Electromagnetic Wave Propagation
Lecture 14: Multilayer film applications

Daniel Sjöberg

Department of Electrical and Information Technology
Outline

1. Multilayer dielectric structures at oblique incidence
2. Perfect lens in negative-index media
3. Antireflection coatings at oblique incidence
4. Omnidirectional dielectric mirrors
5. Reflection and refraction in birefringent media
6. Giant birefringent optics
7. Concluding remarks
1. Multilayer dielectric structures at oblique incidence
2. Perfect lens in negative-index media
3. Antireflection coatings at oblique incidence
4. Omnidirectional dielectric mirrors
5. Reflection and refraction in birefringent media
6. Giant birefringent optics
7. Concluding remarks
Most concepts generalize. Important steps:

- Use transverse field components
- The tangential wave vector is conserved
Oblique propagation in bianisotropic materials

The formulation from lecture 4 can be used for numerical implementation of completely general bianisotropic materials:

\[
\frac{\partial}{\partial z} \left( \begin{array}{c} E_t \\ H_t \times \hat{z} \end{array} \right) = -j\omega W(z) \cdot \left( \begin{array}{c} E_t \\ H_t \times \hat{z} \end{array} \right)
\]

\[
W = \left( \begin{array}{cc} 0 & -\hat{z} \times I \\ I & 0 \end{array} \right) \cdot \left[ \begin{array}{cc} \epsilon_{tt} & \xi_{tt} \\ \zeta_{tt} & \mu_{tt} \end{array} \right] - A(k_t) \cdot \left( \begin{array}{cc} I & 0 \\ 0 & \hat{z} \times I \end{array} \right)
\]

\[
A(k_t) = \left[ \begin{array}{cc} 0 & -\omega^{-1}k_t \times \hat{z} \\ \omega^{-1}k_t \times \hat{z} & 0 \end{array} \right] - \left( \begin{array}{cc} \epsilon_t & \xi_t \\ \zeta_t & \mu_t \end{array} \right)
\]

\[
\left( \begin{array}{cc} \epsilon_{zz} & \xi_{zz} \\ \zeta_{zz} & \mu_{zz} \end{array} \right)^{-1} \left[ \begin{array}{cc} 0 & -\omega^{-1}\hat{z} \times k_t \\ \omega^{-1}\hat{z} \times k_t & 0 \end{array} \right] - \left( \begin{array}{cc} \epsilon_z & \xi_z \\ \zeta_z & \mu_z \end{array} \right)
\]

In this lecture, we focus on isotropic media.
Wave concepts

Tangential phase (Snel’s law):

\[ n_a \sin \theta_a = n_i \sin \theta_i = n_b \sin \theta_b \implies \cos \theta_i = \sqrt{1 - \frac{n_a^2 \sin^2 \theta_a}{n_i^2}} \]

Phase thickness:

\[ \delta_i = k_{z,i} l_i = \frac{\omega}{c_0} n_i l_i \cos \theta_i = 2\pi \frac{f}{f_0} \frac{n_i l_i}{\lambda_0} \cos \theta_i \]

Transverse wave impedance and refractive index:

\[ \eta_{TM} = \frac{k_z}{\omega \epsilon} = \sqrt{\frac{\mu}{\epsilon} \frac{k_z}{\omega \sqrt{\epsilon \mu}}} = \eta \cos \theta = \frac{\eta_0 \cos \theta}{n} = \frac{\eta_0}{n_{TM}} \]

\[ \eta_{TE} = \frac{\omega \mu}{k_z} = \sqrt{\frac{\mu}{\epsilon} \frac{\omega \sqrt{\epsilon \mu}}{k_z}} = \frac{\eta}{\cos \theta} = \frac{\eta_0}{n \cos \theta} = \frac{\eta_0}{n_{TE}} \]
Propagation of split fields

Reflection and transmission coefficients:

\[
\rho_{T,i} = \frac{n_{T,i-1} - n_{T,i}}{n_{T,i-1} + n_{T,i}}, \quad \tau_{T,i} = 1 + \rho_{T,i}
\]

Matching and propagation matrices:

\[
\begin{pmatrix}
E_{T,i+} \\
E_{T,i-}
\end{pmatrix} = \frac{1}{\tau_{T,i}} \begin{pmatrix}
1 & \rho_{T,i} \\
\rho_{T,i} & 1
\end{pmatrix} \begin{pmatrix}
\mathrm{e}^{j\delta_i} & 0 \\
0 & \mathrm{e}^{-j\delta_i}
\end{pmatrix} \begin{pmatrix}
E_{T,i+1,+} \\
E_{T,i+1,-}
\end{pmatrix}
\]

Reflection coefficient recursion:

\[
\Gamma_{T,i} = \frac{\rho_{T,i} + \Gamma_{T,i+1} \mathrm{e}^{-2j\delta_i}}{1 + \rho_{T,i} \Gamma_{T,i+1} \mathrm{e}^{-2j\delta_i}}
\]

initialized by \( \Gamma_{T,M+1} = \rho_{T,M+1} \).
Propagation of total fields

Propagation matrix:

\[
\begin{pmatrix}
E_{T,i} \\
H_{T,i}
\end{pmatrix} =
\begin{pmatrix}
\cos \delta_i & j\eta_{T,i} \sin \delta_i \\
-j\eta_{T,i}^{-1} \sin \delta_i & \cos \delta_i
\end{pmatrix}
\begin{pmatrix}
E_{T,i+1} \\
H_{T,i+1}
\end{pmatrix}
\]

Transverse impedance recursion:

\[
Z_{T,i} = \eta_{T,i} \frac{Z_{T,i+1} + j\eta_{T,i} \tan \delta_i}{\eta_{T,i} + jZ_{T,i+1} \tan \delta_i}, \quad Z_{T,M+1} = \eta_{T,b}
\]

Generalization of multidiel.m:

\[
[\Gamma_1, Z_1] = \text{multidiel}(n, L, \lambda, \theta, \text{pol})
\]
Single dielectric slab

\[ \Gamma_{T,1} = \frac{\rho_{T,1} + \rho_{T,2}e^{-2j\delta_1}}{1 + \rho_{T,1}\rho_{T,2}e^{-2j\delta_1}} = \frac{\rho_{T,1}(1 - e^{-2j\delta_1})}{1 - \rho_{T,1}^2e^{-2j\delta_1}} \]

\[ \delta_1 = 2\pi \frac{f}{f_0} \frac{n_1 \ell_1}{\lambda_0} \cos \theta_1 = \pi \frac{f}{f_1}, \quad f_1 = \frac{f_0}{2 \frac{n_1 \ell_1}{\lambda_0} \cos \theta_1} \]
Frequency shift

- Notch frequency is shifted up
- Reflection level is increased
- Bandwidth is decreased

(Fig. 8.3.2 in Orfanidis)
Frustrated total internal reflection

![Diagram of frustrated total internal reflection](Fig. 8.4.2 in Orfanidis)

\[ k_x = k_0 n_a \sin \theta \]

\[ k_{za} = \sqrt{k_0^2 n_a^2 - k_x^2} = k_0 n_a \cos \theta \]

\[ k_{zb} = \begin{cases} 
  k_0 \sqrt{n_b^2 - n_a^2 \sin^2 \theta} & \text{if } \theta \leq \theta_c \\
  -jk_0 \sqrt{n_a^2 \sin^2 \theta - n_b^2} = -j\alpha_{zb} & \text{if } \theta \geq \theta_c 
\end{cases} \]
Dependence on thickness and angle

(Figs. 8.4.3 and 8.4.4 in Orfanidis)
Power flow

The combined field from two evanescent waves is

\[
E_T(z) = \frac{1 + \rho_a}{1 - \rho_a^2 e^{-2\alpha_{zb}d}} \left[ e^{-\alpha_{zb}z} - \rho_a e^{-2\alpha_{zb}d} e^{\alpha_{zb}z} \right] E_{a+} \\
H_T(z) = \frac{1 - \rho_a}{1 - \rho_a^2 e^{-2\alpha_{zb}d}} \left[ e^{-\alpha_{zb}z} + \rho_a e^{-2\alpha_{zb}d} e^{\alpha_{zb}z} \right] \frac{E_{a+}}{\eta_{Ta}}
\]

The power transported in one evanescent wave is zero,

\[
P_z = \frac{1}{2} \text{Re}\{E_T H_T^*\} = \frac{1}{2} \text{Re} \left\{ \frac{(1 + \rho_a)(1 - \rho_a^*)}{|1 - \rho_a^2 e^{-2j\alpha_{zb}d}|} \frac{|E_{a+}|^2}{\eta_{Ta}} e^{-2\alpha_{zb}z} \right\} = 0
\]

since \((1 + \rho_a)(1 - \rho_a^*) = 1 - |\rho_a|^2 + \rho_a - \rho_a^* = 2j \text{Im}(\rho_a)\). But the combination of two evanescent waves can carry power:

\[
P_z = \frac{1}{2} \text{Re}\{E_T H_T^*\} = \cdots = (1 - |\Gamma|^2) \frac{|E_{a+}|^2}{2\eta_{Ta}}
\]

\[
|\Gamma|^2 = \frac{\sinh^2(\alpha_{zb}d)}{\sinh^2(\alpha_{zb}d) + \sin^2 \phi_a}, \quad \rho_a = e^{j\phi_a}
\]
Outline

1. Multilayer dielectric structures at oblique incidence
2. Perfect lens in negative-index media
3. Antireflection coatings at oblique incidence
4. Omnidirectional dielectric mirrors
5. Reflection and refraction in birefringent media
6. Giant birefringent optics
7. Concluding remarks
\( \epsilon = -\epsilon_0, \mu = -\mu_0, \quad n = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}} = -1, \quad \eta = \sqrt{\frac{\mu}{\epsilon}} = \eta_0. \)

(Fig. 8.6.1 in Orfanidis)

Snel’s law:

\[ n_a \sin \theta_a = n \sin \theta \quad \Rightarrow \quad \theta = -\theta_a \]

Zero reflection since \( \eta = \eta_0. \)
Field solutions

The electric field is (TM polarization)

\[
E = \begin{cases} 
E_0 \left( \hat{x} - \frac{k_x}{k_z} \hat{z} \right) e^{-jk_z z} e^{-jk_x x}, & \text{for } z \leq 0 \\
E_0 \left( \hat{x} + \frac{k_x}{k_z} \hat{z} \right) e^{jk_z z} e^{-jk_x x}, & \text{for } 0 \leq z \leq d \\
E_0 \left( \hat{x} - \frac{k_x}{k_z} \hat{z} \right) e^{-jk_z (z-2d)} e^{-jk_x x}, & \text{for } z \geq d 
\end{cases}
\]

where the longitudinal wave number is

\[
k_z = \sqrt{k_0^2 - k_x^2} = \begin{cases} 
\sqrt{k_0^2 - k_x^2}, & \text{if } k_x^2 \leq k_0^2 \\
-j\sqrt{k_0^2 - k_x^2}, & \text{if } k_0^2 \leq k_x^2 
\end{cases}
\]

Thus, for evanescent waves where \(k_z = -j\alpha\), the negative medium amplifies the transmitted wave.
Amplification of evanescent waves

(Fig. 8.6.2 in Orfanidis)
Influence of non-ideal parameters

Transmittance below is computed as \(10 \log_{10} |T e^{-jk_zd}|^2\), which is not necessarily less than one for evanescent waves.

\[
\text{Peaks correspond to plasmon resonances, where } \quad \Re(k_{x,\text{res}}) \approx \frac{1}{d} \left| \frac{\epsilon_0 - \epsilon}{\epsilon_0 + \epsilon} \right|. \quad \text{This implies a resolution of } \quad \Delta x = \frac{1}{\Re(k_{x,\text{res}})}.\]

(Fig. 8.6.3 in Orfanidis)
1. Multilayer dielectric structures at oblique incidence
2. Perfect lens in negative-index media
3. Antireflection coatings at oblique incidence
4. Omnidirectional dielectric mirrors
5. Reflection and refraction in birefringent media
6. Giant birefringent optics
7. Concluding remarks
Suitably designed thin layers can reduce reflection from interfaces.
What happens at oblique incidence?
Two-layer coating

\[ n = [1, 1.38, 2.45, 1.5]; \quad L = [0.3294, 0.0453]; \]
\[ \text{la0} = 550; \quad \text{la} = \text{linspace}(400, 700, 101); \quad \text{pol} = 'te'; \]
\[ \text{G0} = \text{abs}(	ext{multidiel}(n, L, \text{la}/\text{la0})).^2 \times 100; \]
\[ \text{G20} = \text{abs}(	ext{multidiel}(n, L, \text{la}/\text{la0}, 20, \text{pol})).^2 \times 100; \]
\[ \text{G30} = \text{abs}(	ext{multidiel}(n, L, \text{la}/\text{la0}, 30, \text{pol})).^2 \times 100; \]
\[ \text{G40} = \text{abs}(	ext{multidiel}(n, L, \text{la}/\text{la0}, 40, \text{pol})).^2 \times 100; \]
\[ \text{plot}(	ext{la}, [\text{G0}; \text{G20}; \text{G30}; \text{G40}]); \]
Demo program
Typical effects at oblique incidence

- Frequency shift
- Difference in polarization
- Possible to redesign for specific angle of incidence
- Higher reflection for higher angles
- Brewster angle for TM polarization
 Outline

1. Multilayer dielectric structures at oblique incidence
2. Perfect lens in negative-index media
3. Antireflection coatings at oblique incidence
4. Omnidirectional dielectric mirrors
5. Reflection and refraction in birefringent media
6. Giant birefringent optics
7. Concluding remarks
Dielectric mirrors

Increasing angle of incidence implies

- Frequency shift
- Polarization dependence

Omnidirectionality requires overlap of frequency bands for TE and TM for all angles.
Necessary condition

The angles in all layers are related by Snell’s law

\[ n_a \sin \theta_a = n_H \sin \theta_H = n_L \sin \theta_L = n_b \sin \theta_b \]

and the phase thicknesses are

\[ \delta_H = 2\pi \frac{f}{f_0} L_H \cos \theta_H, \quad \delta_L = 2\pi \frac{f}{f_0} L_L \cos \theta_L \]

where \( L_i = n_i \ell_i / \lambda_0 \) and \( \cos \theta_i = \sqrt{1 - n_a^2 \sin^2 \theta_a / n_i^2} \). To prevent a wave from the outside to access the Brewster angle inside the mirror (which would imply transmission), we need

\[ n_a < \frac{n_H n_L}{\sqrt{n_H^2 + n_L^2}} \]
Bandgap

All procedures from normal incidence are applicable when considering transverse field components. We look for waves propagating with the effective wave number $K$, so that there should exist solutions

\[
\begin{pmatrix}
E_{i,T} \\
H_{i,T}
\end{pmatrix} = \begin{pmatrix}
\cos(\delta_L) & j\eta_{LT} \sin(\delta_L) \\
-j\eta_{LT}^{-1} \sin(\delta_L) & \cos(\delta_L)
\end{pmatrix} \begin{pmatrix}
\cos(\delta_H) & j\eta_{HT} \sin(\delta_H) \\
-j\eta_{HT}^{-1} \sin(\delta_H) & \cos(\delta_H)
\end{pmatrix} \begin{pmatrix}
E_{i+2,T} \\
H_{i+2,T}
\end{pmatrix} = e^{-jK\ell} \begin{pmatrix}
E_{i,T} \\
H_{i,T}
\end{pmatrix}
\]

This is an eigenvalue equation, which can be put in the form

\[
\cos(K\ell) = \frac{\cos(\delta_H + \delta_L) - \rho_T^2 \cos(\delta_H - \delta_L)}{1 - \rho_T^2}
\]

where $\rho_T = \frac{n_{HT} - n_{LT}}{n_{HT} + n_{LT}}$ depends on polarization.
| \cos(K\ell)| \leq 1 \text{ for propagating fields:}

The bandgap increases with contrast, and when increasing the angle:

- The entire bandgap shifts up in frequency for both polarizations (the layers become optically thinner).
- The TM bandgap decreases, and the TE bandgap increases.
Frequency response

\[ n_H = 3, \quad n_L = 1.38, \quad N = 30 \]

Calculated with multidiel.m.

(Fig. 8.8.2 in Orfanidis)
Bandwidths and angle

\[ n_H = 3, \quad n_L = 1.38 \]

\[ n_H = 2, \quad n_L = 1.38 \]

(Fig. 8.8.6 in Orfanidis)

Omnidirectional

Not omnidirectional
The TM polarization travels at the Brewster angle if

\[ n_a \sin \theta_a = n_a \frac{1}{\sqrt{2}} = \frac{n_H n_L}{\sqrt{n_H^2 + n_L^2}} \]

Hence TM is transmitted and TE reflected.
Outline

1. Multilayer dielectric structures at oblique incidence
2. Perfect lens in negative-index media
3. Antireflection coatings at oblique incidence
4. Omnidirectional dielectric mirrors
5. Reflection and refraction in birefringent media
6. Giant birefringent optics
7. Concluding remarks
Effective refractive index

\[ D = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \cdot E, \quad B = \mu_0 H, \quad k = N k_0 \hat{k} \]

\[ N = \frac{1}{\sqrt{\frac{\cos^2 \theta}{n_1^2} + \frac{\sin^2 \theta}{n_3^2}}} \]

\[ \eta_{TM} = \eta_0 \frac{N \cos \theta}{n_1^2} \]

\[ \eta_{TE} = \frac{\eta_0}{n_2 \cos \theta} \]
Snel’s law

Snel’s law is \[ N \sin \theta = N' \sin \theta' \], or, more explicitly,

\[
\frac{n_1 n_3 \sin \theta}{\sqrt{n_1^2 \sin^2 \theta + n_3^2 \cos^2 \theta}} = \frac{n_1' n_3' \sin \theta'}{\sqrt{n_1'^2 \sin^2 \theta' + n_3'^2 \cos^2 \theta'}} \quad \text{(TM)}
\]

\[
n_2 \sin \theta = n_2' \sin \theta' \quad \text{(TE)}
\]

Can be solved for \( \theta \) as function of \( \theta' \) or vice versa. Coded in

\[
\text{thb} = \text{snel}(n_a, n_b, \theta_a, \text{pol});
\]
The reflection coefficients are

$$\rho_{TM} = \frac{\eta'_{TM} - \eta_{TM}}{\eta'_{TM} + \eta_{TM}} = \frac{n_1 n_3 \sqrt{n_3'^2 - N^2 \sin^2 \theta} - n_1' n_3'^2 \sqrt{n_3^2 - N^2 \sin^2 \theta}}{n_1 n_3 \sqrt{n_3'^2 - N^2 \sin^2 \theta} + n_1' n_3'^2 \sqrt{n_3^2 - N^2 \sin^2 \theta}}$$

$$\rho_{TE} = \frac{\eta'_{TE} - \eta_{TE}}{\eta'_{TE} + \eta_{TE}} = \frac{n_2 \cos \theta - \sqrt{n_2'^2 - n_2^2 \sin^2 \theta}}{n_2 \cos \theta + \sqrt{n_2'^2 - n_2^2 \sin^2 \theta}}$$

Can be computed using routine fresnel.

- If $n_3 = n_3'$ we have $\rho_{TM} = \frac{n_1 - n_1'}{n_1 + n_1'}$ independent of $\theta$.
- If $\mathbf{n} = [n_1, n_1, n_3]$ and $\mathbf{n}' = [n_3, n_3, n_1]$ we have

$$\rho_{TE} = \rho_{TM} = \frac{n_1 \cos \theta - \sqrt{n_3^2 - n_1^2 \sin^2 \theta}}{n_1 \cos \theta + \sqrt{n_3^2 - n_1^2 \sin^2 \theta}}$$
Examples of Fresnel coefficients

(a) \( n = [1.63, 1.63, 1.5] \) \( n' = [1.63, 1.63, 1.63] \)

(b) \( n = [1.54, 1.54, 1.63] \) \( n' = [1.5, 1.5, 1.5] \)

(c) \( n = [1.8, 1.8, 1.5] \) \( n' = [1.5, 1.5, 1.5] \)

(d) \( n = [1.8, 1.8, 1.5] \) \( n' = [1.56, 1.56, 1.56] \)
Differences from the isotropic case

- The Brewster angle can be zero (a)
- The Brewster angle may not exist (c)
- The Brewster angle may be imaginary (both $\rho_{\text{TE}}$ and $\rho_{\text{TM}}$ increase monotonically with angle) (d)
- There may be total internal reflection for one polarization but not for the other (b)
- The reflection coefficient can be the same for TE and TM
Outline

1. Multilayer dielectric structures at oblique incidence
2. Perfect lens in negative-index media
3. Antireflection coatings at oblique incidence
4. Omnidirectional dielectric mirrors
5. Reflection and refraction in birefringent media
6. Giant birefringent optics
7. Concluding remarks
Dielectric mirror

(Fig. 8.8.1 in Orfanidis)

The layers are now birefringent with

\[ \mathbf{n}_H = [n_{H1}, n_{H2}, n_{H3}], \quad \mathbf{n}_L = [n_{L1}, n_{L2}, n_{L3}] \]

The code `multidiel.m` can be used to calculate the response.
First design

\[ N = 50, \quad n_H = [1.8, 1.8, 1.5], \quad n_L = [1.5, 1.5, 1.5], \quad n_a = n_b = 1 \]

Reflectance at 0° and 60°

25% thickness gradient

Relatively similar TE and TM, TM is a bit more narrowband.
$N = 30$, $n_H = [1.8, 1.8, 1.5]$, $n_L = [1.5, 1.5, 1.8]$, $n_a = n_b = 1.4$

Identical TE and TM, but depends on angle.

(Fig. 8.13.2 in Orfanidis)
Slight change of second design

\( N = 30, \ n_H = [1.8, 1.8, 1.5], \ n_L = [1.5, 1.5, 1.9], \ n_a = n_b = 1.4 \)

Perturbation at higher angles.

(Fig. 8.13.3 in Orfanidis)
**Fig. 7.** (A) Light transport tubes using (a) GBO broadband mirror, (b) commercial aluminum mirror, and (c) commercial silver mirror. The ratio of length to diameter of the tubes is 17, and white light is used to illuminate the open aperture. (B) A GBO film cavity that is illuminated from the front aperture with white light. Note the change of highly saturated color with observing angle.

GBO reflective polarizer

\[ N = 80, \ n_H = [1.86, 1.57, 1.57], \ n_H = [1.57, 1.57, 1.57], \ n_a = n_b = 1 \]

Mismatch only in one direction.

(Fig. 8.13.4 in Orfanidis)
Giant birefringent optics

- The “giant” refers to the relatively large birefringence
- Commercialized by 3M
- Careful matching of refractive indices may eliminate differences between polarizations
- Particularly, the Brewster angle can be controlled
- Materials can be manufactured from birefringent polymers
Outline

1. Multilayer dielectric structures at oblique incidence
2. Perfect lens in negative-index media
3. Antireflection coatings at oblique incidence
4. Omnidirectional dielectric mirrors
5. Reflection and refraction in birefringent media
6. Giant birefringent optics
7. Concluding remarks
Concluding remarks

This course has dealt with the following subjects in depth:

- Propagation of electromagnetic waves in stratified media
- Material modeling in time and frequency domain, isotropic as well as bianisotropic
- Fundamental wave properties: polarization, wave impedance, wave number
- Tangential fields and tangential wave numbers are continuous across interfaces, and have been used consistently
- Numerical methods and examples have appeared throughout
  - Lots of codes from Orfanidis and on course home page
- Several applications of multilayer structures
Only projects and orals remaining!