of ($*$) it is clear that $\partial \boldsymbol{M}/\partial t$ is orthogonal to $\boldsymbol{M}$, meaning the last term is $-\alpha^2 \partial \boldsymbol{M}/\partial t$ (check the computational rules for double cross products in Orfanidis' appendix). Thus, we can move it to the left side, divide by $1 + \alpha^2$, and obtain

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\frac{\gamma\mu_0}{1+\alpha^2} \boldsymbol{M} \times \boldsymbol{H} - \frac{\alpha\gamma\mu_0}{1+\alpha^2} \frac{\boldsymbol{M} \times (\boldsymbol{M} \times \boldsymbol{H})}{|\boldsymbol{M}|} \qquad (**)$$

This is now an equation on the standard form that matlab can solve (and this is actually the form in which it was originally proposed by Landau). Example scripts are supplied (`compM.m`, `dMdt.m`, and `Hfield.m`), which solve the equation in the simple case of a static magnetic field $\boldsymbol{H}$ and an initial magnetization which is not parallel to $\boldsymbol{H}$.

4.2 Numerical experiments

Make your own experiments with the scripts `compM.m` and `dMdt.m`, changing some of the parameters and see how they influence the solution. In particular, you should be able to see the relaxation phenomenon: as a static field $\boldsymbol{H}$ is applied, the magnetization starts to precess around the magnetic field and finally aligns with it.

Also try to modify the scripts so that you can simulate the response of a time harmonic input, $\boldsymbol{H}(t) = \boldsymbol{H}_0 \cos(\omega_a t)$, where $\boldsymbol{H}_0$ is a fixed vector and $\omega_a = 2\pi f_a$ is the angular frequency of the applied field. Try at least the frequencies 1 GHz, 3 GHz, and 10 GHz. You should be able to observe the influence of the nonlinearity of the constitutive relation, *i.e.*, even though you use a fixed frequency input, your output (the components of $\boldsymbol{M}$) will not be a pure sine wave.