Outline

1. Introduction
2. Multiple dielectric slabs
3. Antireflection coatings
4. Dielectric mirrors
5. Propagation bandgaps
6. Narrow-band transmission filters (Fabry-Perot resonators)
7. Conclusions
5 Narrow-band transmission filters (Fabry-Perot resonators)
6 Conclusions

Daniel Sjöberg, Department of Electrical and Information Technology
Key questions

- How can we analyze multilayer structures?
- What can we build with them?
- How can we design the structures?
1 Introduction

2 Multiple dielectric slabs

3 Antireflection coatings

4 Dielectric mirrors

5 Propagation bandgaps

6 Narrow-band transmission filters (Fabry-Perot resonators)

7 Conclusions
Scattering from multilayer structure

\[
\begin{pmatrix}
E_{1+} \\
E_{1-}
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
1 & \eta_a \\
1 & -\eta_a
\end{pmatrix} \mathbf{P}_1 \cdots \mathbf{P}_M \begin{pmatrix}
\frac{1}{\eta_b} & 1 \\
1 & -\frac{1}{\eta_b}
\end{pmatrix} \begin{pmatrix}
E'_{M+1, +} \\
0
\end{pmatrix}
\]

\[
\mathbf{P}_i = \begin{pmatrix}
\cos(k_i l_i) & j\eta_i \sin(k_i l_i) \\
-j\eta_i^{-1} \sin(k_i l_i) & \cos(k_i l_i)
\end{pmatrix}
\]

(Fig. 6.1.1 in Orfanidis)
Scattering from multilayer structure

\[
\begin{align*}
\left( \frac{E_{1+}}{E_{1-}} \right) &= P'_1 \cdots P'_M \frac{1}{\tau_{M+1}} \begin{pmatrix} 1 & \rho_{M+1} \\ \rho_{M+1} & 1 \end{pmatrix} \begin{pmatrix} E'_{M+1,+} \\ 0 \end{pmatrix} \\
\end{align*}
\]

\[
P'_i = \frac{1}{\tau_i} \begin{pmatrix} e^{jk_i \ell_i} & \rho_i e^{-jk_i \ell_i} \\ \rho_i e^{jk_i \ell_i} & e^{-jk_i \ell_i} \end{pmatrix}
\]
Scattering parameters

The total transfer matrix relation is

\[
\begin{pmatrix}
E_{1+} \\
E_{1-}
\end{pmatrix} = \begin{pmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{pmatrix}
\begin{pmatrix}
E'_{M+1,+} \\
0
\end{pmatrix} = \begin{pmatrix}
T_{11}E'_{M+1,+} \\
T_{21}E'_{M+1,+}
\end{pmatrix}
\]

which implies

\[
T = \frac{E'_{M+1,+}}{E_{1+}} = \frac{1}{T_{11}}
\]

\[
\Gamma_1 = \frac{E_{1-}}{E_{1+}} = \frac{T_{21}E'_{M+1,+}}{E_{1+}} = \frac{T_{21}}{T_{11}}
\]

Thus, having computed the total transfer matrix, the reflection and transmission coefficients correspond to simple rearrangements.
Layer recursion for reflection

With propagation matrices

\[
\begin{pmatrix}
E_{i,+} \\
E_{i,-}
\end{pmatrix} = \frac{1}{\tau_i} \begin{pmatrix}
\rho_i e^{jk_i\ell_i} & \rho_i e^{-jk_i\ell_i} \\
\rho_i e^{jk_i\ell_i} & e^{-jk_i\ell_i}
\end{pmatrix}
\begin{pmatrix}
E_{i+1,+} \\
E_{i+1,-}
\end{pmatrix}
\]

\[
\begin{pmatrix}
E_i \\
H_i
\end{pmatrix} = \begin{pmatrix}
\cos(k_i\ell_i) & j\eta_i \sin(k_i\ell_i) \\
j\eta_i^{-1} \sin(k_i\ell_i) & \cos(k_i\ell_i)
\end{pmatrix}
\begin{pmatrix}
E_{i+1} \\
H_{i+1}
\end{pmatrix}
\]

the reflection coefficient at interface \( i \) can be found by recursion

\[
\frac{E_{i-}}{E_{i+}} = \Gamma_i = \frac{\rho_i + \Gamma_{i+1} e^{-2jk_i\ell_i}}{1 + \rho_i \Gamma_{i+1} e^{-2jk_i\ell_i}}, \quad \Gamma_{M+1} = \rho_{M+1}
\]

and the impedance at interface \( i \) in the same way

\[
\frac{E_i}{H_i} = Z_i = \eta_i \frac{Z_{i+1} + j\eta_i \tan(k_i\ell_i)}{\eta_i + jZ_{i+1} \tan(k_i\ell_i)}, \quad Z_{M+1} = \eta_b
\]

These are equivalent. Thus, the reflection properties can be found from a one-pass calculation, iterating from \( M + 1 \) to 1.
In general, to find also the transmission coefficient, the full cascading technique must be employed. However, in the important case of no losses energy conservation gives us the result

\[
\frac{1 - |\Gamma|^2}{2\eta_a} = \frac{|T|^2}{2\eta_b}
\]

Thus, for lossless structures the number \(1 - |\Gamma|^2\) represents the transmittance.
From now on, we assume the medium is non-magnetic, that is,

\[ k = \omega \sqrt{\varepsilon \mu} = \frac{\omega}{c_0} \sqrt{\varepsilon_r} = k_0 n \]

and

\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{n} \]

Thus, a slab is characterized solely by its physical length \( \ell \) and its refractive index \( n = \sqrt{\varepsilon_r} \).
% multidiel.m - reflection response of isotropic or birefringent multilayer structure

% na | n1 | n2 | ... | nM | nb
% left medium | L1 | L2 | ... | LM | right medium
% interface 1 2 3 M M+1

% Usage: [Gamma,Z] = multidiel(n,L,lambda,theta,pol)
% [Gamma,Z] = multidiel(n,L,lambda,theta) (equivalent to pol='te')
% [Gamma,Z] = multidiel(n,L,lambda) (equivalent to theta=0)

% n = isotropic 1x(M+2), uniaxial 2x(M+2), or biaxial 3x(M+2), matrix of refractive indices
% L = vector of optical lengths of layers, in units of lambda_0
% lambda = vector of free-space wavelengths at which to evaluate the reflection response
% theta = incidence angle from left medium (in degrees)
% pol = for 'tm' or 'te', parallel or perpendicular, p or s, polarizations

% Gamma = reflection response at interface-1 into left medium evaluated at lambda
% Z = transverse wave impedance at interface-1 in units of eta_a (left medium)

% notes: M is the number of layers (M >= 0)
% n = \[na, n1, n2, ..., nM, nb\] = 1x(M+2) row vector of isotropic indices
% \[ na1 n11 n12 ... n1M nb1 \] 3x(M+2) matrix of birefringent indices,
% n = \[ na2 n21 n22 ... n2M nb2 \] = if 2x(M+2), it is extended to 3x(M+2)
% \[ na3 n31 n32 ... n3M nb3 \] by repeating the top row

% optical lengths are in units of a reference free-space wavelength lambda_0:
% for i=1,2,...,M, L(i) = n(1,i) * l(i), for TM,
% L(i) = n(2,i) * l(i), for TE,
% TM and TE L(i) are the same in isotropic case. If M=0, use L=[]
% lambda is in units of lambda_0, that is, lambda/lambda_0 = f_0/f

% reflectance = |Gamma|^2, input impedance = Z = (1+Gamma)./(1-Gamma)
% delta(i) = 2*pi*[n(1,i) * l(i) * sqrt(1 - (Na*sin(theta))^2 ./ n(3,i).^2))]/lambda, for TM
% delta(i) = 2*pi*[n(2,i) * l(i) * sqrt(1 - (Na*sin(theta))^2 ./ n(2,i).^2))]/lambda, for TE
% if n(3,i)=n(3,i+1)=Na, then will get NaN's at theta=90 because of 0/0, (see also FRESNEL)
% it uses SQRTE, which is a modified version of SQRT approriate for evanescent waves
% see also MULTIDIEL1, MULTIDIEL2

% Sophocles J. Orfanidis - 1999-2008 - www.ece.rutgers.edu/~orfanidi/ewa
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An anti-reflection coating can be realized by one or many layers.
Some materials in use for optical anti-reflection coatings are found on page 190 in Orfanidis:

<table>
<thead>
<tr>
<th>material</th>
<th>$n$</th>
<th>material</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>cryolite ($\text{Na}_3\text{AlF}_6$)</td>
<td>1.35</td>
<td>magnesium fluoride ($\text{MgF}_2$)</td>
<td>1.38</td>
</tr>
<tr>
<td>Silicon dioxide ($\text{SiO}_2$)</td>
<td>1.46</td>
<td>polystyrene</td>
<td>1.60</td>
</tr>
<tr>
<td>cerium fluoride ($\text{CeF}_3$)</td>
<td>1.63</td>
<td>lead fluoride ($\text{PbF}_2$)</td>
<td>1.73</td>
</tr>
<tr>
<td>Silicon monoxide ($\text{SiO}$)</td>
<td>1.95</td>
<td>zirconium oxide ($\text{ZrO}_2$)</td>
<td>2.20</td>
</tr>
<tr>
<td>zinc sulfide ($\text{ZnS}$)</td>
<td>2.32</td>
<td>titanium dioxide ($\text{TiO}_2$)</td>
<td>2.40</td>
</tr>
<tr>
<td>bismuth oxide ($\text{Bi}_2\text{O}_3$)</td>
<td>2.45</td>
<td>silicon ($\text{Si}$)</td>
<td>3.50</td>
</tr>
<tr>
<td>germanium ($\text{Ge}$)</td>
<td>4.20</td>
<td>tellurium ($\text{Te}$)</td>
<td>4.60</td>
</tr>
</tbody>
</table>

Not all refractive indices are available, meaning the tolerances of the design cannot be too tight.
How can the parameters of the coating be chosen?

Each slab is characterized by two parameters, thickness $\ell_i$ and refractive index $n_i$.

We may fix the parameter vector $\mathbf{a} = \{\{\ell_i\}_{i=1}^{N}, \{n_i\}_{i=1}^{N}\}$, and compute the response $\Gamma(\mathbf{a})$. Setting up an optimization loop, we typically minimize a penalty function like

$$
\int_{f_1}^{f_2} \left| \Gamma(\mathbf{a}, f) - \Gamma_0(f) \right|^n w(f) \, df
$$

where $\Gamma_0(f)$ is the target frequency response, and the weight function $w(f)$ may emphasize different parts of the spectrum.

- This is straightforward, but as the number of layers increases, it becomes important to have a good start design, and understand what can and cannot be done.
Demo

Oblique reflection from a multilayer structure
Explicit design of a two-layer structure

We wish to match refractive index \( n_b = 1.5 \) to \( n_a = 1 \) using two slabs. The reflection coefficients are

\[
\Gamma_1 = \frac{\rho_1 + \Gamma_2 e^{-2j k_1 \ell_1}}{1 + \rho_1 \Gamma_2 e^{-2j k_1 \ell_1}}, \quad \Gamma_2 = \frac{\rho_2 + \rho_3 e^{-2j k_2 \ell_2}}{1 + \rho_2 \rho_3 e^{-2j k_2 \ell_2}}
\]

Zero reflection requires \( \Gamma_1 = 0 \), or

\[
e^{2j k_1 \ell_1} = -\Gamma_2 / \rho_1.
\]

\[
|\Gamma_2|^2 = \frac{\rho_2^2 + \rho_3^2 + 2 \rho_2 \rho_3 \cos(2k_2 \ell_2)}{1 + \rho_2^2 \rho_3^2 + 2 \rho_2 \rho_3 \cos(2k_2 \ell_2)} = \rho_1^2
\]

which can be transformed to

\[
\cos^2(k_2 \ell_2) = \frac{\rho_1^2 (1 - \rho_2 \rho_3)^2 - (\rho_2 - \rho_3)^2}{4 \rho_2 \rho_3 (1 - \rho_1^2)}
\]

\[
\sin^2(k_2 \ell_2) = \frac{(\rho_2 + \rho_3)^2 - \rho_1^2 (1 + \rho_2 \rho_3)^2}{4 \rho_2 \rho_3 (1 - \rho_1^2)}
\]
Determining materials

Assuming $n_1 < n_2$ and $n_2 > n_b$, we have $\rho_2 < 0$ and $\rho_3 > 0$. The following conditions must be satisfied for the equations to have a solution:

$$\left| \frac{\rho_3 + \rho_2}{1 + \rho_2 \rho_3} \right|^2 < \rho_1^2 < \left| \frac{\rho_3 - \rho_2}{1 - \rho_2 \rho_3} \right|^2$$

which requires

$$n_1 \in [\sqrt{n_b}, n_b] = [1.22, 1.5] \quad \text{and} \quad n_2 \notin [\sqrt{n_b}, n_1 \sqrt{n_b}] = [1.22, 1.69]$$

using $n_1 = 1.38$

Materials:
- Magnesium flouride: $n_1 = 1.38$
- Bismuth oxide: $n_2 = 2.45$

Lengths (target wavelength $\lambda_0 = 55 \text{ nm}$)

$$k_1 \ell_1 = 2.0696 \quad n_1 \ell_1 = 0.3294 \lambda_0 \quad \ell_1 = 131 \text{ nm}$$

$$k_2 \ell_2 = 0.2848 \quad n_2 \ell_2 = 0.0453 \lambda_0 \quad \ell_2 = 10.2 \text{ nm}$$
Results

Some drawbacks:

▶ Smaller bandwidth
▶ Complicated procedure
▶ Small dimensions

(Fig. 6.2.1 in Orfanidis)
For a quarter wavelength slab, \( n_i l_i = \lambda_0 / 4 \), we have

\[
Z_i = \frac{Z_{i+1} + j\eta_i \tan(k_i l_i)}{\eta_i + jZ_{i+1} \tan(k_i l_i)} = \frac{\eta_i^2}{Z_{i+1}}
\]

and for a half wavelength slab, \( n_i l_i = \lambda_0 / 2 \), we have

\[
Z_i = \frac{Z_{i+1} + j\eta_i \tan(k_i l_i)}{\eta_i + jZ_{i+1} \tan(k_i l_i)} = Z_{i+1}
\]

Thus, quarter wavelength slabs inverts the impedance, whereas half wavelength slabs preserves impedance.

- These rules can be used to make simple designs at target wavelength.
An impedance approach to antireflection

The goal is to transform the impedance $\eta_b$ to $\eta_a$. Two possibilities:

First possibility (quarter-quarter):

$$\eta_a = \frac{\eta_1^2}{Z_2} = \frac{\eta_1^2}{\eta_2^2/\eta_b} = \eta_b \frac{\eta_1^2}{\eta_2^2} \Rightarrow \frac{n_a}{n_b} = \frac{n_1^2}{n_2^2}$$

Second possibility (quarter-half-quarter):

$$\eta_a = \frac{\eta_1^2}{Z_2} = \frac{\eta_1^2}{\eta_3^2/\eta_b} = \eta_b \frac{\eta_1^2}{\eta_3^2} \Rightarrow \frac{n_a}{n_b} = \frac{n_1^2}{n_3^2}$$
The design was made for the center wavelength $\lambda = 550\,\text{nm}$, the result for any wavelength is computed with `multidiel.m`:

Note there was no requirement on the half-wavelength slab! The function of this slab is to increase the bandwidth.
Mismatched quarter-quarter, matched quarter, matched quarter-half-quarter, mismatched quarter-half-quarter. The effect of the half-wavelength slab is to add the twist for the red curve, since $\Gamma_\ell = \Gamma_0 e^{-2jk\ell}$. 
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Dielectric mirrors

Instead of anti-reflection, we can design a multilayered structure aimed at significant reflection.

Useful to avoid losses in metal structures, which may absorb a few percent of power at optical frequencies. Typical design issues: thickness and bandwidth.
Design at center wavelength

Alternating quarter wavelength slabs of high \((n_H)\) and low \((n_L)\) refractive index (short hand notation \(AH(LH)^4G\)):

\[
Z_2 = \frac{\eta_L^2}{Z_3} = \frac{\eta_L^2}{\eta_H^2/Z_4} = \left(\frac{n_H}{n_L}\right)^2 Z_4 = \left(\frac{n_H}{n_L}\right)^4 Z_6 = \cdots = \left(\frac{n_H}{n_L}\right)^8 \eta_b
\]

(Fig. 6.3.1 in Orfanidis)
Reflection

The impedance at interface 1 after $N$ bilayers is

$$Z_1 = \frac{\eta_H^2}{Z_2} = \frac{\eta_0^2/\eta_H^2}{\left(\frac{n_H}{n_L}\right)^{2N} \eta_b}$$

which implies the reflection coefficient

$$\Gamma_1 = \frac{Z_1 - \eta_a}{Z_1 + \eta_a} = \ldots = \frac{1 - \left(\frac{n_H}{n_L}\right)^{2N} \frac{n_H^2}{n_a n_b}}{1 + \left(\frac{n_H}{n_L}\right)^{2N} \frac{n_H^2}{n_a n_b}} \to -1, \quad N \to \infty$$

Thus, for many layers, we get high reflection. Without the final $n_H$ layer, the reflection would have the limit $\Gamma \to +1$.

| $n_H$ | $n_L$ | $N$ | $|\Gamma_1|^2$ |
|-------|-------|-----|----------------|
| 2.32  | 1.38  | 1   | 0.7685         |
|       |       | 2   | 0.9111         |
|       |       | 4   | 0.9884         |
|       |       | 8   | 0.9998         |
|       |       | 16  | 1.0000         |

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Propagation on other wavelengths

\[
\begin{align*}
(E_{1+}) & = \frac{1}{\tau_1} \begin{pmatrix} e^{j\delta_H} & \rho_1 e^{-j\delta_H} \\ \rho_1 e^{j\delta_H} & e^{-j\delta_H} \end{pmatrix} \mathbf{F}^N \frac{1}{\tau_2} \begin{pmatrix} 1 & \rho_2 \\ \rho_2 & 1 \end{pmatrix} \begin{pmatrix} E'_{M+1,+} \\ 0 \end{pmatrix} \\
\mathbf{F} & = \frac{1}{1 + \rho} \begin{pmatrix} e^{j\delta_L} & \rho e^{-j\delta_L} \\ \rho e^{j\delta_L} & e^{-j\delta_L} \end{pmatrix} \frac{1}{1 - \rho} \begin{pmatrix} e^{j\delta_H} & -\rho e^{-j\delta_H} \\ -\rho e^{j\delta_H} & e^{-j\delta_H} \end{pmatrix} \\
\delta_H & = k_H \ell_H, \quad \delta_L = k_L \ell_L, \quad \rho = \frac{n_H - n_L}{n_H + n_L}
\end{align*}
\]

(Fig. 6.3.1 in Orfanidis)
Results for different number of layers

Computed using \texttt{multidiel.m} with the parameters $n_a = 1$, $n_b = 1.52$, $n_H = 2.32$, $n_L = 1.38$. Bandwidth is relatively insensitive to the number of layers, ca $\frac{600 - 430}{500} = 0.34$.

For the infinite structure, the bandwidth is (will be shown later)

$$\frac{\Delta \lambda}{\lambda_0} = \frac{\pi}{2} \left( \frac{1}{\arccos(\rho)} - \frac{1}{\arccos(-\rho)} \right) = 0.336, \quad \rho = \frac{n_H - n_L}{n_H + n_L} = 0.254$$
Different optical thicknesses

Computed using `multidiel.m` with the parameters $n_a = 1$, $n_b = 1.48$, $n_H = 4.6$, $n_L = 1.6$. The optical thicknesses are $n_H \ell_H = 0.2944 \lambda_0$ and $n_L \ell_L = 0.2112 \lambda_0$, and $N = 4$.

- Note that $n_H \ell_H + n_L \ell_L = 0.5 \lambda_0$, this is the important period.
- Higher bandwidth ($\Delta \lambda / \lambda_0 = 0.73$) due to higher contrast ($\rho = 0.48$).
Multiband reflectors

Short-hand notation for the dielectric mirror: $A H (L H)^8 G$, where $A$ denotes the air medium, $H$ the high-index medium, $L$ the low-index medium, and $G$ is the glass. Multiband reflectors can be made by inserting extra multiples of one or both of $H$ and $L$ slabs:

Note fundamental period of $\Delta f / f_0 = 2$. 

(Fig. 6.3.4 in Orfanidis)
Short- and longpass reflectors

The frequency dependence can be manipulated by the outmost layers. Original design is $AH(LH)^8G$ centered at 500 nm.

**Shortpass:** $A(0.5L)H(LH)^8(0.5L)G$

**Longpass:** $A(0.5H)L(HL)^8(0.5H)G$

Note how the roles of H and L are dual in the two designs. Dotted curve is original design centered at the corresponding $\lambda_0$. (Fig. 6.3.5 in Orfanidis)
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Consider the limit case \( N \to \infty \). We look for waves propagating with the effective wave number \( K \), so that there should exist solutions (where \( \delta_L = k_L \ell_L, \delta_H = k_H \ell_H, \) and \( \ell = \ell_L + \ell_H \))

\[
\begin{pmatrix}
E_i \\
H_i
\end{pmatrix} = \begin{pmatrix}
\cos(\delta_L) & j\eta_L \sin(\delta_L) \\
(j\eta_L^{-1} \sin(\delta_L) & \cos(\delta_L)
\end{pmatrix} 
\cdot \begin{pmatrix}
\cos(\delta_H) & j\eta_H \sin(\delta_H) \\
(j\eta_H^{-1} \sin(\delta_H) & \cos(\delta_H)
\end{pmatrix} 
\begin{pmatrix}
E_{i+2} \\
H_{i+2}
\end{pmatrix} = e^{-jK\ell} \begin{pmatrix}
E_i \\
H_i
\end{pmatrix}
\]

This is an eigenvalue equation, which can be put in the form

\[
\cos(K\ell) = \frac{\cos(\delta_H + \delta_L) - \rho^2 \cos(\delta_H - \delta_L)}{1 - \rho^2}
\]

where \( \rho = \frac{n_H - n_L}{n_H + n_L} \). \( K \) is called the Bloch wavenumber.
Bandgaps

With $\delta_H = \delta_L = \delta$, the equation is

$$\cos(K\ell) = \frac{\cos(2\delta) - \rho^2}{1 - \rho^2}$$

Propagating waves correspond to real $K$, which implies $|\cos(K\ell)| \leq 1$.

Inside the band gap, the waves are exponentially attenuated.
The band edges are given by the condition

\[-1 = \frac{\cos(2\delta) - \rho^2}{1 - \rho^2} \Rightarrow \cos^2 \delta = \rho^2\]

Since \(\delta = \frac{\pi}{2} \lambda_0/\lambda\), where \(\lambda_0\) is the center wavelength, the band edges are

\[\lambda_1 = \frac{\lambda_0 \pi/2}{\arccos(-\rho)} \quad \lambda_2 = \frac{\lambda_0 \pi/2}{\arccos(\rho)}\]

and the bandwidth is

\[\frac{\Delta \lambda}{\lambda_0} = \frac{\pi}{2} \left( \frac{1}{\arccos(\rho)} - \frac{1}{\arccos(-\rho)} \right)\]

\[\frac{\Delta f}{f_0} = \frac{4}{\pi} \arcsin(\rho)\]

The bandwidth is zero for \(\rho = 0\), and \(\Delta f/f_0 = 2\) for \(\rho = 1\) (\(\lambda_2 \to \infty\) and \(f_2 \to 0\) when \(\rho \to 1\)), where

\[\rho = \frac{n_H - n_L}{n_H + n_L}\]
Bandgaps generalize to 3D structures

Manufactured and measured at FOI in Linköping.
Propagation bandgaps may appear in any periodic structure:
- Crystals
- Gratings
- Periodically loaded transmission lines

Often analyzed by solving for the Bloch waves, propagating with factor $e^{-jK \cdot r}$, which leads to eigenvalue problems.

Looking for solutions $[E(r), H(r)]e^{-jK \cdot r}$ where $E(r)$ and $H(r)$ are periodic functions, turns Maxwell’s equations into

$$
\begin{pmatrix}
0 & -(\nabla - jK) \times I \\
(\nabla - jK) \times I & 0
\end{pmatrix}
\begin{pmatrix}
E(r) \\
H(r)
\end{pmatrix}
= -j\omega
\begin{pmatrix}
\epsilon(r) & \xi(r) \\
\zeta(r) & \mu(r)
\end{pmatrix}
\begin{pmatrix}
E(r) \\
H(r)
\end{pmatrix}
$$

with periodic boundary conditions. For some bands of $\omega$, there are no solutions with real-valued $K$-vectors.
Multidimensional bandgap structures

See http://ab-initio.mit.edu for free text book, lecture notes and computational software MPB (eigensolver) and Meep (FDTD).
Photonic Crystals: Periodic Surprises in Electromagnetism

Steven G. Johnson

a one-week seminar (five 1.5-hour lectures)
MIT MRS Chapter, 2003 IAP tutorial series, organized by Ion Bita
(with subsequent supplements)

See also: Photonic Crystals: Molding the Flow of Light (second edition)
— our new (2008) textbook, available online at no cost

See also: MIT Fall Semester 2005: 18.325 — Mathematical Methods in Nanophotonics

See also: MIT Spring Semester 2008: 18.369 — Mathematical Methods in Nanophotonics
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Having designed a dielectric mirror, we can use it to construct a Fabry-Perot resonator, which is a narrow-band transmission filter.

Since a half wavelength slab preserves impedance, we can eliminate such slabs at the design frequency. By inserting a low-index material between two bilayer stacks, we obtain

\[(HL)^N L (HL)^N = (HL)^{N-1} HLLHL (HL)^{N-1}\]
\[\rightarrow (HL)^{N-1} HHL (HL)^{N-1}\]
\[\rightarrow (HL)^{N-1} L (HL)^{N-1} \rightarrow \cdots \rightarrow L\]

Adding another outer layer \(L\), the final structure \((HL)^N L (HL)^N L \rightarrow 2L\) allows perfect transmission at the design wavelength.
Example of a Fabry-Perot resonator design

Computed using `multidiel.m` with parameters $n_a = n_b = 1.52$, $n_L = 1.4$, and $n_H = 2.1$. (Fig. 6.5.1 in Orfanidis)
Two FPRs

\[ G | (HL)^{N_1} L (HL)^{N_1} | (HL)^{N_2} L (HL)^{N_2} | G \]

The resonance bandwidth is controlled by the number of layers.

(Fig. 6.5.2 in Orfanidis)
Three FPRs

\[ \frac{G}{(HL)^{N_1} L (HL)^{N_1}} \left( \frac{(HL)^{N_2} L (HL)^{N_2}}{(HL)^{N_3} L (HL)^{N_3}} \right) L G \]

Note an extra \( L \) layer is added. The ripple is decreased by slight increase of middle FPR.

(Fig. 6.5.3 in Orfanidis)
Four FPRs

\[ G | (HL)^{N_1} L (HL)^{N_1} | (HL)^{N_2} L (HL)^{N_2} | (HL)^{N_3} L (HL)^{N_3} | (HL)^{N_4} L (HL)^{N_4} | G \]

No extra \( L \) layer needed. The ripple is decreased by slight increase of the middle FPRs.
Control of the design

The resonance bandwidth is controlled by the number of layers.

The isolation of the resonance is controlled by the contrast.
Conclusions

- Multilayer structures are easily analyzed by cascading techniques.
- Typical designs are based on quarter wavelength and half wavelength slabs.
- A stack of high/low index slabs can form a dielectric mirror.
- Insertion of extra layers can form a Fabry-Perot resonator.
- High bandwidth and high attenuation requires many layers.